Performance of Linear Programming in Optimizing the Energy Schedule of a Grid-connected Hybrid System Compared to Particle Swarm Optimization

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***Abstract*—This study aims to find a rapid and efficient method for managing the energy of a Grid-connected hybrid system. Thus, two optimization strategies, the Linear Programming (LP) and the Particle Swarm Optimization (PSO), have been suggested to minimize the operating cost of the hybrid system while respecting the constraints of all the system components. Then, a comparative study has been made between these two methods (i.e., LP and PSO). Consequently, the operating cost obtained using PSO algorithm is close to the one provided by the LP algorithm. However, the PSO algorithm is slower than the LP algorithm and requires different parameters to be chosen. Finally, the impact of the battery initial state of charge on the operating cost is studied.**

Keywords—Linear Programming; Particle Swarm Optimization; Optimization; Grid-connected hybrid system.

I. INTRODUCTION

The energy consumption of access networks represents a principal part of telecommunications operators' energy bills. Several works [1] [2] have been initiated on sources, energy storage, and their management to reduce this consumption and the carbon footprint. Green production is a promising way to overcome this fossil energy issue [3] [4]. Besides, it is necessary to develop acceptable management methods and technical tools guaranteeing network reliability [5]. In this perspective, the notion of microgrid has appeared to resolve part of this management problem. Indeed, it is an intelligent system composed of green and local production as well as a storage system to ensure the reliability of the system. An energy management system provides an optimal configuration and sizing with the economic management of exchanged energy within. The maximization of economic efficiency and reliability is undoubtedly the top of all research targets [6].
Different studies have been made to achieve this purpose. For instance, in [6] a multi-objective optimization problem of optimizing the schedule of sources, as well as the import/export power with the grid, has been solved using an optimization-based approach called Branch and Bound method. In addition, optimal energy management of microgrid, which constitutes of a PV system and a storage system with minimum of cash flow using dynamic programming technique has been suggested in [7] [8]. A comparative study has been presented in [9] to illustrate the efficiency of Linear Programming (LP) compared to PSO and adaptive dynamic programming for an intelligent home energy resources scheduling in the presence of uncertain data. Hossain et al. [10] present a particle swarm optimization for real-time application energy management to find optimal battery control of a community microgrid. In [11], a fuzzy logic-based energy management system for a residential grid-connected system including renewable energy sources and storage capability is suggested. The difference between this study and the studies cited above occurs in the problem formulation and the constraints to be respected. To optimize the energy scheduling/management in a connected microgrid, there are two types of methods: an exact optimization methods that guarantee finding an optimal solution (e.g., LP) and heuristic optimization methods that don't guarantee that the solution founded is optimal (e.g., PSO). In this study, both algorithms LP and PSO are applied to find the optimal energy scheduling of a grid-connected hybrid system. Furthermore, three different scenarios are considered to provide a comparison between these two algorithms. It is shown that the LP algorithm is faster and does not require parameters to be tuned which is not the case for PSO algorithm. On the other hand, the impact of the initial state of charge of the battery on the operating cost is studied for the LP algorithm.

The work is organized as shown: system description and energy models of the architecture components are introduced in Section II. These models will be used to calculate the required parameters for optimization approaches. Section III

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suggests two different methodologies for optimizing energy planning with a minimum operating cost. In this section, we introduce the objective function and constraints adopted in each method. The results obtained by these approaches are compared in Section IV. In addition, some assessment has been established to show the robustness of the proposed strategy.

II. System Description

The architecture studied is a grid-connected hybrid system composed of renewable energy sources, i.e., Photovoltaic and Wind Turbine, batteries, and DC load as shown in Figure 1. To reduce the energy loss, we assume that the battery should be charged only by the remaining energy, this means when the energy produced by renewable sources is greater than the load demand. Otherwise, the energy left will be exchanged with another local site. On the other hand, if the consumption exceeds the production of the renewable source, the battery will discharge to meet the remaining energy.

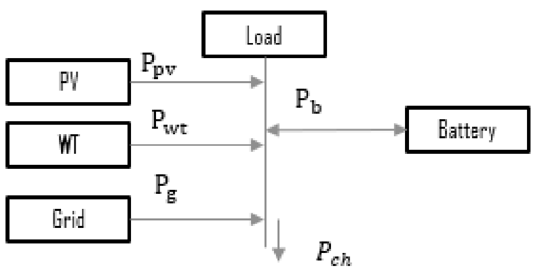

Figure 1. Grid-connected hybrid system architecture

A. Photovoltaic model

The mathematical model for estimating the output power of a PV module is a linear function of the solar radiation and the ambient temperature [12]. It can be calculated as follows:

$$P_{pv} = P_p f_c G(\frac{1+\beta(T_c - T_{ref})}{G_r}) \tag{1}$$

$$T_c = T_a + G(\frac{(NOCT-20)}{800}) \tag{2}$$

where P_p is the rated power under standard test conditions (kW), f_c is PV derating factor (88%), G is solar radiation (W/m^2), β is the temperature coefficient of efficiency (-0.41%/°C), G_r is the standard amount radiation (1000 W/m^2), T_{ref} is standard test temperature (25°C), T_c is the cell temperature (°C), T_a is the ambient temperature (°C) and NOCT is the nominal operating cell temperature (45°C).

B. Wind Turbine model

To model the wind turbine, the mechanical power, which is directly extracted from it, can be given by [13]

$$P_{wt} = \begin{cases} 0.5 C_p S \varphi V^3, & \text{if } V_i \leq V \leq V_n \\ P_r, & \text{if} V_n \leq V \leq V_o \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where V_n, V_i and V_o are the rated (11m/s), the cut-in (3.5m/s), the cut-out (25m/s) wind speeds respectively, C_p is the power coefficient, S is the turbine blades swept area (10.87m^2), φ is the air density (1.225kg/m^3), V is the wind speed at hub height H and P_r is the rated power. For the purpose of adjusting the wind profile according to the height, the following equation can be used [14]

$$V = V_0(\frac{H}{H_0})^{\alpha} \tag{4}$$

where V_0 is the wind speed measured at the reference height H_0. α is the power law exponent depends on the nature of terrain (0.14).

C. Battery model

Batteries are used to store excess power in the microgrid and operate when the system has deficit power. At any hour, the battery stored energy is related to the previous one and the energy production and consumption situation of the system during the time from $t-1$ and t as used in [10].

- Charging mode

$$W_b(t) = W_b(t-1) + (W_{pv} + W_{wt} - W_l) \tag{5}$$

- Discharging mode

$$W_b(t) = W_b(t-1) - (W_l - W_{wt} - W_{pv}) \tag{6}$$

where W_l is the energy consumption (kWh). W_{pv}, W_{wt} represent the energy production by Photovoltaic module and Wind turbine (kWh) respectively.

III. Optimization algorithm

The main goal of this paper is to minimize the operating cost of the energy exchanged with the grid to obtain an optimal energy schedule of the grid-connected hybrid system. The optimization algorithm should ensure that the discharging of the battery will be done during high demand, while the charging will be done during high production, moreover, the state of charge should be within upper and lower limits. In addition, the battery must return to its initial state of charge at the end of the optimization horizon to ensure that the system has a stabilized energy balance for one cycle. To solve this issue, we presume two methods of optimization explained below:

A. Linear programming

This approach is based on the linear programming paradigm that consists in minimizing or maximizing a given function according to the following constrained scheme [9]:

$$\begin{cases} \max f(x) = c^T x \text{ or } \min f(x) = c^T x \\ \text{subject to:} \quad Ax \leq b \; or \; Ax \geq b \; or A_{eq}x = b_{eq} \\ \text{Where:} \quad x \geq 0, x \in \mathbb{R}^{n\times 1}, A \in \mathbb{R}^{m\times n}, b \in \mathbb{R}^{m\times 1}, \\ c \in \mathbb{R}^{n\times 1}, A_{eq} \in \mathbb{R}^{p\times n}, b_{eq} \in \mathbb{R}^{p\times 1} \end{cases} \tag{7}$$

In this approach, the decision variables for the economic dispatch problem are as follows:

- The battery energy of charging and discharging (W_c and W_d).

- The exchanged energy with the grid utility (W_g).
- The exchanged energy with another site (W_{exch}).
- The battery state of charge (SOC).

This means, in our case the vector x is chosen as follows: $[W_g, \quad W_c, \quad W_d, \quad SOC, \quad W_{exch}]^T$. The vector c is formed using (8)-(9). The right part of (10)-(12) represents the elements of the matrix A_{eq}, and the left part of these equations are used to form the vector b_{eq}. Matrix A contains the coefficients multiply the decision variables in (13)-(18). The vector b includes the upper and the lower limits of each variable.

The objective function in (8) aims to minimize the cost of the energy purchased from the grid W_g when the consumption is greater than the production as well as make profit by exchanging the remaining energy with other site W_{exch}. And, C_g in (8) represents the purchased energy price (0.2€/kWh).

$$\min C = \sum_{i=1}^{T} W_g(i) C_g k(i) \tag{8}$$

Where T is the energy management system period (e.g., one day) and i is time interval (e.g., 1h). The parameter k is a binary variable for the charging state of the battery. In fact, k represents two constraints to be respected which are the battery should not be discharged when the system has an excess energy and vice versa. k is calculated in terms of the load demand and renewable energy production, as described in (9), where W_l, W_{pv} and W_{wt} are the load, solar PV and Wind Turbine energy (kWh)

$$k(i) = \begin{cases} 1, & \text{if } d(i) \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{9}$$

$$d(i) = W_l(i) - W_{pv}(i) - W_{wt}(i) \tag{10}$$

Equation (11) guarantees that the battery will be charged mostly by the renewable energy.

$$d(i) = k(i)(W_g(i) - W_d(i)) + (k(i) - 1)(W_c(i) - W_{exch}(i)) \tag{11}$$

Equation (12) calculates the battery state of charge SOC in each slot time i to maintain its values within the given limitations in (13).

$$SOC(i+1) = SOC(i) + \frac{(1 - k(i))W_c + k(i)W_d}{E_c} \tag{12}$$

$$SOC_{min} \leq SOC(i) \leq SOC_{max} \tag{13}$$

Where E_c is the nominal energy of the battery[kWh]. The inequalities (14)-(17) indicate the lower and upper bounds that should be respected for the exchanged energy with the grid, the external consumers, and the battery, respectively.

$$0 \leq W_g(i) \leq W_{gmax} \tag{14}$$

$$W_{exch}(i) \geq 0 \tag{15}$$

$$0 \leq P_c(i) \leq P_{cmax} \tag{16}$$

$$P_{dmax} \leq P_d(i) \leq 0 \tag{17}$$

The ε refers to an admitted tolerance in the constraints concern the charging of the battery in the end of the period of the optimization T.

$$|SOC(T) - SOC(1)| \leq \varepsilon \tag{18}$$

B. Particle swarm optimization

Its concept is based on the behavior of birds to compute global optimization functions [10]. In PSO, each possible solution is modeled as a particle that moves through the input hyperspace, which can have numerous dimensions [15]. First and foremost, each solution takes a random position with a random velocity in the search space. At each iteration, the particles move towards their best position, and therefore that of their neighborhood, which corresponds to the optimum position, by updating their velocity [16]. In this section, the objective is similar, whereas the objective function has presented differently. The first term in (19) refers to a penalty applied in the case of the battery charged with the grid. The decision variable in this methodology is only the exchanged energy with the battery. For this strategy, the cost function is described by (19) to reduce the electricity bill by minimizing the exchanged energy with the grid. Indeed, according to the difference between the load demand and the renewable energy production, as well as the sign of the decision variable, the penalty is applied to avoid the following scenarios:

- Charging or discharging the battery when the system is in a steady state.
- Discharging the battery even if there is an excess of energy.
- Energy left is not sufficient to charge the battery.

$$\min C = P + \sum_{i=1}^{T} W_g(i) C_g \tag{19}$$

The grid will meet the load when the consumption is greater than the production. On the other hand, if the suppliers transcend the load demand, the energy left will be exchanged with other consumers.

$$d(i) = W_l(i) - W_{pv}(i) - W_{wt}(i) \tag{20}$$

$$W_g = \begin{cases} W_l(i) - d(i) + W_b(i), & \text{if } d(i) \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{21}$$

$$W_{exch} = \begin{cases} -d(i) - W_b(i), & \text{if } d(i) \leq 0 \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

At each point of time, the program ensures that the solution respects the constraints presented as follows:

$$P_{bmin} \leq P_b(i) \leq P_{bmax} \tag{23}$$

$$SOC_{min} \leq SOC(i) \leq SOC_{max} \tag{24}$$

$$0 \leq W_g(i) \leq W_{gmax} \tag{25}$$

$$W_{exch}(i) \geq 0 \tag{26}$$

$$|SOC(T) - SOC(1)| \leq \varepsilon \tag{27}$$

IV. Simulation and discussion

The considered system includes a DC load with a constant rated power of 5 kW, a PV with an installed peak power of 69 kW, a Wind turbine with a rated power of 16 kW, and a battery with a rated energy of 74 kWh.

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TABLE I. PSO PARAMETERS

PSO	Number of variables	24
	Number of iterations	300
	population size	1000
	Inertia coefficient	1
	Damping ratio of inertia coefficient	0.99
	Personal acceleration coefficient	2
	Social acceleration coefficient	2

A. Comparative study

In this subsection, the performance of both algorithms to provide the energy management for this aforementioned system is compared through simulation results using Matlab. Indeed, three scenarios have been tested (see Figures 2-4) to find which approach is more efficient regarding some parameters such as the operating cost, the computational time, and the energy exchanged with other site. These scenarios present the meteorological data at Lannion for three different months July, May and October. In the simulations which follow, the period is fixed as 24 hours, the initial state of charge is chosen as $SOC(1) = 80\%$, and the error between the final and the initial values of SOC is taken as 3%, i.e., $\varepsilon = 3\%$ in Eqs.(18) and (27). Moreover, the values of $SOC(min)$ and $SOC(max)$ have been selected as SOC(min)=30% and SOC(max)=100%. The optimization parameters for PSO algorithm are given in Table I. In Figures 2-4, it can be observed that the energy dispatch proposed by both methods for the three scenarios, are globally similar. Indeed, when the PV and Wind turbine production is more important than the load demand, the two approaches suggest to charge the battery and transfer the remaining energy to other consumers. But, the difference occurs when the production is less than the demand. In this case, PSO algorithm proposes to use the grid and the battery to meet the load demand. Conversely, linear programming suggests meeting the load by discharging only the battery. Besides, the two strategies respect obviously the constraints about the final value of the battery state of charge in all cases. Regarding the computational time, the linear programming can find the optimal solution within one minute. However, the PSO algorithm takes more than one hour to find it, since its convergence depends on the number of iterations and population size which have been chosen big enough. Here, it should be mentioned that if the number of iterations and population size have not been adequately chosen, the convergence of PSO algorithm cannot be ensured. The operating cost proposed by LP is less expensive than the one proposed by PSO algorithm for all scenarios, as it can be shown in TableII. On the other hand, the PSO algorithm offers to exchange more energy compared to other one since the battery discharges less than the first technique as shown in the Figures 2-4.

To sum up, the LP finds quickly and efficiently the optimal schedule of the considered system compared to the PSO algorithm. Furthermore, as soon as the numbers of the decision variables increase, the use of the PSO becomes avoidable. That is due to the reason that, the PSO algorithm requires a lot of parameters to be tuned.

TABLE II. OPERATING COST AND EXCHANGED ENERGY PROPOSED BY TWO APPROACHES.

Scenario	**PSO**		**LP**	
	C (€)	W_{exch} (kWh)	**C** (€)	W_{exch} (kWh)
Case 1	1.85	32	0.68	24
Case 2	2.61	27	1.18	18
Case 3	2.38	18.5	1.2	10

Where case 1, case 2, and case 3 represent the meteorological data in July, May, and October, respectively.

B. Sensitivity analysis

In this part, some parameters will be analyzed using linear programming. It treats the impact of the initial state of charge on the operational cost and the energy sold to other sites over 24h.The values of the SOC_{min}, SOC_{max}, and ε are similar to those used in the subsection A. However, the data that will be used in this subsection are presented in Figure 2. Table III shows that the minimum operational cost has been obtained in the case where the battery initial level of energy is 70%. Indeed, the purchased energy from the grid is zero as well as the 70% of energy stored is sufficient to meet the load and respect the constraint about the final value of the SOC. Consequently, the exchanged energy with the other consumers is the minimum because the most excess energy is used to charge the battery. At the beginning of the optimization, if the battery is fully charged or discharged, the operational cost and the exchanged energy for the solution obtained would be greater than the other scenarios. In other words, the battery is less used in these cases in order to respect the constraint about the final SOC. Figure 5, it represents the energy schedule of the system aforementioned considering the optimal value of the $SOC(t_0)$. As can be shown in the Figure 5, the battery has completely discharged in the state of deficit when the renewable sources production is insufficient. Besides, it is remarkable that the excess energy has been sufficient to charge completely the battery. For that reason, the system has respected the constraints without using the grid energy to meet the load like the other cases. To conclude, the economic scenario to adopt is with SOC(t_0) equals to 70% since the energy exchanged with the grid in this case is zero.

TABLE III. OPERATING COST AND EXCHANGED ENERGY FOR DIFFERENT VALUES OF THE INITIAL BATTERY STATE OF CHARGE USING LP.

SOC_{int} (%)	**100**	**90**	**80**	**70**	**60**	**50**	**40**
C (€)	2.61	1.64	0.68	0	0.46	1.42	2.39
W_{exch} (kWh)	39	31	24	19	22	30	37

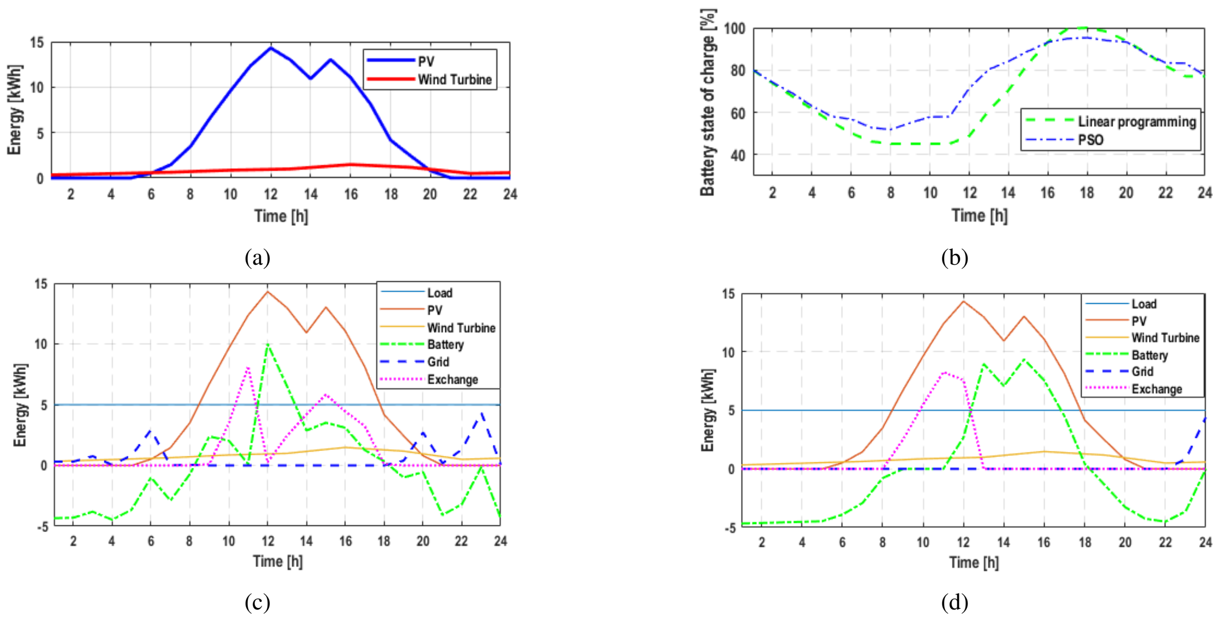

Figure 2. (a) The PV and Wind turbine energy profile on 1st of July in Lannion. (b) The variation of the battery state of charge proposed by PSO and LP. The energy scheduling on 1st of July using: (c) PSO and (d) LP.

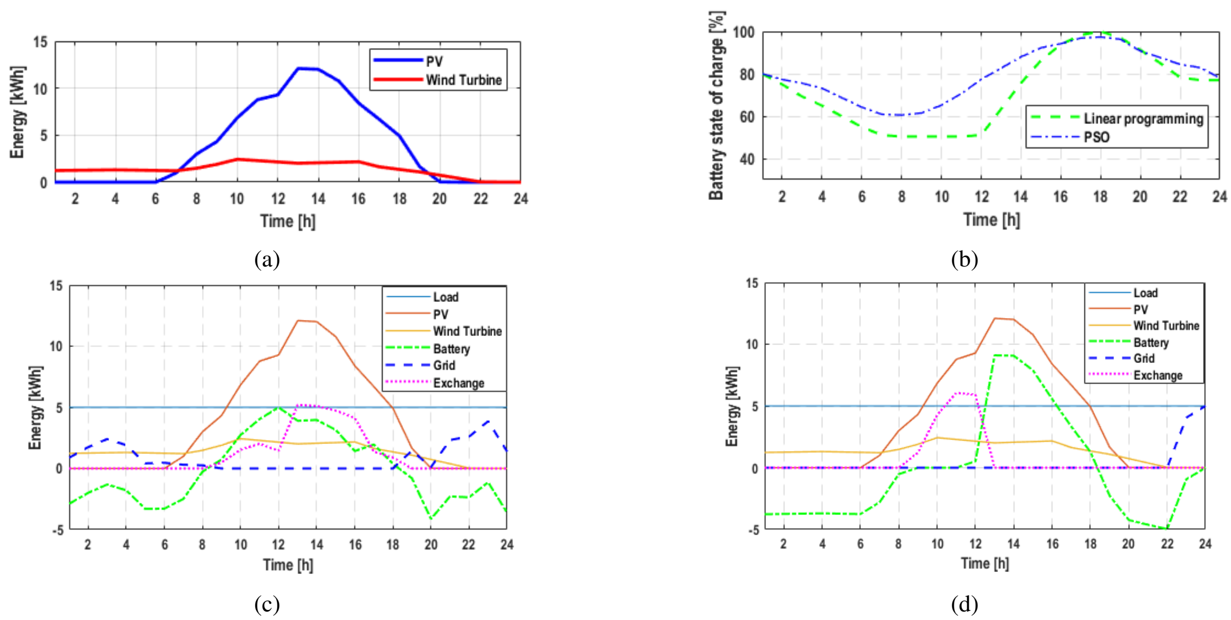

Figure 3. (a) The PV and Wind turbine energy profile on 1st of May in Lannion.(b) The variation of the battery state of charge proposed by PSO and LP. The energy scheduling on 1st of May using: (c) PSO and (d) LP.

V. Conclusion

This paper has applied two approaches for the optimal energy scheduling of a Grid-connected hybrid system which are the LP and the PSO algorithms. Then, a comparison has been made to confirm the effectiveness and the rapidity of the LP in front of the PSO algorithm in terms of computational time and operational cost. Moreover, it is shown that the PSO algorithm requires some parameters to be tuned to achieve the convergence which is not the case for the LP. On the other hand, a sensitivity analysis for the LP has been studied also. The obtained results confirm that the LP will be more effective if the battery starts with an initial state of charge equal to 70%. As future works, a comparative study between two exact optimization methods (i.e., LP and Mixed-Integer linear programming (MILP)) will be studied. Furthermore, the LP-based energy management will be combined with a sizing algorithm to optimize the configuration of a grid-connected hybrid system. Moreover, a

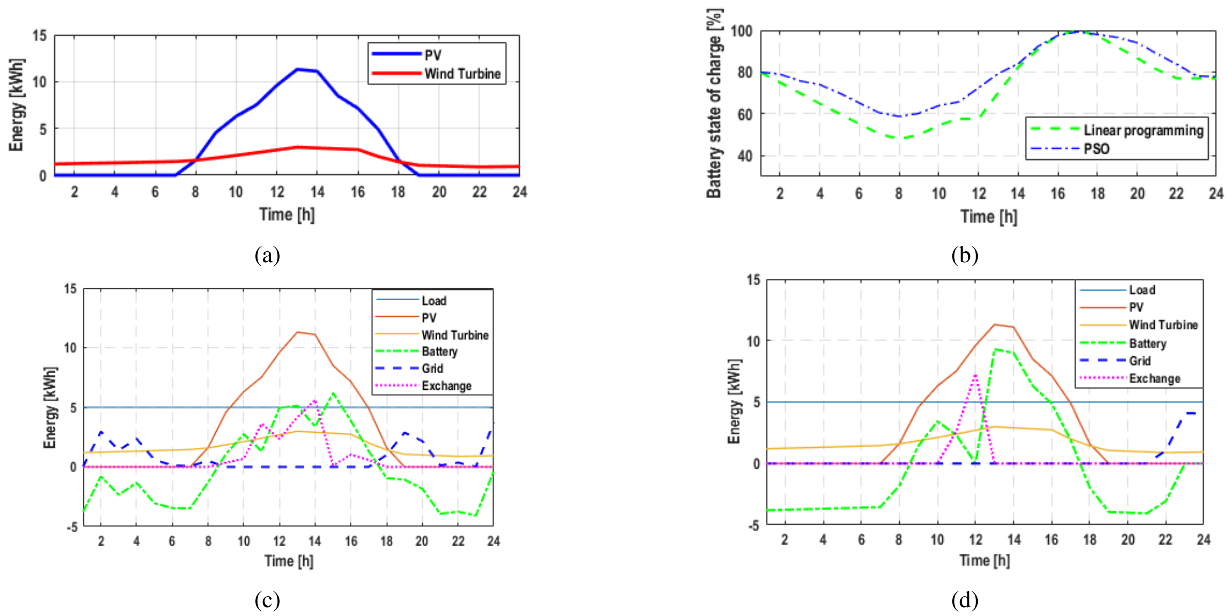

Figure 4. (a) The PV and Wind turbine energy profile during on 1st of October in Lannion. (b) The variation of the battery state of charge using PSO and LP. The energy scheduling on 1st of October using: (c) PSO and (d) LP.

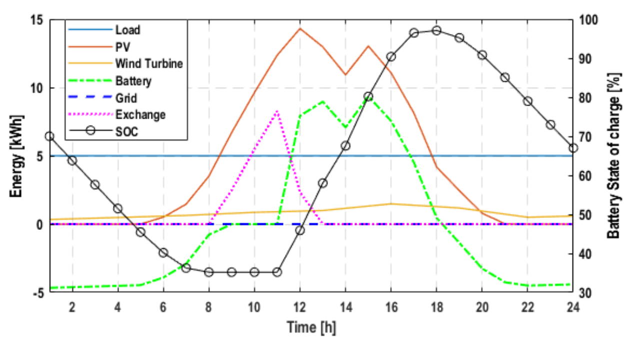

Figure 5. The energy scheduling of the system studied on 1st of July in Lannion

comparative study with the existing sizing algorithms will be performed.

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