Probabilities of Detection and False Alarm in MTM- Based Spectrum Sensing for Cognitive Radio Systems

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***Abstract*— The paper presents closed-form expressions for the detection, and false alarm probabilities for spectrum sensing detection based on the Multitaper Spectrum Estimation Method (MTM) using Neyman-Pearson criterion. The MTM spectrum sensing is a powerful technique in Cognitive Radio (CR) systems. It tolerates problems related to bad biasing, and large variance of estimates, that are the main drawbacks in the periodogram (i.e., energy detector). The performance of the MTM spectrum sensing system is controlled by parameters, such as the chosen half time bandwidth product, Discrete Prolate Slepian Sequence (DPSS) (i.e., tapers), DPSS' eigenvalues, and the number of tapers used. These parameters determine the theoretical probabilities of detection and false alarm, which are used to evaluate the system performance. The paper shows a good match between the theoretical and numerical simulation results.**

***Keywords*— cognitive radio; spectrum sensing; multitier spectrum estimation.**

I. INTRODUCTION

Cognitive radio is an innovative new technology in wireless communications, which was firstly proposed by Mitola in 1999 [1]. It allows secondary users (CRs), to opportunistically use the vacant spectrum subbands that are licensed already to primary users (PRs), at a specific time and geographical location. By full exploitation of the vacant spectrum subbands while keeping the PR users protected for harmful interference, CR technology provides efficient new spectral opportunities for next generations of wireless applications. It represents a new paradigm of spectrum allocation that helps reduce spectrum scarcity, and underutilization. Additionally, it can provide communications anywhere at any time [2].

A CR system should be capable of scanning through a given spectrum to find vacant bands to operate. The accurate CR system decision about the availability of vacant bands is totally dependent on the quality of the sensing techniques used. Clearly, CR technology can only be useful if an accurate sensing scheme is used.

Although the matched filtering and the cyclostationary feature detector have high performance as spectrum sensing techniques in CR, such techniques require prior information about the PR signaling [3-5].

On the other hand, the energy detector does not require prior information about the PR signaling and has low complexity. Such advantages come at the expense of moderate performance due to the use of single rectangular windows' tapering [6].

Multi taper spectrum estimation (MTM) [7], uses orthonormal tapers; known as the Discrete Prolate Slepian Sequence (DPSS) [8]. It produces a single spectrum estimate with minimum spectral leakage and good variance. MTM is an approximation of the optimal spectrum estimate; the Maximum-likelihood method but at reduced computation [9], [10]. Haykin, on the other hand, suggested the use of MTM as an efficient method for spectrum sensing in CR [2].

Using Neyman-Pearson criterion [11], theoretical derivations of probabilities of false alarm, and detection for the MTM spectrum sensing optimal detector are necessary to evaluate its performance. Furthermore, MTM detection system includes parameters, such as time bandwidth product, the DPSS, and their associated eigenvalues that control the quality of the spectrum estimate. Consequentially, a set of different parameters and thresholds can be chosen that maximizes the performance of the detection.

Although MTM was first studied by Thomson in 1982, statistics and probabilistic theoretical work are still an open research issue. In [12], the authors derived the probabilities of detection and false alarm formulae based on the spectrum estimate characteristic function (CHF) by formulating the MTM spectrum detector as a quadratic function of Gaussian vector.

In this paper, we present closed-form formulae for the probabilities of detection and false alarm for the MTM-based spectrum detector. The probability density function (PDF) of

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the MTM spectrum estimate (decision statistic) is approximated to Gaussian. The mean and the variance of the PDF have been derived for both hypotheses, and used in the calculation of the probabilities.

Our theoretical work presented in this paper, includes two cases: firstly, the PR signal is known as a modulated signal, and secondly, the PR signal is unknown and assumed as Gaussian random variable.

The rest of the paper is organized as follows: Section II defines the model for the system under consideration and reviews MTM technique Section III presents the theoretical work of the MTM detector. Section IV presents the results and Section V concludes the paper.

II. SYSTEM MODEL

In our system model, we consider OFDM signaling scheme for the PR user. The PR transmitter with N subcarriers (N-IFFT/FFT) transmits OFDM-QPSK signal with energy E_s over each subcarrier. The CR transceiver is supported by (N-IFFT/FFT) processor as well so as to perform both tasks of communications, and sensing. Additionally, MTM spectrum detector is added to the CR receiver for spectrum sensing.

The received PR signal, at CR receiver, is sampled to generate a finite discrete time samples series $\{x_t; t = 0,1,\dots,N-1\}$, where t is time index. The discrete time samples are 'dot multiplied' with different tapers $v_{(t,k)}(N,W)$ (tapers are Discrete Prolate Slepian Sequences). The associated eigenvalues of the k^{th} taper is $\lambda_k(N,W)$. The product is applied to a Fourier Transform to compute the energy concentrated in the bandwidth $(-W,W)$ centered at frequency f. The half time bandwidth product is NW, and the total number of generated tapers is $2NW$. For K orthonormal tapers used in the MTM, there will be K different eigenspectrums produced and defined as [7]:

$$Y_k(f_i) = \sum_{t=0}^{N-1} v_{(t,k)}(N,W) x_t e^{-j2\pi f_i t} \quad (1)$$

where, $f_i = 0, \frac{1}{N}, \frac{2}{N}, \dots, \frac{N-1}{N}$ are the normalized frequency bins. The spectrum estimate given by Thomson theoretical work is defined as [7]:

$$S_{MTM}(f_i) = \frac{\sum_{k=0}^{K-1} \lambda_k(N,W) |Y_k(f_i)|^2}{\sum_{k=0}^{K-1} \lambda_k(N,W)} \quad (2)$$

On the other hand, the energy detector, when the samples are taken at uniform time spacing, gives the power spectrum density estimation as [6]:

$$S_{ED}(f_i) = \frac{1}{N} \left| \sum_{t=0}^{N-1} x_t e^{-j2\pi f_i t} \right|^2 \quad (3)$$

In order to evaluate the performance of the MTM spectrum detector, we considered the probability of detection $P_d(f_i)$, the probability of false alarm $P_f(f_i)$, and the probability of miss detection $P_m(f_i)$ at each frequency bin f_i based on the Neyman-Pearson (NP) criterion. $P_d(f_i)$ is the probability that CR detector decides correctly the presence of the PR's signal, $P_f(f_i)$ is the probability that CR detector decides the PR's signal is present when it is absent, and $P_m(f_i)$ is the probability that CR fails to detect the PR's signal when it is present.

The binary hypothesis test for CR spectrum sensing at the lth time is given by:

$$\begin{aligned} \mathcal{H}_0&: \quad x_t(l) = w_t(l) \\ \mathcal{H}_1&: \quad x_t(l) = s_t(l) + w_t(l) \end{aligned} \quad (4)$$

where l = 0,1,…,L-1 is OFDM block's index, $x_t(l)$, $w_t(l)$, and $s_t(l)$ denote the CR received, noise, and PR transmitted samples. The transmitted PR signal is distorted by the zero mean additive white Gaussian noise $w_t(l) \sim \mathcal{CN}(0, \sigma_w^2)$. The signal to noise ratio (SNR) is $SNR = \frac{E_s}{\sigma_w^2}$.

The time instant l comes from the samples over different OFDM blocks; and time instant t comes from the samples from the same OFDM block (i.e., IFFT/FFT samples). Thus, the spectrum sensing time in second is $(L)(N)(T_s)$, where T_s represents symbol duration, L represents the number of OFDM blocks that used in sensing, and N is the number of samples per OFDM block (i.e., FFT size). The decision statistic over L OFDM blocks using MTM is defined as follows:

$$DEC_{MTM}(f_i) = \sum_{l=0}^{L-1} \frac{\sum_{k=0}^{K-1} \lambda_k(N,W) \left| \sum_{t=0}^{N-1} v_{(t,k)}(N,W) x_t(l) e^{-j2\pi f_i t} \right|^2}{\sum_{k=0}^{K-1} \lambda_k(N,W)} \quad (5)$$

III. DECISION STATISTIC PROBABILITY DENSITY FUNCTION

For large L at low SNR, we approximate the PDF of the eigenspectrum absolute square $|Y_k(f_i)|^2$ from chi-square to Gaussian, then the decision statistic ($DEC_{MTM}(f_i)$) is represented by the sum of K correlated Gaussian samples (i.e., eigenspectrum absolute square $|Y_k(f_i)|^2$).

Thus, the decision statistic using MTM detector is approximately normal Gaussian distributed as expected due to MTM linear processing.

We now consider the mean (E), and the variance (VAR) of the decision statistic $DEC_{MTM}(f_i)$ for both hypotheses. (i.e., $E(DEC_{MTM}(f_i)/\mathcal{H}_0$, $E(DEC_{MTM}(f_i)/\mathcal{H}_1)$, $VAR(DEC_{MTM}(f_i)/\mathcal{H}_0)$, and $VAR(DEC_{MTM}(f_i)/\mathcal{H}_1)$).

The probability of detection, and the probability of false alarm at frequency bin f_i $P_d^{MTM}(f_i)$, and $P_f^{MTM}(f_i)$, respectively, for the decision statistic with Gaussian distribution are defined as:

$$\begin{aligned} P_d^{MTM}(f_i) &= P(DEC_{MTM}(f_i) > \gamma / \mathcal{H}_1) \\ &= Q\left(\frac{\gamma - E(DEC_{MTM}(f_i)/\mathcal{H}_1)}{\sqrt{VAR(DEC_{MTM}(f_i)/\mathcal{H}_1)}}\right) \end{aligned} \quad (6)$$

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$$P_f^{MTM}(f_i) = P(DEC_{MTM}(f_i) > \gamma / \mathcal{H}_0)$$

$$= Q\left(\frac{\gamma - E(DEC_{MTM}(f_i)/\mathcal{H}_0)}{\sqrt{VAR(DEC_{MTM}(f_i)/\mathcal{H}_0)}}\right) \quad (7)$$

The probability of miss detection can be defined as:

$$P_m^{MTM}(f_i) = P(DEC_{MTM}(f_i) < \gamma / \mathcal{H}_1)$$

$$= 1 - Q\left(\frac{\gamma - E(DEC_{MTM}(f_i)/\mathcal{H}_1)}{\sqrt{VAR(DEC_{MTM}(f_i)/\mathcal{H}_1)}}\right) \quad (8)$$

the term $Q(\xi)$ is given by the tails of the distribution, and γ represents the threshold. Note that γ can be controlled based on σ_w^2.

When only noise is present for $\mathcal{H}_0$ case at frequency bin f_i, and based on the linearity property of the FFT process, the mean of the decision statistic $E(DEC_{MTM}(f_i)/\mathcal{H}_0)$ can be defined for K Gaussian samples as:

$$E(DEC_{MTM}(f_i)/\mathcal{H}_0) =$$

$$C.\sum_{l=0}^{L-1}(\sum_{k=0}^{K-1}(\lambda_k(N,W).K\sum_{t=0}^{N-1}\sum_{t'=0}^{N-1}E(v_{(t,k)}(N,W) .v_{(t',k)}(N,W).w_t(l)w_{t'}(l)))) \quad (9)$$

where $C = E\left(\frac{1}{\sum_{k=0}^{K-1}\lambda_k(N,W)}\right) = \frac{1}{\sum_{k=0}^{K-1}\lambda_k(N,W)}$ (10)

It can be shown that (9) can be simplified as:

$$E(DEC_{MTM}(f_i)/\mathcal{H}_0) = \sum_{l=0}^{L-1}\sum_{t=0}^{N-1} v_{(t,k)}^2(N,W).K.E\big(w_t(l)w_{t'}(l)\big) \quad (11)$$

From the definition of the Discrete Prolate Slepian Sequence (DPSS), we have [8]:

$$\sum_{t=0}^{N-1} v_{(t,k)}(N,W).v_{(t,k')}(N,W) = \begin{cases}1, & k = k'\\ 0, & k \neq k'\end{cases} \quad (12)$$

The orthonormality of the sequences can be used to simplify (11), when $t = t'$ as follows:

$$E(DEC_{MTM}(f_i)/\mathcal{H}_0) = \sum_{l=0}^{L-1}.K.E\big(w_t^2(l)\big) =$$

$$LK\Big(\big(E(w_t(l))\big)^2 + VAR(w_t(l))\Big) = LK(0 + \sigma_w^2) = LK\sigma_w^2 \quad (13)$$

When the PR signal is present for $\mathcal{H}_1$ case at frequency bin f_i, the mean of the decision statistic $E(DEC_{MTM}(f_i)/\mathcal{H}_1)$ can be defined following the same steps of $\mathcal{H}_0$ as:

$$E(DEC_{MTM}(f_i)/\mathcal{H}_1) = K\sum_{l=0}^{L-1}E\big(s_t^2(l) + 2s_t(l)w_t(l) + w_t^2(l)\big)$$

$$= LK(E_s + \sigma_w^2) \quad (14)$$

where $E(s_t^2(l)) = E(E_s) = E_s$, and $E(w_t^2(l)) = VAR(w_t(l)) + (E(w_t(l)))^2 = \sigma_w^2$, and $E\big(2s_t(l)w_t(l)\big) = 0$.

We now consider the variances of the hypotheses in the next stage of the derivation. In order to simplify our derivation, we redefine (5) using decision statistic coefficients α_k, $k = 0,1,2,\dots,K-1$, as follows:

$$DEC_{MTM}(f_i) = \sum_{l=0}^{L-1}\sum_{k=0}^{K-1}\alpha_k(f_i) \quad (15)$$

where coefficient α_k is defined as follows:

$$\alpha_k(f_i) = \frac{\lambda_k(N,W)|Y_k(f_i)|^2}{\sum_{k=0}^{K-1}\lambda_k(N,W)}, k = 0,1,2,\dots,K-1 \quad (16)$$

Then, the variance of $\alpha_k(f_i)$ can be defined as follows:

$$VAR(\alpha_k(f_i)/\mathcal{H}_p) = \frac{\lambda_k^2(N,W).VAR(|Y_k(f_i)|^2/\mathcal{H}_p)}{(\sum_{k=0}^{K-1}\lambda_k(N,W))^2}, p = 0,1 \quad (17)$$

The variance of the $\mathcal{H}_0$ hypothesis where the noise only is present for K correlated Gaussian samples (i.e., eigenspectrum absolute square $|Y_k(f_i)|^2$) $VAR(DEC_{MTM}(f_i)/\mathcal{H}_0)$, can be defined as follows:

$$\begin{aligned}VAR(DEC_{MTM}(f_i)/\mathcal{H}_0) = &\sum_{l=0}^{L-1}(\sum_{k=0}^{K-1}VAR(\alpha_k(f_i)/\mathcal{H}_0) + 2\rho_{01}\sqrt{VAR(\alpha_0(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_1(f_i)/\mathcal{H}_0)} +\\ &2\rho_{02}\sqrt{VAR(\alpha_0(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_2(f_i)/\mathcal{H}_0)} + \cdots +\\ &2\rho_{0K-1}\sqrt{VAR(\alpha_0(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_{K-1}(f_i)/\mathcal{H}_0)} +\\ &2\rho_{12}\sqrt{VAR(\alpha_1(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_2(f_i)/\mathcal{H}_0)} +\\ &2\rho_{13}\sqrt{VAR(\alpha_1(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_3(f_i)/\mathcal{H}_0)} + \cdots +\\ &2\rho_{1K-1}\sqrt{VAR(\alpha_1(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_{K-1}(f_i)/\mathcal{H}_0)} + \cdots +\\ &2\rho_{K-2K-1}\sqrt{VAR(\alpha_{K-2}(f_i)/\mathcal{H}_0)}\sqrt{VAR(\alpha_{K-1}(f_i)/\mathcal{H}_0)})\end{aligned} \quad (18)$$

where $\rho_{ij} \approx 1$, is the correlation coefficient between α_i, and α_j, and since $VAR(|Y_k(f_i)|^2/\mathcal{H}_p) = VAR(|Y_k(f_i)|^2/\mathcal{H}_p)$, for $i,j = 0,1,2,\dots,K-1$ applying the orthonormality in (12). Then (18) can be rewritten using (10) and (17) as follows:

$$\begin{aligned}VAR(DEC_{MTM}(f_i)/\mathcal{H}_0)\\ = C^2.VAR(|Y_k(f_i)|^2\\ /\mathcal{H}_0).\sum_{l=0}^{L-1}\Bigg(&\sum_{k=0}^{K-1}\lambda_k^2(N,W) + 2\lambda_0(N,W)\lambda_1(N,W)\\ &+ 2\lambda_0(N,W)\lambda_2(N,W) + \cdots\\ &+ 2\lambda_0(N,W)\lambda_{K-1}(N,W)\\ &+ 2\lambda_1(N,W)\lambda_2(N,W) + 2\lambda_1(N,W)\lambda_3(N,W)\\ &+ \cdots + 2\lambda_1(N,W)\lambda_{K-1}(N,W) + \cdots\\ &+ 2\lambda_{K-2}(N,W)\lambda_{K-1}(N,W)\Bigg)\end{aligned} \quad (19)$$

when $t = t'$, and since the variance of Gaussian random variable W, $VAR(W^2) = 2(VAR(W))^2$, then $VAR(|Y_k(f_i)|^2/\mathcal{H}_0)$, can be defined as follows:

$$\begin{aligned}VAR(|Y_k(f_i)|^2/\mathcal{H}_0)\\ = \left(\sum_{t=0}^{N-1}v_{(t,k)}^2(N,W)\right)^2.VAR((w_t^2(l))\\ = (2\sigma_w^4)\end{aligned} \quad (20)$$

Finally, $VAR(DEC_{MTM}(f_i)/\mathcal{H}_0)$ over L can be rewritten using (19) and (20) as follows:

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$$VAR(DEC_{MTM}(f_i)/\mathcal{H}_0) = 2C^2 L\lambda_\Sigma \sigma_w^4 \qquad (21)$$

where λ_Σ, is defined as follows:

$$\lambda_\Sigma = \sum_{k=0}^{K-1} \lambda_k^2(N,W) + 2\lambda_0(N,W)\lambda_1(N,W) + 2\lambda_0(N,W)\lambda_2(N,W) + \cdots + 2\lambda_0(N,W)\lambda_{K-1}(N,W) + 2\lambda_1(N,W)\lambda_2(N,W) + 2\lambda_1(N,W)\lambda_3(N,W) + \cdots + 2\lambda_1(N,W)\lambda_{K-1}(N,W) + \cdots + 2\lambda_{K-2}(N,W)\lambda_{K-1}(N,W) \qquad (22)$$

When the PR signal is present-for $\mathcal{H}_1$ case at frequency bin f_i, the variance of the decision statistic $VAR(DEC_{MTM}(f_i)/\mathcal{H}_1)$ is:

$$VAR(DEC_{MTM}(f_i)/\mathcal{H}_1) = C^2.L.\lambda_\Sigma.VAR(|Y_k(f_i)|^2/\mathcal{H}_1) \quad (23)$$

when $t = t'$, and since $VAR(s_t^2(l)) = 0$ in this case, and $VAR\big(2s_t(l)w_t(l)\big) = 4E_s\sigma_w^2$, then

$$VAR(|Y_k(f_i)|^2/\mathcal{H}_1) = VAR\,(s_t^2(l) + 2s_t(l)w_t(l) + w_t^2(l)) = (2\sigma_w^4 + 4E_s\sigma_w^2) \qquad (24)$$

Finally, (23) can be written as follows:

$$VAR(DEC_{MTM}(f_i)/\mathcal{H}_1) = C^2 L\lambda_\Sigma \left(2\sigma_w^4 + 4E_s\sigma_w^2\right) \qquad (25)$$

The probabilities formulae in (6), (7), and (8) can now be rewritten as follow:

$$P_d^{MTM}(f_i) = Q\left(\frac{\gamma - LK(E_s+\sigma_w^2)}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^2(\sigma_w^2+2E_s)}}\right) \qquad (26)$$

$$P_f^{MTM}(f_i) = Q\left(\frac{\gamma - LK\sigma_w^2}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^4}}\right) \qquad (27)$$

$$P_m^{MTM}(f_i) = 1 - Q\left(\frac{\gamma - LK(E_s+\sigma_w^2)}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^2(\sigma_w^2+2E_s)}}\right) \qquad (28)$$

It is clear that, the main processing difference between the energy detector and the MTM detector is simply multiplying the signal by a number of orthonormal tapers; the *DPSS* to produce a single estimate, while the multiplication in the energy detector is by a single rectangular taper. Thus in order to see the effect of this difference, we use the probabilities formulae of the energy detector which can be defined for the same system conditions as follow [13], [14]:

$$P_d^{ED}(f_i) = Q\left(\frac{\gamma - L(E_s+\sigma_w^2)}{\sqrt{2L\sigma_w^2(\sigma_w^2+2E_s)}}\right) \qquad (29)$$

$$P_f^{ED}(f_i) = Q\left(\frac{\gamma - L\sigma_w^2}{\sqrt{(2L\sigma_w^4)}}\right) \qquad (30)$$

$$P_m^{ED}(f_i) = 1 - Q\left(\frac{\gamma - L(E_s+\sigma_w^2)}{\sqrt{2L\sigma_w^2(\sigma_w^2+2E_s)}}\right) \qquad (31)$$

The number of OFDM blocks L, which is needed to achieve predefined probabilities of detection $P_d^{MTM}(f_i)$, and false alarm $P_f^{MTM}(f_i)$ in the MTM technique can be written using (26) and (27) to be as follows:

$$L = \left(\frac{\sqrt{2C^2\lambda_\Sigma\sigma_w^4}\,Q^{-1}\left(P_f^{MTM}(f_i)\right) - \sqrt{2C^2\lambda_\Sigma\sigma_w^2(\sigma_w^2+2E_s)}\,Q^{-1}\left(P_d^{MTM}(f_i)\right)}{KE_s}\right)^2 \qquad (32)$$

which can be written in (dB) to be as follows:

$$L(dB) = 10log_{10}(L)$$

The probabilities formulae when the PR's signal is modeled as Gaussian random process are listed in the appendix.

In multipath fading environment, the binary hypothesis test in (4) can be redefined for $\mathcal{H}_1$ to be as follows:

$$x_t(l) = \sum_{m=0}^{M-1} h_m s_{t-m}(l) + w_t(l) \qquad (33)$$

where the discrete channel impulse response between the PR's transmitter and CR's receiver is represented by h_m, $m = 0,1,\ldots,M-1$, and M is the total number of resolvable paths. The discrete frequency response of the channel is obtained by taking the N point FFT, with $N \geq M$ as follows [14]:

$$H(f_i) = \sum_{m=0}^{M-1} h_m e^{-j2\pi f_i m} \qquad (34)$$

In this case, the formulae in (26), and (28) can be written as follow:

$$P_d^{MTM}(f_i) = Q\left(\frac{\gamma - LK(|H(f_i)|^2E_s+\sigma_w^2)}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^2(\sigma_w^2+2|H(f_i)|^2E_s)}}\right) \qquad (35)$$

$$P_m^{MTM}(f_i) = 1 - Q\left(\frac{\gamma - LK(|H(f_i)|^2E_s+\sigma_w^2)}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^2(\sigma_w^2+2|H(f_i)|^2E_s)}}\right) \qquad (36)$$

The SNR can be redefined here to be as follows:

$$SNR = \frac{|H(f_i)|^2E_s}{\sigma_w^2} \qquad (37)$$

The same steps can be followed to rewrite the formulae (29), (30), and (31) for the energy detector case.

In this paper, we assume that the channel gain between the PR's transmitter and the CR's receiver is constant during the spectrum sensing duration, and $|H(f_i)|^2 = 1$. In practice, $|H(f_i)|^2$ can be estimated priori during the time that PR's transmitter occupies a specific band with specific power [14].

IV. SIMULATION RESULTS

We evaluate our theoretical work by running a simulation program where the PR's signal is QPSK with normalized energy equal to 1 over each subcarrier. Both-CR and PR users employ 64-IFFT/FFT digital signal processing in their communications with sampling frequency 20 MHz/ T_s=0.05µs, where T_s represents the symbol duration, the MTM parameters used are NW=4, and 5 tapers, and the results obtained over 1000000 realizations. Additionally, we compare the performance of MTM spectrum detector system to that of the energy detector under the same conditions. We used theoretical and simulation results for a chosen frequency bin at the CR FFT to examine the hypotheses $\mathcal{H}_1$, and $\mathcal{H}_0$.

Fig. 1 shows the probability of detection P_d versus probability of false alarm P_f using MTM detector with *NW*=4 and 5 tapers (simulation and theory) and the energy detector at AWGN with SNR=−10dB and $L = 20$ OFDM blocks. Note that, the total number of samples used is $(L = 20) \times (N = 64) = 1280$, which approximately corresponds to sensing time $(L = 20) \times (N = 64) \times (Ts = 0.05\mu s) = 64\mu s$. By comparing the theoretical to the simulation in the MTM case, we note that the theoretical results match well the simulation one. At the same system conditions, the probability of detection P_d of MTM outperforms that for energy detector by 30%, when the probability of false alarm is P_f=10%, and the miss detection P_m in MTM is lower than that in energy detector case by 40%.

Fig. 2 shows the theoretical results of the number of OFDM blocks (L) required to achieve $P_d = 99\%$, and $P_f = 1\%$ at AWGN environment with different SNR using MTM with *NW*=4 and 5 tapers compared to the energy detector. It is clear that the number of OFDM blocks used in the sensing process in the MTM system is lower than that for the energy detector. For example, at SNR=−15dB, the L required by the MTM is 33dB, and the energy detector is 47dB. These two values correspond to 1995 and 5012 OFDM blocks for MTM and the energy detector, respectively, in the linear scale. Thus, the energy detector requires 2.5 times as many samples compared to MTM in order to achieve the same probabilities at the same SNR. Such a large number of the samples for sensing in CR system might hinder the opportunistic use of the vacant channels, as it is the main objective of the developing of CR systems.

Fig. 3 shows the probabilities of detection P_d that gives probabilities of false alarm $P_f = 5\%$, and 10% versus the SNR at AWGN using MTM with *NW*=4 and 5 tapers and $L = 50$. For both of predefined probabilities of false alarm the probabilities of detection are almost 100% for SNR=−7dB or higher with unnoticeable change for $P_f = 10\%$ curve, which is reasonable. Both probabilities of detection curves start to decrease with the decrease in the SNR with noticeable outperforming of the $P_f = 10\%$ curve. At SNR=−25dB, $P_d = 11\%$ for $P_f = 10\%$ curve, and $P_d = 6\%$ for $P_f = 5\%$ curve.

Fig. 4 shows the threshold versus probabilities of false alarm and detection using MTM with *NW*=4 and 5 tapers at AWGN with SNR=−7dB (i.e., $\sigma_w^2 = 5.0119$) and $L = 50$. Such a figure presents the range of the threshold that should be chosen in order to meet specific probability of false alarm and detection at defined SNR level and L used in the spectrum sensing. As an example, for threshold=5, the probability of false alarm and detection pair (P_f, P_d) is $(50\%, 100\%)$. By increasing the threshold level to 5.3, the pair becomes $(10\%, 100\%)$. This figure can be revaluated at different SNR and L conditions using (26) and (27).

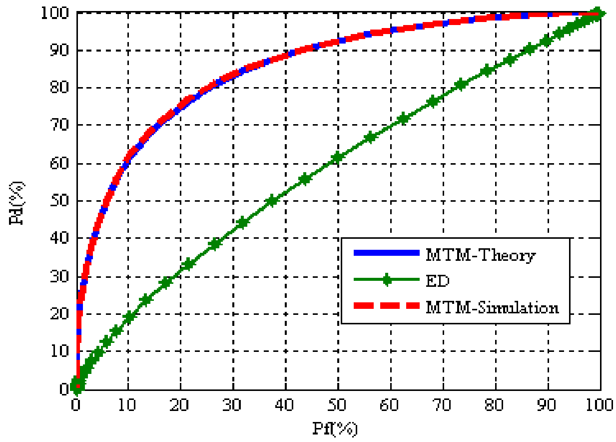

Fig. 1. Probability of detection versus probability of false alarm using MTM with NW=4 and 5 tapers (simulation and theory) and the periodogram (energy detector) at AWGN with SNR=−10dB and $L = 20$.

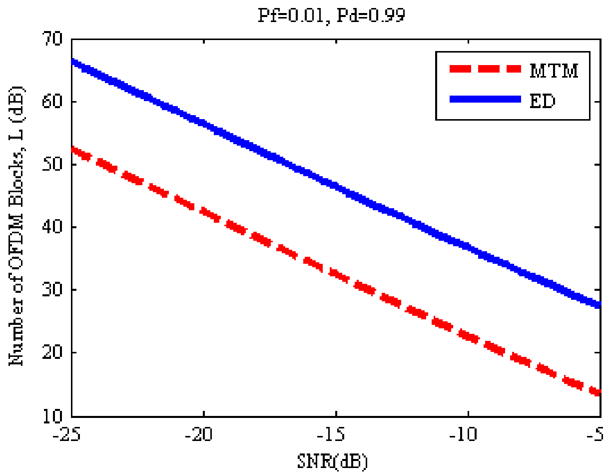

Fig. 2. Comparison between the number of OFDM blocks (L) required to achieve $P_d = 99\%$, and $P_f = 1\%$ at AWGN with different SNR using MTM with NW=4 and 5 tapers and the energy detector.

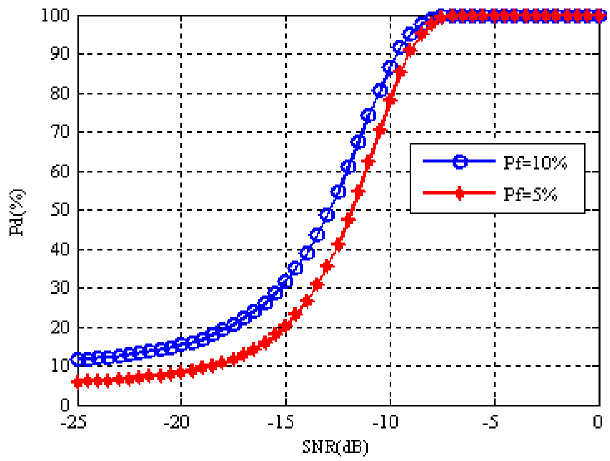

Fig. 3. Probability of detection that meets $P_f = 5$ and 10% versus the SNR at AWGN using MTM with *NW*=4 and 5 taper and $L = 50$ samples for spectrum sensing.

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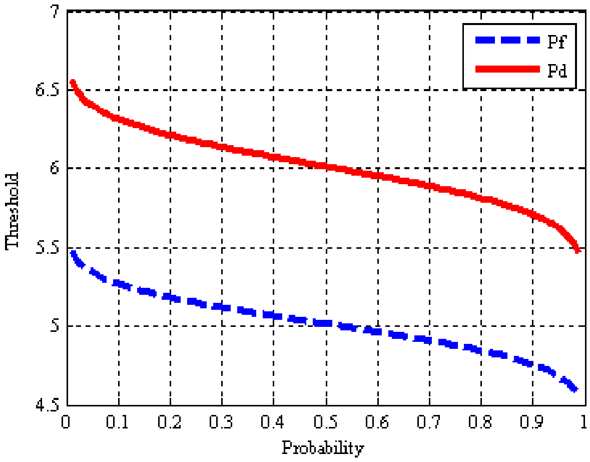

Fig. 4. Threshold versus probabilities of false alarm and detection using MTM with NW=4 and 5 tapers at AWGN with SNR=−7dB and $L = 50$ samples are used in the spectrum sensing.

V. CONCLUSION

In this paper, we have derived closed-form formulae for the probabilities of false alarm, detection, and miss detection as functions of the parameters of the MTM spectrum detector such as threshold, number of sensed blocks L, number of tapers, eigenvalues of the DPSS, PR signal power, and the noise power. These probabilities control the performance of the MTM-based spectrum sensing detector. Additionally, MTM probabilities can be used to choose the appropriate threshold that maximizes the probability of detection at fixed probability of false alarm.

In the process of the derivation, we defined the PDF of the MTM decision theory. Statistical parameters, such as the mean, and the variance of the distribution have been derived for different PR signals.

Comparing the performance of the MTM spectrum sensing detector to that for the energy detector, we found the MTM detector outperforms the performance of the energy detector by about 40% increase in the probability of detection at fixed probability of false alarm 10%. Furthermore the energy detector requires 2.5 times the number of samples to achieve the same probabilities of detection and false alarm given by the MTM detector operating at the same conditions.

APPENDIX

For the case when the PR's signal $s_t(l)$, is modeled as a random Gaussian variable with zero mean and variance σ_s^2 (i.e., $s_t(l) \sim \mathcal{CN}(0, \sigma_s^2)$) [15]. Following the same derivation steps of the modulated signal case, it can be proved that the probabilities formulae are defined as follow:

$$P_d^{MTM}(f_i) = Q\left(\frac{\gamma - LK(\sigma_w^2+\sigma_s^2)}{\sqrt{2LC^2\lambda_\Sigma(\sigma_w^2+\sigma_s^2)^2}}\right) \quad (38)$$

$$P_f^{MTM}(f_i) = Q\left(\frac{\gamma - LK\sigma_w^2}{\sqrt{2LC^2\lambda_\Sigma\sigma_w^4}}\right) \quad (39)$$

$$P_m^{MTM}(f_i) = 1 - Q\left(\frac{\gamma - LK(\sigma_w^2+\sigma_s^2)}{\sqrt{2LC^2\lambda_\Sigma(\sigma_w^2+\sigma_s^2)^2}}\right) \quad (40)$$

and the number of samples L (i.e., OFDM blocks) is defined as follows:

$$L = \left(\frac{\sqrt{2C^2\lambda_\Sigma\sigma_w^4}\,Q^{-1}\left(P_f^{MTM}(f_i)\right) - \sqrt{2C^2\lambda_\Sigma(\sigma_w^2+\sigma_s^2)^2}\,Q^{-1}\left(P_d^{MTM}(f_i)\right)}{K\sigma_s^2}\right)^2 \quad (41)$$

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