A Knowledge Extraction from Epidemic Control Simulation

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***Abstract*—In a social simulation of infection control policies using an epidemic model, it is necessary to consider the latent perspectives of stakeholders focusing on various costs or effects. Therefore, we classify the simulation results produced by various combinations of control policies from the perspective of latent evaluation. In this paper, we use the data envelopment analysis (DEA) method to classify the results of epidemic control simulations. DEA is an analytic method that measures and compares the efficiency of multiple input multiple output systems' performance data. From various latent evaluation perspectives, DEA classification allows us to endogenously extract knowledge about the superiority of each control policy over the others from various latent evaluation perspectives. This approach is a good example of classification and knowledge extraction for a set of social simulation logs.**

Keywords- epidemic model; Data Envelopment Analysis; DEA classification

I. INTRODUCTION

It is impractical to evaluate the results of a simulation using only a specific objective function when simulating the control of a society with various stakeholders. In this paper, we use an infectious disease model as an example to classify a set of simulation output logs without assuming a specific objective function. Furthermore, we extract the knowledge about which output logs are dominant over others under which the objective function, and which logs are not dominant under any objective function.

Since Kermack-McKendrick's proposal of the proposal of the Susceptible-Infectious-Recovered (SIR) model [1][2], there have been numerous studies on epidemic modeling and simulations describing the spread of infectious diseases in society. The study of infectious disease control using those epidemiology models has roughly directed two mutually related directions. One direction is theoretical research, which refines the social structure and diffusion process of infectious diseases and discovers ways to control them. The other direction is empirical research, which uses data on infectious diseases incidence to estimate key parameters for the spread and control of infectious diseases.

Many studies that use epidemic models focus on the number of infected people as the primal state variable and effective reproduction number as a key parameter. However, for practical research, we must consider more state variables as the model sophistication. We should also consider various control variables such as policy costs for infection control and economic side effects. For these multiple variables, various stakeholders may have their objective functions with different weights from different standpoints.perspectives. Our problem is to extract the knowledge related to the superiority or inferiority of simulation results for the various possible objective functions.

This study does not target for a specific infection, such as Corona-virus Disease (COVID). However, this methodology is open to study into the actual control of current and future infectious diseases.

Our approach is as follows. In Section II, we briefly outlook data envelopment analysis (DEA), which is a method for analyzing the efficiency of multiple inputs and outputs systems, to classify the results of epidemic simulation runs. In Section III, considering that the epidemic control simulation has multiple control and state variables, we consider each run of the simulation as a multiple inputs and outputs system. In Section IV, DEA allows us to extract knowledge about which simulation runs are dominant over others under what objective function, as well as which set of runs is dominated by which dominant run.

II. DATA ENVELOPMENT ANALYSIS

DEA [3][4], proposed by Charnes, Cooper, and Rhodes in 1978, is a method for efficiency comparison among decision making units (DMUs) that have multiple inputs and multiple outputs. In DEA, each DMU is characterized by a reference set consisting of DMUs with greater optimal efficiency. The envelopment generated by connecting the reference sets allows us to compare the relative positions of DMUs.

A. Concept of Data Envelopment Analysis

Charnes Cooper and Rhodes (CCR) method [3] is the most basic DEA method, its concept is as follows: a) weighted efficiency of DMUs with multiple inputs and

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outputs, b) efficiency comparison among DMUs through optimization of the evaluative weights of the various DMUs, and c) characterization by a reference set, which is a collection of DMUs with greater optimal efficiency than the DMUs under consideration.

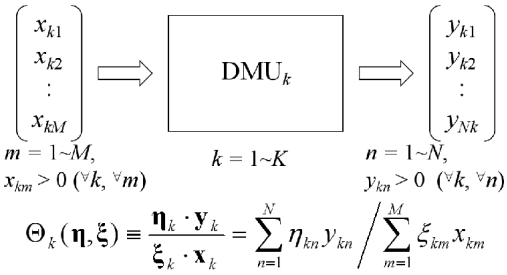

Figure 1. DMUs and efficiency Θ

The first step is to use a reference set to characterize the DMUs. First, we assume that each DMU$_k$ (k = 1~K) of the K DMUs generates an N-component output $\mathbf{y}_k$ $\{y_{kn} > 0 \mid n = 1\sim N\}$ from an M-component input $\mathbf{x}_k = \{x_{km} > 0 \mid m = 1\sim M\}$. At this point, the efficiency Θ_k of each DMU$_k$ can be defined as $\Theta_k(\boldsymbol{\eta}_k, \boldsymbol{\xi}_k) \equiv (\boldsymbol{\eta}_k \cdot \mathbf{y}_k) / (\boldsymbol{\xi}_k \cdot \mathbf{x}_k) = \sum_{n,m}\{(\eta_{kn} y_{kn}) / (\xi_{km} x_{km})\}$ with the evaluative weights $\boldsymbol{\xi}_k = \{\xi_{km} > 0 \ (m = 1\sim M)\}$ and $\boldsymbol{\eta}_k$ ($\eta_{kn} > 0$ (n = 1~N)) attached to the input $\mathbf{x}_k$ and output $\mathbf{y}_k$ (see Figure 1).

In this case, the evaluative weights $\boldsymbol{\xi}_k$ and $\boldsymbol{\eta}_k$ can be moved as much as desired, changing the efficiency Θ_k. The evaluative weights $\boldsymbol{\xi}_k$ and $\boldsymbol{\eta}_k$ are optimized so that the efficiency Θ_k of each DMU$_k$ is the most advantageous while maximizing the efficiency across all DMUs ($\Theta_k \leq 1$). However, the key point is optimizing the evaluative weights $\boldsymbol{\xi}_k$ and $\boldsymbol{\eta}_k$ so that no efficiency of other DMU$_h$ surpasses a value of 1 ($\Theta_h \leq 1$). Therefore, DEA is formulated as the fractional programming problem shown below (see Figure 2).

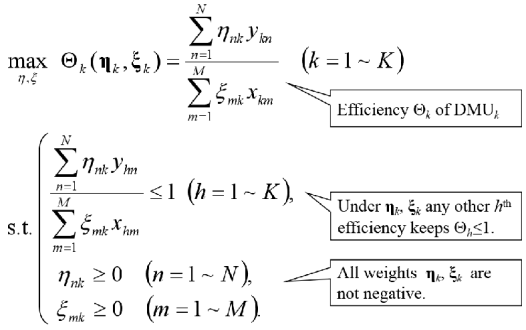

Figure 2. Optimization problem to find the efficiency Θ_k of the k^{th} DMU

If no optimization of the evaluative weights can bring the efficiency Θ_k to 1 and it is less than the $\Theta_{k'}$ values of other DMU$_k$s, then DMUk is inefficient. In this case, the DMU$_{k'}$ whose optimal efficiency is 1, is referred to as reference set E_k of DMU$_k$, and it can be used as a reference for improving the efficiency of DMU$_k$ (see Figure 3).

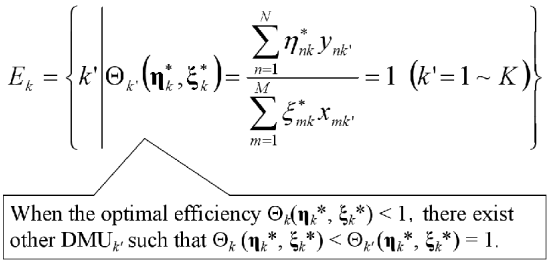

Figure 3. Reference set E_k of DMU$_k$.

In Figure 2, the DEA optimization is formulated as a fractional programming problem. In practice, it can be solved by converting it into the equivalent linear programming problem shown in Figure 4.

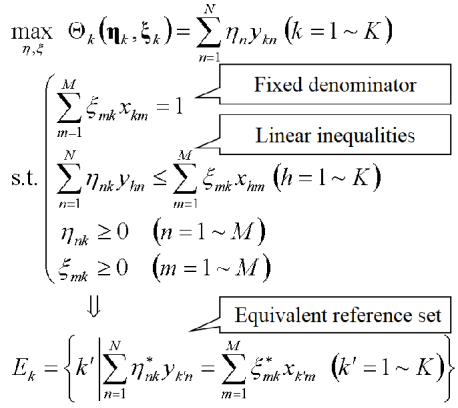

Figure 4. The equivalent forms of the optimization (Figure 2) and the reference set (Figure 3) as linear programming problem

One of the advantages of using DEA is that its optimization process (described above) automatically solves the problem of determining weights when measuring the efficiency of a multiple inputs multiple-outputs system.

In addition to empirical research on organizational efficiency, DEA is applied to simulation research for optimization of production systems, etc. [5]-[8].

B. Classification by Data Envelopment Analysis

DEA is also used for data classification.[9] As a data classification methodology, DEA has a different approach than other distance-based methodologies such as cluster analysis.

In DEA, the reference set E_k of a given DMU$_k$ is an object of reference for improving DMU$_k$, the reference set of an efficient DMU is itself. If we draw a scatter plot of DMUs in the space of input-output variables, the envelopes of the DMUs are shaped by all efficient DMUs. The minimal convex region containing the efficient DMU and the origin also contains inefficient DMUs whose reference set is the efficient DMU. Therefore, this set of DMUs can be classified as sharing the same reference set (see Figure 5).

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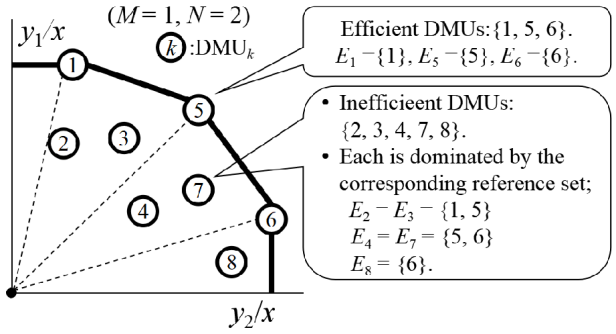

Figure 5. Dividing the envelope by efficient DMUs (reference set) allows classification of inefficient DMUs by common reference sets.

DMUs with the same reference set have similar weighting to improve efficiency. Those weightings are generated by DEA. Thus, the advantage of using DEA is that it allows for endogenous knowledge extraction on the relationship between the DMUs dominating the others in the group (the reference set and the optimal weights) and other subordinated DMUs.

Another advantage of using DEA as a classifier is that it is simple to consider group similarity. The number of overlapping elements of two reference sets indicates the similarity between those groups. The more common reference sets are shared between two groups, the closer they are.

DEA is applicable to data classification in fields other than the social sciences. For example, Hoshino [10] attempts to apply DEA to the classification of gene expression data.

III. EPIDEMIC CONTROL SIMULATION

Here, we present a simple epidemic control simulation to demonstrate knowledge extraction using DEA. As an epidemic model, we use the SIR model for mortal infection by Kermack-McKendrick [2]. Furthermore, we add input variables for infection control and medical care, to implement multiple-input multiple-output decision-making.

A. Epidemic Control Model

As an example, the epidemic control model used is a system of difference equations (1)–(2). Figure 6 shows the state transitions. Equation (3) shows the effects of social infection suppression and medical care on patient survival.

Equation (1) shows the time variation of the state variable, whereas (2) shows the initial values of the dependent and state variables. Equation (3) shows the time variation of the coefficients due to the control variable.

The state variables are $S(t)$ for the uninfected susceptible population, $I(t)$ for the infected population, $R(t)$ for the recovered non-susceptible population, and $D(t)$ for the deceased population, where the dependent variable $P(t)$ is the surviving population. The infection rate c, healing rate h, and mortality rate m are variables that can be changed by the control variables $u(t)$, $v(t)$, and $w(t)$. When there is no control input, the positive constants c_0, h_0, and m_0 denote the values of $c(t)$, $h(t)$, and $m(t)$, respectively.

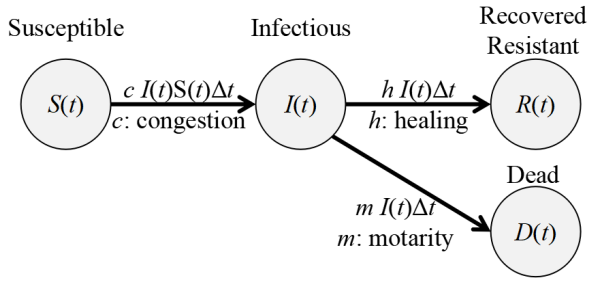

Figure 6. State transitions of the epidemic model

$$\begin{cases} S(t+1) = S(t) - c(t)S(t)I(t), \\ I(t+1) = I(t) + c(t)S(t)I(t) - h(t)I(t) - m(t)I(t), \\ R(t+1) = R(t) + h(t)I(t), \\ D(t+1) = D(t) + m(t)I(t). \end{cases} \quad (1)$$

$$\begin{cases} P(t) = S(t) + I(t) + R(t), \quad \text{population alive}, \\ P(0) = P_0, \quad \text{initial population}, \\ S(0) = P_0 - \varepsilon, \quad I(0) = \varepsilon, , \; R(0) = 0, \; D(0) = 0. \end{cases} \quad (2)$$

$$\begin{cases} c(t) = \dfrac{c_0}{1 + \alpha_u \cdot u(t)}, & : \text{contagion}, \\ h(t) = h_0(1 + \alpha_v \cdot v(t)), & : \text{healing}, \quad \begin{cases} u(t), v(t), w(t) \ge 0 \\ u(t) + v(t) + w(t) \le 1 \end{cases} \\ m(t) = \dfrac{m_0}{1 + \alpha_w \cdot w(t)}, & : \text{mortality}. \end{cases} \quad (3)$$

Among the control variable $u(t)$, $v(t)$, and $w(t)$, $u(t)$ represents control measures to suppress the infection rate $c(t)$. With increasing $u(t)$, $c(t)$ asymptotically decreases. The positive constants α_u represents the sensitivities of $u(t)$ to $c(t)$. The larger α_u, $c(t)$ steeper decreases with respect to $u(t)$.

The variable $v(t)$ represents control measures to improve the healing rate $h(t)$, which is an increasing function of $v(t)$. The positive constants α_v is the sensitivities of $v(t)$ to $h(t)$. The larger α_v means steeper increasing $h(t)$ with respect to $v(t)$.

Furthermore, $w(t)$ represents control measures to suppress the mortality rate $m(t)$. With increasing $w(t)$, $m(t)$ asymptotically decreases. The positive sensitivity constants α_w means that the larger α_w changes $m(t)$ steeper with respect to $w(t)$.

Although the costs and effects of these measures differ, the differences are expressed only in terms of the sensitivities α_u, α_v, and α_w without losing generality by standardizing on costs, according to the cost constraint $u(t) + v(t) + w(t) \le 1$.

B. Epidemic Control Simulation

The model is then run to obtain simulation data for epidemic control. Table 1 shows the common initial conditions and coefficients for the simulation run. Table 2 shows the simulation execution (RUN) settings following the 10 different control input assignments.

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TABLE I. COMMON SIMULATION SETTING

Common Simulation Setting				
Initial Conditions	$S(0)$: Susceptibles	$I(0) = \varepsilon$: Infectives	$R(0)$: Recovered	$D(0)$: Dead
	9990	10	0	0
	$P(0)$: Population Alive =10000			
Coefficients	c_0 Contagion	h_0 Healing	m_0 Mortality	
	0.00002	0.01	0.01	
Control Sensitivities	α_u	α_v	α_w	
	10	15	20	

TABLE II. SIMULATION RUN SETTING(CONTROL VARIABLES)

Simulation Run Setting													
Time stage		***stage*-0** t = 0~29			***stage*-1** t = 30~89			***stage*-2** t = 90~179			***stage*-3** t = 180~500		
Control variables		$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$	$u(t)$	$v(t)$	$w(t)$
RUN # (DMU #)	0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.4	0.3	0.3
	1				1.0	0.0	0.0	1.0	0.0	0.0			
	2				1.0	0.0	0.0	0.0	1.0	0.0			
	3				1.0	0.0	0.0	0.0	0.0	1.0			
	4				0.0	1.0	0.0	1.0	0.0	0.0			
	5				0.0	1.0	0.0	0.0	1.0	0.0			
	6				0.0	1.0	0.0	0.0	0.0	1.0			
	7				0.0	0.0	1.0	1.0	0.0	0.0			
	8				0.0	0.0	1.0	0.0	1.0	0.0			
	9				0.0	0.0	1.0	0.0	0.0	1.0			

The simulation summary values of the execution results obtained from the above settings are shown in Table 3. The summary values were used because the number of runs in this example was small. Note that DEA requires that the output variables be positive, and the larger output variables, the better the characteristics.

Two control inputs and three outputs are used as summary variables. The two control inputs are Cs and Cm, and the three outputs are BtmActP, AvgActP, and AlvPE. Here, Cs denotes the time average of the contact infection control measure and its sensitivity ($u \cdot \alpha_u$) for population $P(t)$, which means the social cost of measures (such as lockdown or vaccination) to reduce the contact infection rate $c(t)$. Cm denotes the time average of cures and their sensitivity plus life-saving measures and their sensitivity ($v \cdot \alpha_v + w \cdot \alpha_w$) for the infected population $I(t)$. It represents the medical cost of promoting recovery and reducing deaths.

Furthermore, BtmActP is the minimum value of the sum of the uninfected population $S(t)$ and the recovered population $R(t)$. It represents the depth of the trough of social and economic activities. Furthermore, AvgActP is the time average of the active population ($P(t) - I(t)$). It represents the average of social and economic activities over the period. Finally, AlvPE is the minimum value of the population $P(t)$. It represents the population that survived in the end.

TABLE III. SIMULATION RESULTS (SUMMARY VALUE DATA)

		Simulation Result (Summary Values)				
		Input:		***OutPut:***		
		Cs	Cm	BtmActP	AvgActP	AlvPE
		Social Cost:	Medical Cost:	Bottom Active Population:	Average Active Population:	Alive Poplation at the end:
		Avg[P(t)·αu·u(t)]	Avg[I(t)(αv·v(t)+αw·w(t))]	Min[P(t)-I(t)]	Avg[P(t)-I(t)]	Min[P(t)]
Run # (DMU #)	0	13,574	231	1,915	5,112	5,296
	1	49,693	340	8,176	8,590	8,638
	2	34,737	617	8,371	9,011	9,072
	3	34,481	19,220	2,514	7,974	8,967
	4	41,879	912	8,650	9,560	9,603
	5	24,652	919	8,650	9,575	9,619
	6	24,328	6,776	6,505	9,147	9,488
	7	33,203	15,244	1,906	6,589	7,235
	8	24,088	15,932	1,906	8,663	9,399
	9	24,418	29,615	1,906	8,082	9,524

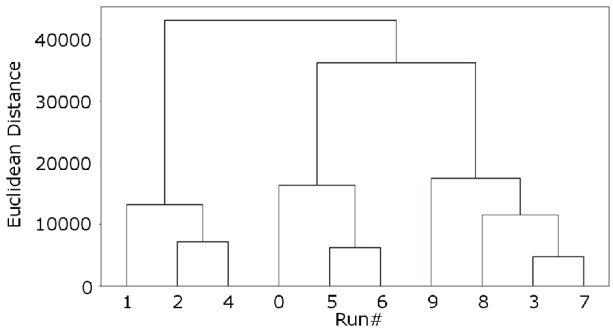

Figure 7. Clustering of input–output summary values

C. *Classification by Clustering*

Before applying DEA, the simulation run was classified by clustering. The results of hierarchical clustering of the simulation run shown in Table 3 using the Ward method with Euclidean metric are shown in Figure 7. Intuitively, as shown, the runs with similar summary value data are clustered together.

Nowadays, cluster analysis is a commonly used and convenient method, and it is also used to classify results of simulation runs [11][12]. However, when defining the distances between data with multiple variables in cluster analysis, we require exogenous weights on the variables. Furthermore, the relationship between data in the same cluster is explained only by the distance between them.

In empirical research, DEA and cluster analysis are compared in studies of the efficiency of educational systems [13].

IV. KNOWLEDGE EXTRACTION BY DATA ENVELOPMENT ANALYSIS

There are thousands of studies that use DEA. Some of those DEA studies include optimization with simulation models and clustering of data. However, in research on epidemics and infectious diseases, the application of DEA is limited to several empirical studies [14]-[20].

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TABLE IV. DEA RESULTS FOR THE SUMMARY VALUES IN TABLE III

DEA_CCR		DEA Score Θ	Reference Set	Input Weights ξ		Output Weights η		
				ξ Cs	ξ Cm	η BtmActP	η AvgActP	η AlvPE
Run # (DMU #)	0	1.0	{0, 1, 2}	0.4	22.4	1.8	0	1.2
	1	1.0	{1}	0	29.4	1.2	0	0
	2	1.0	{1, 2}	0.1	8.4	1.2	0	0
	3	0.6665	{8, 5}	0.3	0	0	0	0.7
	4	0.7903	{0, 2, 5}	0.1	5.3	0.5	0.4	0
	5	1.0	{5}	0.4	0	1.2	0	0
	6	0.9995	{8, 5}	0.4	0	0	0	1.1
	7	0.5584	{8, 5}	0.3	0	0	0	0.8
	8	1.0	{8}	0.4	0	0	0	1.1
	9	0.9997	{8}	0.4	0	0	0	1.0

Using DEA, we attempt to extract knowledge from the results of the previous section's epidemic control simulation runs. The knowledge to be extracted here is, first, the classification, reflecting the characteristics of the simulation run results, and second, the information on which run results are better than others under what evaluative weights.

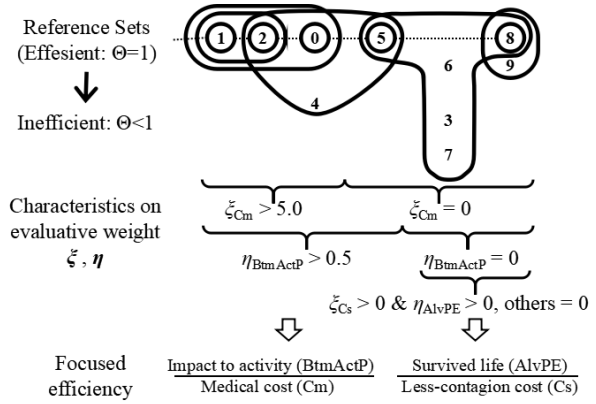

Figure 8. Classification and weight characterization by DEA

Table 4 shows the optimal efficiency Θ (DEA Score), reference set (reference set), and optimized weights for the input and output summary values (Input Weight, Output Weight) obtained by DEA (CCR method) from the simulation input-output summary value data (Table 3). For calculation of the CCR method, we used the Python code by Namiki [21].

The DEA results (Table 4) show that runs {0, 1, 2, 5, 8} are efficient (Θ = 1) while runs {3, 6, 7} are inefficient (Θ < 1). Because those inefficient runs share runs {5} and {8} as common reference sets, the weights ξ, η tend to be close in value.

As for the efficient runs {0, 1, 2, 5, 8}, runs 8 and 5 serve as reference sets for other classification groups. Runs 0, 1, and 2 serve as reference sets for one another, indicating that there is little difference between them. However, a large difference is evident between runs {8} and runs {0} and {1}, as no common reference set shares between them.

Evaluative weight ξ, η characterize that these reference sets are efficient under what posture. The runs {0, 1, 2} are efficient under the posture that focuses on the lowest number of the uninfected population divided by the medical cost. However, run {8} is efficient under the posture that focuses on the number of finally survived population divided by the infection prevention cost.

These results are summarized in Figure 8. The execution results are grouped according to their shared reference set. This also reveals which data can be used as a reference to improve group efficiency within the group.

This efficiency also characterizes the groups in terms of what posture is focused on. Thus, by endogenously determining these evaluative weights without prior knowledge, DEA enables us to extract knowledge of latent evaluative weights and the superiority or inferiority of simulation results under the weights.

V. CONCLUSION

In this paper, we used DEA to extract knowledge from the results of epidemic control simulation. The DEA classified epidemic simulation results using a method other than cluster analysis. Furthermore, DEA endogenously discovered evaluative weights that define the efficiency of simulation results and shows the potential evaluative weights and superiority of epidemic simulation results based on them. This method was a good example of classification and knowledge extraction for a set of social simulation results.

As a future research, we are going to apply DEA classification to simulation log analysis for agent-based social simulation and business gaming.

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