Addressing Prerequisites for STEM Classes Using an Example of Linear Algebra for a Course in Machine Learning

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Abstract—**While teaching Science, Technology, Engineering, and Mathematics (STEM) subjects, we frequently encounter situations where we have several prerequisites for a particular course. We anticipate that students will have different levels of knowledge in these prerequisites. A prerequisite (Linear algebra for Machine Learning course) was implemented as an interactive online course using Jupyter Notebooks and nbgrader. A preliminary survey shows a preference by students and instructors for this interactive implementation.**

Keywords- prerequisites; machine learning; linear algebra; interactive self-study course; Jupyter Notebooks.

I. INTRODUCTION

While teaching STEM subjects, we frequently encounter situations where we have several prerequisites for a particular course. We expect these students will have different levels of knowledge regarding these prerequisites. In most cases, a conceptual understanding and an ability to apply the prerequisite material are sufficient for most students. Students are not expected to know details, such as proofs, etc.

We encountered one such situation while teaching a Machine Learning (ML) course to first-year graduate students. An ML course relies on knowledge of linear algebra, multi-dimensional calculus and probability. The standard approach is to provide material for student self-study in addition to refresher material, so called crash course material given during the course. The advantage here is that students get at least the minimum amount of the required material, with an option for additional self-learning if desired.

We also encounter multiple disadvantages, however, with such an approach. For one, time needed for the main subject is spent on prerequisites. Review time for prerequisites should be limited as it is very challenging to cover necessary material at a sufficiently high level. While students have the option to self-study, learning with an instructor is significantly more effective and efficient. Another disadvantage: neither students nor instructors could verify whether the necessary level of understanding and application of prerequisite material had been achieved. This may be remedied with quizzes or tests, which in turn require additional precious instruction time.

To address these issues, we decided to use an available teaching technology: we would organize prerequisite material in the form of interactive online self-study. We used Jupyter Notebook [1] - based technology flexible enough to create an interactive course with proper mathematical typesetting as well as programming support (Python) in case we had to do modifications which we assumed should allow us to address these issues.

Thus, instead of providing generic self-study materials for prerequisites in the form of a book or pdf, we provided a concise Interactive Online self-study course that covers prerequisites and offers Concept Inventory (IOCI) based short tests, which evaluate students' understanding of the main concepts and their ability to apply the material. Hence, we precisely target the goals of the course prerequisites.

We implemented the course using iPython Notebook [2] software with additional course management support provided by the nbgrader plugin [16]. The course was developed on Amazon's c9 cloud and was available to students online. The course works in an automated or semi-automated way, allowing the instructor to see test results by topic or intervene and comment on student answers.

In our specific case, we started with linear algebra (LA) prerequisite material for the ML graduate course. We developed prerequisite self-study course material with CI-based tests. Students can return to topics already studied, advance upon completion of an appropriate test, or skip tests altogether and concentrate on study material alone.

Our course offers a two-part novelty: making prerequisite material in the form of interactive online course; incorporating quizzes and homework in the form of Concept Inventory (CI), which addresses only required for prerequisite understanding of concepts and notions and ability to apply the material in the main course context. To the best of our knowledge, such combinations were not used before. The course was also translated into Russian and deployed at two universities: St John's University (New York) and the National University of Science and Technology, MISIS (Moscow). It covered two experimental groups with a total of 30-plus students. According to the preliminary survey, both students and instructors prefer the interactive Jupyter Notebook-based study approach to the standard prerequisite classes.

This paper proceeds as follows. In Section II, we describe existing CIs and state-of-the-art Interactive Online Systems. In Section III, we proceed to a description of LA as a prerequisite material to the Machine Learning course. We

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show how CI addresses the requirement of the specific prerequisite material. In Section IV, we describe the cloud system used for the initial implementation of the course as well as hardware requirements for running a test experiment of about 200 software simulated test students. In Section V, we provide a preliminary (proof of concept) evaluation of our approach. We end our paper with a conclusion and discussion of future work.

II. STATE OF THE ART

The purpose of a prerequisite class differs from a "normal" class. It prepares a student for another class, not directly for a future career. Hence, it is often perceived as something less necessary. As observed in [10][13], students often see prerequisites as a waste of time and avoidable. If handled appropriately, a prerequisite course would solve motivational issues. One way to minimize time and resources spent is to make it self-paced so that a student goes through it at a comfortable pace and when time is available.

The first part of the outlined program – teaching only the material actually needed - is course-specific and should be addressed on case-by-case basis.

The second part about level and form of material taught, however, can be answered in general, at least for STEM classes.

A. *Notion of Concept Inventory*

While teaching STEM classes, as we observed in most cases, a conceptual understanding and an ability to apply the prerequisite material are sufficient. Students are not expected to know details, such as proofs, etc. The CI is the best existing approach to assessing conceptual understanding rather than memorization of a set of facts. CI as a form of an assessment is based on checking if a student understands basic concepts of a given subject as opposed to reciting a number of subject specific facts, equations, etc. As David Hestenes states in his paper, *Force Concept Inventory*, [17] CI Assessment is "not a test of intelligence" but rather, "it is a probe of belief systems".

An immediate advantage of CI is that it can be used for any student. That is, it does not matter, what the subject specific background of the student is, since, as stated above, CIs do not test formal knowledge but rather understanding of basic concepts. For example, as was demonstrated in [11], there is no significant difference observed between the test results even if the class time, class readiness, or type of class are different. That includes even classes that lack traditional lectures, such as Matematica-based classes. Typically, CIs are created and delivered as multiple-choice tests. However, as opposed to standard tests CIs are not comparison tests but norm-referenced tests.

The main goal of CIs, as stated above, is to test the students understanding of basic concepts. However, a typical CI test also checks for typical misconceptions.

There are two typical types of misconceptions: general scientific misconceptions and misconceptions introduced during the teaching process – so-called didaskalogenic misconceptions. The tool CIs use for testing misconceptions is known as distractors. Basically, distractors are the answer choices, which are specifically designed to imitate typical misconceptions. Summarizing, a CI test is a multiple-choice test consisting of problems with "distractors" as incorrect options that represent typical misconceptions. Typical multiple-choice problems of this type would be:

The following are temperatures for a week in August: 94, 93, 98, 101, 98, 96, and 93.
By how much could the highest temperature increase without changing the median?
A. Increase by 8°
B. Increase by 2°
C. It can increase by any amount
D. It cannot increase without changing the median.

To answer this question, a student needs nothing more than to understand the concept of median. Yet, at the same time, the problem does check for typical misconceptions, providing possible answers that conform to concepts of midrange or mean. Indeed, option D would be true if the question would be about midrange or mean, not about median and is, therefore, a typical example of a "distractor."

The first CI was developed and published by David Hestenes in 1992 [17]. It is known now as the Force Concept Inventory (FCI) and covers Newtonian Mechanics concepts. It had immediate success and was recognized and accepted by thousands of educators. Hestenes coined the term "modeling" to describe the conceptual approach to teaching – as opposed to the traditional factual approach. By now "modeling" approach covers well over 100,000 students each year. As a result of CI's popularity, the American Modeling Teachers Association (AMTA) was created and grew into a nationwide community. Moreover, CIs began in various fields of engineering, science and mathematics.

CI assessment in introductory and prerequisite classes was studied, in particular in [8][9][12][20][22]. With CI the subject specific background of a given student is not significant as stated above because CIs do not test formal knowledge, but rather test the student's understanding of related concepts, which is the student's working knowledge.

An understanding of related concepts is exactly what is needed in prerequisite classes. Mastering prerequisite material at a working knowledge level in order to apply it to the upcoming class.

Another advantage of using Cis in that they are already developed for a wide variety of the subjects including, but not limited to:

1) Natural Sciences:
    a) Physics
        i) Force and Motion
        ii) Electricity and Magnetism
        iii) Statics
    b) Chemistry
    c) Geoscience

2) Engineering
    a) Material Sciences
    b) Fluid Mechanics

3) Life Sciences:
    a) Basic Biology
    b) Natural Selection
    c) Genetics

4) Mathematics & Statistics:
    a) Calculus
    b) Statistics

Therefore, there already exist large depositories of test problems for many subjects in case a need to create a prerequisite class for one of such subjects.

The last aspect – the interactive, self-paced form of the class – can be addressed only through the use of technology.

B. Existing Interactive Online Systems

By now numerous Interactive Online Systems exist, including ALEKS [24], Cengage WebAssign [25], Knewton [26], Pearson MyMathLab Study Plan [27], Acrobatiq [28], Adapt [29], etc. All these systems offer self-paced automatically graded classes for various subjects. Typically, each such class offers an Initial Assessment and then, based on the output each student gets, activities and learning material to work on with regular re-assessments to check on progress. Such re-assessment outputs, in turn, are again used to adjust the assigned activities and learning material.

For instance, ALEKS provides the following self-description: “ALEKS uses adaptive questioning to quickly and accurately determine exactly what a student knows and doesn't know in a course. ALEKS then instructs the student on the topics she is most ready to learn. As a student works through a course, ALEKS periodically reassesses the student to ensure that topics learned are also retained. ALEKS courses are very complete in their topic coverage and ALEKS avoids multiple-choice questions. A student who shows a high level of mastery of an ALEKS course will be successful in the actual course she is taking.”

According to [18], “When asked if there are pieces of the traditional classroom setting that are lost in an online course, the overwhelming response by all recipients was the lack of professor to student and student to student interaction and communication.”

However, the classes based on such systems have several advantages over traditional classes. Such advantages include flexibility, adjustability to a student’s knowledge base, pace, availability of various learning tools, timely feedback, etc. And as stated in [18], “All respondents unanimously answered that they would take an online course in the future, regardless of the challenges that they may have experienced.”

The largest summary of online vs. classroom comparison research [19] concludes that “students in online conditions performed modestly better, on average, than those learning the same material through traditional face-to-face instruction. Learning outcomes for students who engaged in online learning exceeded those of students receiving face-to-face instruction, with an average effect size of +0.20 favoring online conditions.”

At the same time, the same source states that “instruction combining online and face-to-face elements had a larger advantage relative to purely face-to-face instruction than did purely online instruction. The mean effect size in studies comparing blended with face-to-face instruction was +0.35, $p < .001$.” The existing systems, however, all emulate traditional classes in terms of curricula and syllabi. The only difference is the form in which the material and assessment are presented.

On the one hand, it makes the comparison quoted above reliable since there is an objective expected output for each curriculum – and the only difference is the form of presenting the material. Indeed, according to the study itself “analysts examined the characteristics of the studies in the meta-analysis to ascertain whether features of the studies’ methodologies could account for obtained effects. Six methodological variables were tested as potential moderators: (a) sample size, (b) type of knowledge tested, (c) strength of study design, (d) unit of assignment to condition, (e) instructor equivalence across conditions, and (f) equivalence of curriculum and instructional approach across conditions. Only equivalence of curriculum and instruction emerged as a significant moderator variable ($Q = 6.85$, $p < .01$).”

On the other hand, simply emulating the existing traditional classes does not allow the online interactive form to use completely its intrinsic advantages. We do believe that prerequisite classes can benefit more from advantages that the online interactive form offers.

While a variety of platforms exist for creating online accessible interactive classes, Jupyter Notebook looks to be one of the best fits here. Jupyter Notebook makes it easy to start, further develop, and support a class. It is also quite easy to create interactive auto-graded assignments using Jupyter Notebook.

As stated in [1], “Project Jupyter is three things: a collection of standards, a community, and a set of software tools. Jupyter Notebook, one part of Jupyter, is software that creates a Jupyter Notebook. A Jupyter Notebook is a document that supports mixing executable code, equations, visualizations, and narrative text. Specifically, Jupyter Notebooks allow the user to bring together data, code, and prose, to tell an interactive, computational story. Whether analyzing a corpus of American Literature, creating music and art, or illustrating the engineering concepts behind Digital Signal Processing, the notebooks can combine explanations traditionally found in textbooks with the interactivity of an application."

To summarize, Jupyter Notebook allows putting together a comprehensive custom-tailored text using both newly written lectures and excerpts from existing textbooks while also supplementing the text with interactive auto-graded assignments.

Putting these three aspects together facilitates the creation of prerequisite classes that cover only the material really needed and taught in a conceptual form, assessed using the CI approach and put in a form of a self-paced interactive online class using Jupyter Notebook, or a similar platform.

III. LINEAR ALGEBRA AS A PREREQUISITE COURSE FOR MACHINE LEARNING

The LA prerequisite class for Machine Learning class is an online interactive self-paced class built on the Jupyter Notebook platform. The lectures are based on "Linear Algebra Review and Reference" by Zico Kolter and consist of four chapters:

1. Basic Concepts and Notation
2. Matrix Multiplication
3. Operations and Properties
4. Matrix Calculus

The material presents basic definitions and concepts of LA necessary for studying Machine Learning. Each chapter is divided into smaller sections. For example, the "Matrix Multiplication" chapter is divided as follows:

2.1 Vector-Vector Products

2.2 Matrix-Vector Products

2.3 Matrix-Matrix Products

Each section is supplemented by an auto-graded assessment based on CI principles.

A typical problem for Basic Concepts would be:

Find the dimensions of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

A. 2x3 (*)

B. 3x2

C. 1x6

D. 6x1

Option A is a key since the matrix has two rows and three columns.

Option B is a distractor that checks for a misconception that mixes rows with columns.

Option C is a distractor that checks for a misconception that considers a matrix as one long row with six elements.

Option D is a distractor that checks for a misconception that considers a matrix as one long column with six elements.

Another typical example:

Matrix

$$\begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$$

has eigenvalues:

A. -1 and 0

B. -1 and 2 (*)

C. 0 and 2

D. It has no eigenvalues

Option B is a key since (-1-x)(2-x)-0·0=0 has two roots:

-1 and 2.

Option A is a distractor that checks for a misconception that defines the eigenvalues as the values of the first row elements.

Option C is a distractor that checks for a misconception that defines the eigenvalues as the values of second row elements.

Option D is a distractor that checks for a misconception that defines a characteristics polynomial as -1·2-(0-x) (0-x).

In the final version assessments will be based on a sufficiently large pool of problems and will be randomly generated for each student and for each attempt.

A student is able to take this class any time before taking the Machine Learning class, at the pace that fits her or his schedule and degree of prior knowledge. In addition to the lectures, we include the option of having students ask the instructor questions or discussing any aspect of the class

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with other classmates. Each assessment is auto graded but also can be graded by the instructor in case a student challenges the grade.

IV. System implementation

The system was initially implemented on Cloud 9 (currently Amazon c9) virtual machines with 20 Gb. hard-drive and 2 Gb RAM running Ubuntu v. 14, with Python 3.6, miniconda and installation of JupyterHub with nbgrader.

Installation was almost straightforward, the only issue being restriction on use of miniconda instead of full anaconda installation. This is due to restriction of the provided hard-drive size. The main benefit of the system was its low cost: VMs are available for free from AWS. We would like to thank Amazon for providing Cloud based virtual hardware. This essentially made our work possible.

While sufficient for development, the system nonetheless had performance issues. Thus, we had a choice either to proceed to paid Cloud based virtual machines or moved to dedicated home hosted hardware. Our choice was to move the developed system to a Lenovo P-520C workstation with Intel Xeon 6 core W-2133 Processor with vPro, 32 Gb. of RAM with dual hard-drive 512 Gb SSD and 2 Tb. HDD and 2 GB Nvidia P2000. This PC configuration proved to be sufficient to run up to 200 test students. We did not try IOCI to stress the system to run for more students.

V. Evaluation of the approach

We evaluated standard and interactive approaches by running parallel classes for over 30 graduate students taking the Machine Learning course. Half of the students studied the LA prerequisite material in the form of provided reading material. Another half used the interactive Jupyter/nbgrader online system, with a built-in auto-graded CI based tests provided for both self and regular assessment. We ran pre- and post- preparation CI-based tests that check the required comprehension of the LA material as well as a one-question survey for both instructors and students. The survey seeks to discover if the student/instructor prefers reading material or an interactive prerequisite course. An outline of the measurements approach may be found in [19]-[21][23].

Both classes offered a sample that shows prerequisite materials used by their counterparts. Both tests and survey showed a statistically significant preference of interactive prerequisite materials for students with 5% significance level.

Tests results analysis is summarized in Table 1 and uses standard t – test with a different standard deviation for testing if one of the means is larger than the other. The value of the test t shows statistical significance with a confidence level of $\alpha = 5\%$. Here the value *df* is degree of freedom, *d* is value of statistics, *t* is value of t-test corresponding values *d* and *df*.

TABLE I. ONE SIDED TWO MEANS T-TEST FOR GRADES IOCI VS READING

	IOCI	Read
N	16	15
mean	88	84
std	6	6.75

df (degree of freedom)	28.05503
d (see formula (1))	1.739542
t	0.046462

Survey preference is analyzed in Table 2 using small samples t-test for population proportion, see [14][15]. A summary of analysis is offered below in the Table 2. Here, the value of *N-2* is the degree of freedom, the value *d* is calculated as [14][15]:

$$d = (ae - bc)\left(\frac{N-2}{N(nac+mbe)}\right)^{\frac{1}{2}} \tag{1}$$

and values of the variables *a, e, b, c, N, n, m* used in the formula are the corresponding ones in the numerical data below.

TABLE II. SMALL SAMPLES T- TEST FOR POPULATION PROPORTIONS COMPARISON

	IOCI Users	Read Users	Total
Prefer IOCI	a = 14	b = 8	s = 22
Prefer Read	c = 2	e = 7	f = 9
Total	m = 16	n = 15	N = 31

N-2	29
d	2.186271331
t	0.018506791

A similar implementation with similar results (translation of the material into Russian) was done at the National University of Science and Technology, MISIS (Moscow).

VI. Conclusion and future work

The issue of prerequisites impacts many STEM courses because many major courses require a deep understanding of Mathematics, Statistics, etc. This may be challenging in situations where graduate students wish to enroll in major courses at the start of their studies. We encountered such a situation with Machine Learning courses, which require knowledge (or at least a conceptual understanding and hands-on ability) of LA, Matrix Calculus, Probability and Statistics. Standard approaches require that students wait a

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year during which they complete all prerequisites or attack prerequisites as reading material. As the latter approach has several disadvantages, we decided to make prerequisite material more attractive by implementing it using JupyterHub and nbgrader as a self-study interactive course with auto-grading. CIs are used to check how well students understand the material. Students have access to self-check exercises and feedback; instructors can monitor student' success and, if needed, recommend some adjustments. We ran it on an experimental group of students, and both students and instructors prefer this form of study over reading material.

We would like to emphasize that this approach can by no means compare in depth and outcome to regular courses on the topic. As we saw in multiple cases, this approach is used mainly because of students schedule conflicts or a desire to expose students to major courses as soon as possible.

We plan to run the LA prerequisite course by larger numbers of instructors and students and incorporate comments and suggestions from all participants. We further intend to offer the course as open source available to anyone.

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