Detection and Classification Method for a Temporary Change in Walking

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***Abstract*—The global elderly population has grown in recent years and one of the difficulties the elderly face is an increased vulnerability to falls. One of the ways to alleviate this problem is to identify actions that cause falls and to prevent falls using the detection result. Temporary change in walking (i.e., stumble and a stagger) is a typical action. However, existing studies only classify walking or other activities and recognize walking speed, and these studies do not focus on a temporary change in walking. In this paper, we propose a detection method for a change in walking using a change point detection (i.e., anomaly detection for the time series data) and a classification method for the multiple types of change. During the evaluation, four types of anomalous walking videos (i.e. anomaly represents a temporary change.) are used (the total number of videos is 240). As a result, our method can detect anomalous walking in 91.7% of cases and classify four types of detected anomalous walking into three clusters in 89.1% of cases on the basis of each characteristic.**

***Keywords*–Walking recognition; Classification; Anomaly detection; Human activity recognition.**

I. INTRODUCTION

The global elderly population has grown in recent years and one of the difficulties the elderly face is an increased vulnerability to falls [1] [2]. Elder people are more likely to be seriously injured by falls. In addition, this issue also leads to an increase in the medical costs, specifically, the cost of falls was approximately $50 billion in the USA in 2015 [1]. Human activity recognition is a research field that is focused on alleviating this problem [3]. Two types of activity recognition can be used to address this problem. The first type is fall detection, which identifies falls soon after they occur. There are already several studies on the topic, which show that this approach prevents injuries due to falls from becoming severe [4] [5]. The second approach is to detect actions that cause falls to prevent falls. Such actions include a stumble and a stagger. These actions are caused by a temporary change in walking. Thus, the recognition of the change is needed. In addition, the classification of change is needed because the change includes actions that are not related to the falls such as a standstill. However, existing studies on walking recognition can only distinguish walking and other activities [6], recognize walking speed [7], and do not focus on a temporary change in walking. Most of the existing studies recognize activity from common features between multiple persons if there is a clear difference between target activities (e.g., walking and sitting). Various methods can be used to identify these activities. However, it is difficult to recognize a change in walking because the difference in each person is larger than the difference of each action. A stumble detection system for powered artificial legs has been proposed in [8]; however, it is difficult to apply it to cases without artificial legs.

In this paper, we propose a detection method for a change in walking using change point detection and anomaly detection for time series data. Furthermore, we propose the classification of multiple types of change using a method that uses clustering of the results of the change point and anomaly detection.

The rest of the paper is organized as follows. Section II presents related works. Section III presents our proposed recognition method. Section IV evaluates our proposed method, and Section V concludes the paper.

II. RELATED WORK

A. Walking Recognition

Walking recognition is the field of human activity recognition. Human activity recognition is defined as the ability to recognize human activities using sensor data [3] [9] [10]. Each recognition method has two steps that are data collection by sensors and the estimation of human activity on the basis of data.

Currently, there are several studies on walking recognition, which focus on the differences in sensors or estimation methods. Khan et al. have detected walking and other five activities (i.e., sitting, standing, washing hands, driving, running) from accelerometer data using change point detection [6]. The abovementioned study uses a genetic algorithm to optimize the parameters of change point detection to improve the accuracy of detection. Thus, it has the high accuracy of 99.4%-99.8% [6]. Trung et al. have classified five types of walking (i.e., walking on flat ground, up/down stairs, and up/down a slope) with a 90.4% accuracy on the basis of accelerometer data using a support vector machine [11]. Davis and Taylor have recognized walking speeds (i.e., normal speed, half of normal speed, and double of normal speed) and classified walking and other eleven activities (e.g., running and skipping) from video-based four joint coordinates data [12]. Haescher et al. have classified walking speed into four classes (i.e., 1, 2.5, 4, and 5 km/h) from capacitive sensor data [7]. The application of this walking recognition includes automated surveillance, monitoring systems to identify people that may be injured or require assistance, and the estimation of the amount of activity [7] [12].

However, contrary to this study, these studies did not focus on a temporary change in walking. The abovementioned studies and our proposed study can be used together to detect walking in several activities using existing methods and to recognize a change in walking using our method.

B. Image-based Human Posture Estimation

In our proposed method, human joint coordinates are used for the input of change point detection. The coordinates can be extracted using the existing method. In this paper, OpenPose [13], which has been proposed by Cao et al. is used to extract the coordinates. OpenPose outputs two-dimensional 25-joint

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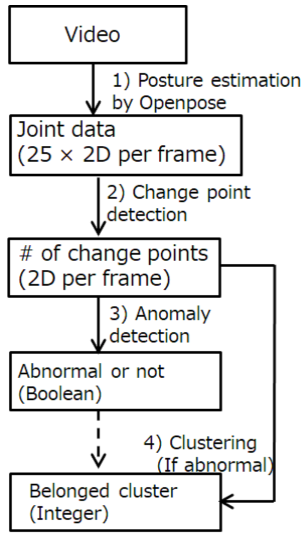

Figure 1. Overview of the proposed method using dataflow.

data per image by the deep learning-based posture estimation. Our proposed method is not constrained by OpenPose. If the coordinates can be acquired, the output of another method can be applied. For example, there are some image-based posture estimation methods such as ArtTrack [14] and DeepCut [15]. Our method can be applied for non-image-based methods such as motion capture or a depth sensor.

III. Recognition Method of A Temporary Walking Change

A. Overview of the Proposed Method

The input of the proposed method is the coordinate data of human joints during walking. The method has three steps. It distinguishes anomaly walk which exhibits behavior that temporarily differs from daily walking and classifies anomaly. Annomaly walking detection can be used to analyze the causes of falls because actions that can cause falls (e.g. stumble and stagger) include the anomalous walking. The input is acquired by videos using OpenPose, as mentioned in Section II. As described in Section IV-A, in this paper, we use a video ($1,920 \times 1,020$ resolution and 30-fps frame rate) of a walking person from the front. A picture of a person is acquired from the front, while the person is walking from the back to the front.

Fig. 1 shows the overview of the proposed method using the dataflow from the video data to classify anomalous walking. The preprocessing of the proposed method is the posture estimation by OpenPose. OpenPose detects a human and estimates posture from the video. OpenPose outputs coordinates of joints, which are two-dimensional 25-point data from one frame of the video.

These joints data are the inputs of the proposed method, and the three steps of the method are shown below.

1) Change point detection determines whether each joint point or relationship between two joints is a change point or not (the meaning of the relationship will be explained in the next section.). The output is the number of change points in each frame because the detection is processed for each joint point and relationship of each frame.
2) Anomaly detection detects anomalus walking using the number of change points. The result is output per the overall walking data (data from one video).
3) Clustering classifies the anomalous walk detected by the second step. It is processed only when input walking data are detected as anomaly.

The processing of each step is performed by unsupervised machine learning. It means that the data labeled by a human are not needed. However, the first and second steps require some preprocessing to determine the parameter of each machine learning algorithm. Preprocessing is the preparation of the standard of the parameter determination. The standard is the result of the change point detection using daily walking data. One or two minutes of walking data are required for each subject. In the evaluation, we use 20 videos of the daily walking data for each subject, and the average length of videos is approximately three seconds. Thus, the total time of the walking data is approximately 1 min. To distinguish daily walking (i.e., input data given by a human and the output data detected by the proposed method), the terms “daily walk” and “normal walk” are used. The details of each step are explained in the next sections.

B. Change Point Detection for the Human Joint Data

Change point detection is processed in each frame data using the target frame and the previous frames consecutive to the target. The data format of the output data of OpenPose is 2-dimensional 25 points data per frame. There are two methods to deal with the data. The data are treated as one 50-dimensional data, or the data are treated as 2-dimensional 25-point data. If the data are processed as 50-dimensional data, the relationships between joints can be considered. However, there is a problem that when some joints considerably change, they affect the entire result. However, if the data are processed as 25-point data, the problem does not occur, however, the relationships cannot be considered. To solve the problem, we propose to use the difference data of all pairs of joints in addition to the 25-point data so that we can deal with the relationships using the sets of two-dimensional data. The total number of data is 325 because the number of pairs is 300. The result of change point detection from these data is output as the number of change points. In this method, we divide the result of joints and differences data to distinguish a change in the movement of joints and the relationships of joints. Thus, the output is one two-dimensional data per a frame.

Fig. 2 shows the change point detection flow as the dataflow. The input data are the joints data of 25 points and the difference data consist of all pairs. We adopt the Multivariate Exponentially Weighted Moving Average (MEWMA) algorithm as the change point detection algorithm because it uses only the target walking data and is not affected by individual differences [16]. The MEWMA algorithm uses the data from the target frame and from previous frames consecutive to the target. If the number of frames is n (i.e., nth frame is the target frame), the MEWMA vector is defined by (1).

$$Z_i = \lambda X_i + (1 - \lambda) Z_{i-1} | i = 1, 2, 3, \ldots n, Z_0 = 0 \quad (1)$$

X_i is the input vector, which is the coordinate of the joint or the difference of each frame. The change point can be detected

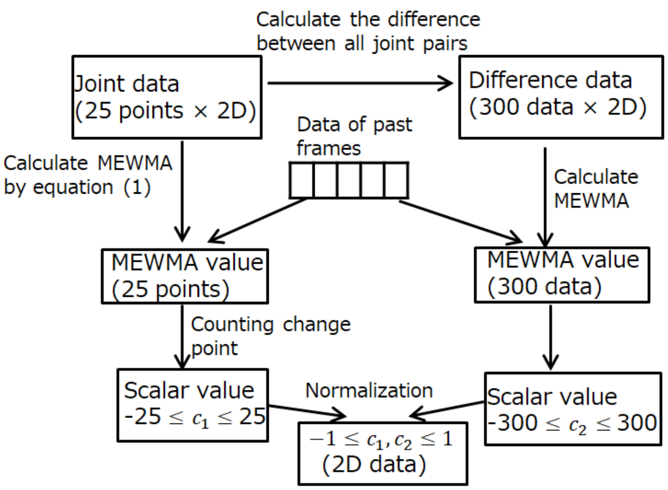

Figure 2. Dataflow of the change point detection.

in equation (2) using the MEWMA vector.

$$T^1 < h_1, T^2 > h_2 | T^2 = Z_n^T \Sigma_n^{-1} Z_n \tag{2}$$

Z_n is the MEWMA vector and Z_n^T its transpose. Σ_n^{-1} is the variance covariance matrix of Z_n. h_1 and h_2 are thresholds ($0 < h_1 < h_2$). h_1 is the case in which a change in the movement during walking becomes small and h_2 is the case when a change in the movement during walking becomes large. The counter of change point is decremented when $T^2 < h_1$, and the counter is incremented when $T^2 > h_2$ to distinguish the cases in which the movement becomes small or large. Using the MEWMA algorithm for all joints and differences, the range of the counter value is $-25 < counter < 25$ (joints) and $-300 < counter < 300$ (differences). Finally, the counting results are normalized by dividing by 25 or 300. The output of the change point detection is the two-dimensional data, and the form is suitable for the anomaly detection mentioned in the next section.

C. Anomaly Detection for the Number of Change Points of Walks

The input data are two-dimensional data per frame obtained from the change point detection. The anomaly is defined as a temporary change in the movement in the daily walk. Thus, the data of daily walking are needed for each person. The data are prepared as a set of the result of the change point detection for the daily walk for a few minutes, and the set is called "normal data".

Anomaly detection is performed in each frame by comparing the normal data and the target frame. Thus, anomaly detection is repeated in each frame by adding the target frame data and removing it after detection. We use a Local Outlier Factor (LOF) as anomaly detection because it can be calculated in each data, and the feature is suitable for repeating [17]. LOF is calculated by comparing local densities. The local density is calculated by reachability distance defined by (3).

$$reachability{-}distance_k(p,q) = max(k{-}distance(q), d(p,q)) \tag{3}$$

where p and q are the points of two-dimensional data, d(p,q) is the Euclidean distance between p and q, k-distance(q) denotes the Euclidean distance between q and the kth nearest neighbor of q. The local density, which is termed Local Reachability Density (LRD) is defined as (4).

$$LRD(p) = \left(\frac{\Sigma_{q \in kNN(p)} reachability - distance_k(p,q)}{k}\right)^{-1} \tag{4}$$

where kNN(p) denotes the set of k nearest neighbor of p. LOF is defined using the LRD by (5).

$$LOF(p) = \frac{\Sigma_{q \in kNN(p)} \frac{LRD(q)}{LRD(p)}}{k}. \tag{5}$$

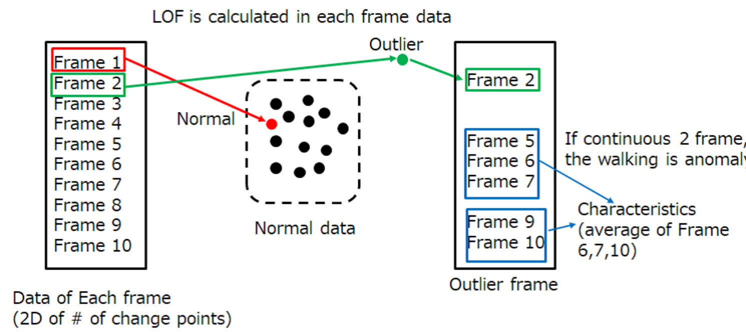

Figure 3. Example of the anomaly detection method.

The value of LOF is large when the target is an outlier. LOF shows whether the target frame is an anomaly or not; however, the result of the frame is not used for the direct detection of anomalous walking, because an anomaly frame may appear in the misestimation estimations of OpenPose. Therefore, in this method, the condition of the anomaly walk is the walk that includes two or more continuous anomaly frames. To classify anomaly walks, the characteristics of the walk are defined as the average number of change points of the continuous two or more anomaly frames.

Fig. 3 shows an example of the anomaly detection. In the example, there are ten-frame data. LOF is calculated for each frame and Fig. 3 shows the case of frames 1 and 2. In the field of calculation of LOF, each frame data is deleted after calculation. The frames identified as outliers are collected if there are 2 or more continuous frames and walking is identified as anomalous walking. The characteristics of anomalous walking are the average values of continuous frames excluding the first frame of the continuous frame.

D. Anomaly Walk Classification

The two-dimensional characteristic data of the anomaly walk are acquired from one video. When there are multiple anomalous walking videos, clustering algorithms can be performed for the characteristics and the walk can be classified. In this paper, we use the K-means algorithm, which is a typical clustering algorithm. In the K-means algorithm, the parameter given by a human is only the number of cluster k. In K-means clustering, first, each point, which means the characteristics of walking, is randomly classified into k clusters. Next, the below two steps are repeated until convergence.

1) The center of gravity of each cluster is calculated.

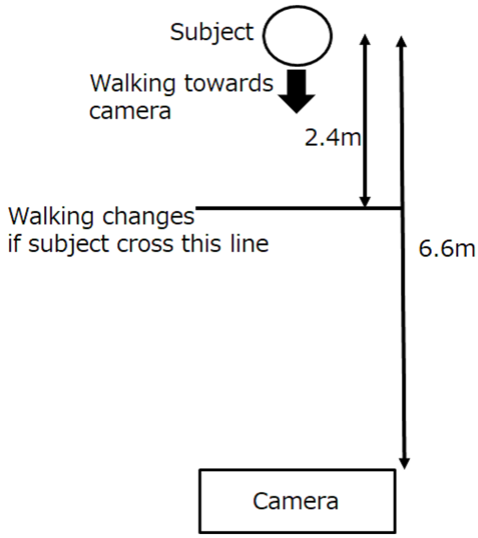

Figure 4. Walking during the evaluation.

2) Each point is reclassified into the cluster which has the nearest center of gravity from the point.

Using processing, the detection of anomaly walks and the classification of the walks is completed from the walking videos. In this method, the longer the video, the higher the probability of the video to include multiple anomalous walking instances, because anomaly detection handles the video as one data unit. The classification method assumes that there is only one anomaly walk per video. Thus, a long video should be divided every several seconds. However, Videos that are too short are also not suitable for the method and the minimum length of the video is two steps because walking has periodicity every two steps.

IV. Evaluations

A. *Evaluation Method and Environment*

We evaluate the accuracy of the detection and classification of anomaly walks in the proposed method. Thus, it is necessary to prepare anomalous walking data for which the correct answer is known. Therefore, we prepare the data of daily walks and change of walking from daily walk to anomaly walk. The subjects are three adults and Fig. 4 shows the details of walking. The walks start 6.6 m away from the camera and change to anomaly walks after 2.4 m of walking. There are the following four anomaly walks.

- Back: Go back one step and start walking again.
- Side: Walk 40 cm from side to side.
- Stop: Stop and start walking again.
- Wide: Walk one large step (1 m).

The resolution of the videos is $1,920 \times 1,020$ and the framerate is 30 fps. Each subject performs 20 times each type of walk (daily and four anomaly walks). Thus, the number of data of each type is 60. Table I shows the average number of frames of each type of walk.

TABLE I. Average number of frames in each walking type.

Walking type	Daily walk	Anomaly walk
Daily	91	0
Back	49	83
Side	39	67
Stop	50	58
Wide	39	25

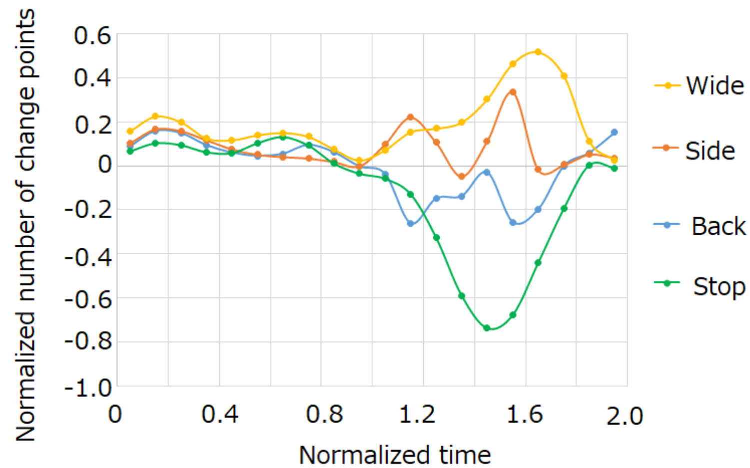

Figure 5. Transitions of the number of change points of the joint data.

B. *Transition of the Number of Change Points in Change Point Detection*

In this section, the transition of the number of change points is shown. It is an interim result of the proposed method; however, it shows an overall trend of each type of anomalous walk. There are five parameters of the MEWMA algorithm. These parameters are detected by a greedy algorithm with a high F value. The order of the parameters of the greedy algorithm is the number of frames, h_1 of joints, h_2 of joints, h_1 of the difference, and h_2 of the difference. The number of frames is 15 (0.5 seconds). The thresholds of the change point are 1/800 (h_1) and 2 times (h_2) the average MEWMA for

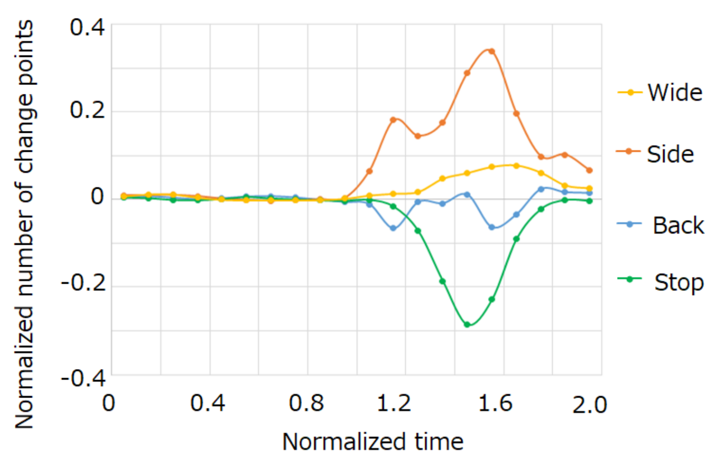

Figure 6. Transitions of the number of change points of the difference data.

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TABLE II. RESULT OF ANOMALY DETECTION FOR EACH ANOMALOUS WALKING TYPE.

Walking type	Subject A		Subject B		Subject C		Total	
	TP	FP	TP	FP	TP	FP	TP	FP
Back	14	0	20	6	20	4	54	10
Side	18	0	20	2	20	2	58	4
Stop	20	2	20	1	20	2	60	5
Wide	17	4	18	6	13	2	48	12
Total	69	6	78	15	73	10	220	31

joints and 1/1200 and 11 times for differences data. The data of daily walk is used only for the determination of parameters, and change point detection is performed for the four types of anomaly walks. Fig. 5 and 6 show the transitions of the number of change points. These figures show the average number of change points for 60 walks by the three subjects. The x-axis shows normalized time; 0 to 1 indicates a daily walk, and 1-2 indicates anomalous walks. The boundary between daily and anomaly walks is determined by human confirmation in every frame. The y-axis shows the normalized number of change points. The data are plotted every 0.1 point on the x-axis as the average number of change points in the range. The positive values in the result indicate a change in walking movements to intense movements and the negative values indicate a change in movements to slow movements. Even in daily walk, the values of joints are positive because the foot movement periodically changes. However, the relationships between joints do not change, and the values of difference are almost 0. The figure shows that the tendency of number of the change points changes between daily and anomaly walks and it is different in each type of anomaly walks. In Side and Wide, both of the values are positive; however, a change in the difference in Wide is smaller than that in Side. The change in Wide is only a change in stride, and the change in the positional relationship between the joints of the body is smaller than that in Side. In Stop and Back, the values of Stop are smaller than Back and Stop is characterized by the temporally stopping of the movement of all joints. Thus, the method can detect the difference between each anomaly walk. This means that the method can be used to analyze the trend of each walking type, and the result of change point detection can be used for anomaly detection and classification. However, in actual operation, the types are not given. Thus, analysis should be done after this classification if the method is used for the analysis

C. Anomalous Walking Detection

Table II shows the result of the anomaly detection for each anomalous walking type. The threshold of LOF is 4.5, which is detected by the greedy algorithm after the parameter of MEAWA is detected. The table shows TP (TP is True Positive, which means the detection of anomaly walks as an anomaly) and FP (FP is False Positive, which means the detection of daily walks as an anomaly) of each subject and of all subjects. Each rate of TP and FP is 91.7% and 12.9% in total; TP has high accuracy ($> 90\%$), and FT has medium accuracy ($> 80\%\ and < 90\%$) for standards in [3]. By focusing on individual results of each type (e.g., TP is 100% in Stop case), it is determined that there are differences in the accuracy. This observation shows that there is a possibility of improving the accuracy by optimizing the parameters that we focused on, such as the threshold or window size of each subject and walking type. The optimization is considered effective for the long-term observation of individuals. This will be evaluated in detail in the future.

TABLE III. RESULT OF THE CLASSIFICATION INTO FOUR CLUSTERS FOR EACH ANOMALOUS WALKING TYPE.

Walking type	Cluster 1	2	3	4
Back	46	0	7	1
Side	0	52	4	2
Stop	50	0	10	0
Wide	0	2	13	33

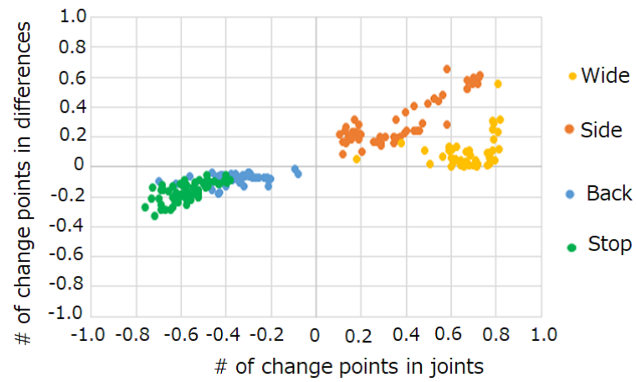

Figure 7. Characteristics of anomaly walks of each walking type.

D. Anomalous Walking Classification

In this section, clustering is performed in the case of TP in anomaly detection. Table III shows the result of clustering into four clusters. The number of clusters is the same as the number of walking types. Each cluster is shown by a number because clustering is unsupervised. In this result, clustering is performed separately for each subject, and we group each cluster of each subject with the highest match rate. Clustering does not classify four types of walks into four clusters. Back and Stop are grouped, and the clusters look divided such as Back and Stop, Side, Wide, and Others. To explain this result, the characteristics of each walking type are plotted in Fig. 7. As shown in the figure, Back and Stop overlapped and created the same cluster. This indicates that the tendencies of movement of Back and Stop are the same, and the difference is only the magnitude of the value. The trend is probably caused by the stopping motion because Back includes the motion of Stop because stopping is needed to switch the front step to the back step. The differences indicate the length of stopping time.

We perform clustering into three clusters owing to the grouping, and Table IV shows the result. In this result, anomaly walks are classified into three clusters (Back and Stop, Side, and Wide), and the accuracy rate is 89.1%. The rate is medium in [3], and it classified walks better than the existing walking recognition methods.

As with anomaly detection, clustering may show individual differences. Fig. 8 shows the plots of the same data of Fig. 7 colored by each subject. For Side and Wide cases, the figure shows that subjects have different tendencies. On the basis of this result, when considering the classification of anomaly walks, it is determined that clustering should not target too

TABLE IV. RESULT OF THE CLASSIFICATION INTO THREE CLUSTERS FOR EACH ANOMALOUS WALKING TYPE.

Walking type	Cluster 1	2	3
Back	46	0	8
Side	0	54	4
Stop	58	0	2
Wide	0	10	38

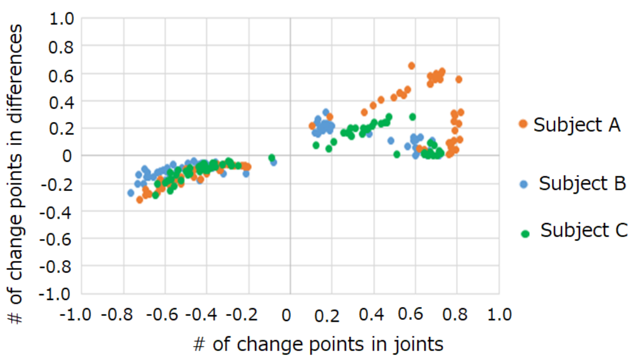

Figure 8. Characteristics of anomaly walks for each subject.

many people at the same time. The result shows the possibility of other applications of the proposed method, if the method can detect or classify the tendency of walking of each subject, it can be used to diagnose diseases, such as paralysis, to measure the effects of rehabilitation.

V. CONCLUSIONS

The detection of a temporary change in walking that causes falls is a way to alleviate the problem. However, existing methods recognize only walking speed or whether a person is walking or not. In this paper, we proposed an anomalous walking detection and classification method by three processing, which are the change point detection, anomaly detection, and clustering. Thus, anomaly walks were detected in 91.7% of cases using 240 videos including change from daily walk to anomaly walk. Furthermore, the detected anomaly walks can be classified into three clusters in 89.1% of cases. The average length of video is 87 frames (30 fps) for the longest motion. The result shows the proposed method can detect and classify a temporary change in walking. This capability can be applied to the analysis of the action that causes falling. In this paper, we evaluated the method by video data, while the required input of the method is the coordinate data of joints. Thus, the method has a wide application range. For example, the method is applicable to wearable technology such as motion sensors, if it can acquire the coordinates of a sufficient number of joints.

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