Comparison of Machine Learning Algorithms on Smartphone Energy Consumption Modeling Issue Based on Real User Context Data

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***Abstract*—Nowadays, billions of smartphones are used worldwide. The energy consumption is a critical issue when using such devices. In this context, smartphone power modeling is a mandatory step to better understand energy drain. On that way, the most widespread methods are based on specific hardware/software level analysis. As opposed to these classical approaches, we propose, in this paper, an alternative method aimed at constructing smartphones power models based on user context data provided by Device Analyzer, an Android application developed by the University of Cambridge. From a very large-scale smartphone usage data, we extract the energy-related events. Then, the energy-related context is formulated as the input features of the energy models. So as to predict the energy consumption of a smartphone, we compare four different machine learning models: a Linear Regression model, an AdaBoosted Decision Trees model, a Gradient Boosted Regression Tree model and a Random Forest model. The proposed energy models are then validated on a real user dataset in different usage scenarios.**

***Index Terms*—smartphone data; user behavior; energy modeling; regression model.**

I. INTRODUCTION

Since the embedded system related technology is progressing rapidly in recent years, mainly thanks to the support of numbers of applications and advanced hardware, smartphone devices have now become platforms integrated with various functions. Obviously, users not only can use their handsets to make phone calls and send SMS as before, but also can browse webs, play video games, listen to music and take photos, etc. Meanwhile, these various functions are tied to a huge growth in energy costs. Despite the progresses made in battery technology with an increase of energy storage, the smartphone usage is still limited by the energy drain of the components and of software needs. Hence, profiling, modeling and characterizing energy drain in the smartphone have become an important research topic in recent years.

In literature, many research work studied smartphone energy consumption focusing on either hardware or operating systems points of view [1]–[4]. Some other work studied how applications are consuming energy [5] or how the network environment changes affect smartphone energy drain [6]. Although this works proposed various accurate smartphone energy models, it should be aware of that the discharging process of a smartphone is a complex process, i.e., due to their unique hardware properties and usage context, different smartphone devices may have different discharging characteristics even when they are of the same type. Thus, one other reasonable alternative for modeling smartphone energy drain is to develop a personalized energy model for one specific smartphone device/user couple.

The purpose of our study is to find the relationship between user behavior and the energy drain in smartphone devices. This method can be a universal way to build energy models for smartphones because it does not need to measure the hardware components energy consuming properties or develop a specialized kernel tools as in precedent research [1], [3]. Our approach treats the energy consumption as the consequence of all the users' operation on their smartphone devices. Based on that, we develop a methodology to model the smartphone energy drain based on user context data. First, the energy-related events are chosen and extracted from the raw usage data. Then according to the selected events, the input features for energy models are generated. Afterwards, a series of machine learning models, the Linear Regression model, the AdaBoosted Decision Tree model, the Gradient Boosted Regression Tree model and the Random Forest model, are tested and compared in different usage scenarios.

The paper is organized as follows. Section II discusses some earlier related research work in this field and introduces the dataset and machine learning algorithms adopted. Section III presents the machine learning pipeline of our energy modeling process. Section IV describes how the features are generated and selected from the raw collected user data. Then, in Section V, the experiments for different scenarios and their associated results are presented. Finally, the conclusions and perspectives are given in Section VI.

II. RELATED WORK AND BACKGROUND

A. Power modeling issue

Smartphone power modeling is to use some energy-related factors, e.g., hardware properties (battery voltage, battery current, CPU utilization, etc.), system calls, user context(screen-on time, phone-call time, WiFi status, etc.), as input variables of models to predict smartphone energy consumption [7]. A general smartphone energy model can be described as follows, where E is the energy consumption of the handset:

$$E = P_{base} \cdot D + \sum_i \beta_i \cdot x_i \cdot d_i \quad (1)$$

The first term of the sum represents the fixed amount of energy consumption for the total experiment time D, and P_{base} is the basic energy consumption per time unit. The second term of the sum represents the variable amount of energy consumption. x_i is a value which represents the property of energy-related factors, for instance, for hardware elements it can be the CPU utilization and for user context it can be the WiFi status. β_i is a scalar indicates the energy-drain effect of x_i and d_i is influencing duration of x_i.

According to (1), from the point of view of hardware, smartphones devices are made of a series of sub-hardware components, CPU, display, cell module, WiFi module, audio, camera, etc. If the energy consumption of each component can be known, then naturally if we adopt a reductionist point of view, the total energy consumption of whole smartphone handset can be regarded as the sum of energy drain of all hardware components. According to the results in [1], this method can estimate smartphone energy drain accurately, but it needs external equipment to measure the power of each hardware, which is not always practical to implement. Besides, this method did not consider the interaction between the components which can represents 10% of the total energy drain [1]. Another frequently used method of modeling smartphone energy consumption is to trace the system calls of the smartphone [3]. As the different system calls are related to different power states in the smartphone, once the benchmarks of different power states are determined, the relationships between power states and system calls can be represented by a Finite State Machine (FSM) energy model.

From the aspect of smartphone usage context, a several research works analyzed how the user behavior and the network environment affect the handset energy drain [4] [6] [8]. Moreover, with the fast-growing crowdsourcing technology, researchers can have the access to a very large amount of user context data from a large number of users, which provides the researchers a new approach to study the interaction between users and smartphone [9] [10]. For instance, [11] [12] presented user-data based methods to predict the smartphone energy consumption and [13] proposed a big data combined with machine learning method to predict handset's battery life. In a similar way, we planned to exploit user context data to develop the smartphone energy model and to profile how the energy consumption cause by user behavior in our research work.

B. Device Analyzer

So as to study how user behavior affects energy drain in the handset, naturally, the first step of the process is to collect user context data in a large-scale. The dataset presented in this paper is the Device Analyzer dataset [14], which contains comprehensive and detailed information about the Android smartphone usage, e.g., events types, event values and relevant time stamps. Considering the trade-off between overhead and performance, the Device Analyzer application records the sampled events, such as battery levels, network traffic and screen brightness levels, at a time interval of 5 minutes, while records the immediate events, such as phone-calls, screen-on/off and WiFi on/off when they occur. Table I presents a list of the collected events. The exhaustive usage information are selected and formulated as the input features of our proposed energy models and the change of battery level of smartphones over 5 minutes of sampling interval is treated as the modeling targets. By using the context data as inputs and energy consumption as outputs, our task is to find proper energy models to imply how user behavior leads to energy drain in smartphone devices.

TABLE I PART OF COLLECTED EVENTS

Event category	Event name
Setting	Airplane mode Ring mode Audio volume, etc.
CPU	Number of cores Maximum frequency Minimum frequency, etc.
Battery	Battery level Battery voltage Charging status Battery temperature, etc.
Screen	Brightness level Screen state Screen size
Application	APP installed APP updated APP service APP foreground APP background, etc.
Phone call	Roaming state Network location Signal type Calling state, etc.
SMS	SMS received SMS sent, etc
Data transfer	Received bytes Received packets Send bytes Send packets
WiFi	WiFi scan WiFi connected WiFi state, etc.
Sensor	Type Max range Values Delay, etc.

C. Machine Learning Algorithms

As explained before, our research purpose is to predict energy consumption based on user behavior. Mathematically, it can be regarded as a regression problem, which is to find a function that matches real outputs from inputs accurately. Due to the powerful modeling ability of machine learning techniques, we resort to different advanced machine learning regression models. The basics of the models is described as below.

1) Linear Regression: Though the Linear Regression (LR) model is considered as the simplest regression model, it can obtain satisfying results in many research cases [13]. For this reason, the LR model is used in our research as the benchmark models to make comparisons to other tree-based models.

2) Decision Trees: The smartphone energy consumption is a continuous process, but we also assume that it consists of a series of power state change caused by different usage context. In this case, the regression decision tree models can be the appropriate models. Classification And Regression Decision Trees (CART) models [15] are tree-shaped models, it can be applied to either classification tasks or regression tasks. In our regression tasks, to approximate continuous target values, the CART algorithm uses the Least Square Difference (LSD) or the Least Absolute Difference (LAD) as criterion to split nodes. To improve the performance of tree models, the CART models are used as the basic estimators, combined with two different classes of ensemble methods: the boosting method and the bagging method.

- AdaBoosted Decision Tree (ADT): The boosting ensemble method aims to reduce the bias of each tree. AdaBoosted Decision Tree model [16] combines a series of weak learners with different weights which are calculated by their predicting accuracies to build a stronger learner.
- Gradient Boosting Decision Tree: The other Boosted Decision Tree model we utilize is Gradient Boosting Decision Tree (GBDT) [17]. GBDT uses the residuals between predictions and targets of former decision trees as new training targets to adapt its parameters.
- Random Forest (RF): The other class of ensemble method is the bagging method and a frequently-used model of bagging method is Random Forests. To overcome the over-fitting of each tree, Random Forests utilize the bootstrap aggregating technique [18] to decrease the variance of each tree.

In Section V, the performance of the four different models will be compared on the smartphone energy modeling issue.

III. METHODOLOGY

In order to construct the smartphone battery level predicting model from raw usage data, we propose a machine learning based modeling methodology. Though, the original data from the Device Analyzer dataset records extremely detailed information about users' usage, most of the information is not useful for our research purpose. Thus, the first step we take in the modeling process is to preprocess the raw data to extract the energy-related events. After the energy-related usage events are selected, according to their properties, the chosen contextual data are formulated as multidimensional matrix space consisting of a series of vectors. Each vector represents the occurrences and values of each energy-related events over a certain time range. We set the time range as 5 minutes because the battery level value is sampled every 5 minutes by the Device Analyzer application. Afterwards, the generated vectors are feed to the machine learning models as input variables. Finally, after the training process, the machine learning energy models are examined on the testing data and their results are analyzed. The pipeline of our methodology is shown in Figure 1. Four types of machine learning model are adopted in our research: Linear Regression, AdaBoosted Decision Trees, Gradient Boosted Regression Trees and Random Forests.

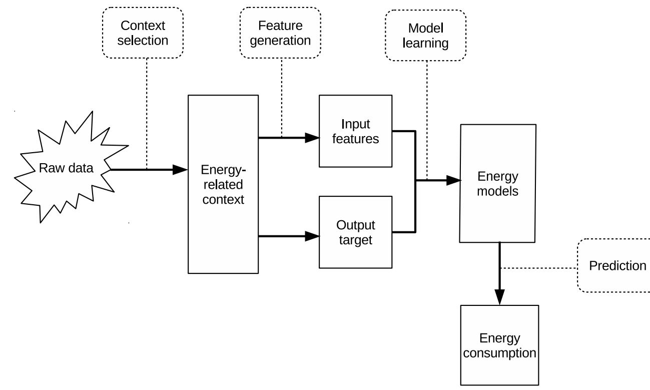


Figure. 1 Pipeline of smartphone energy prediction

IV. USER CONTEXT

A. Event Selection

The dataset for the experiments is collected from one Android smartphone user. It records the user's information for 80 days, and contains 13716 instances. Although the dataset that we use is from one individual user, our methodology is not limited to one specific user or one specific smartphone device. Hence, it can be a re-usable paradigm for smartphone energy consumption modeling. To exploit the useful context for the energy models, it needs to extract energy-related events from the raw dataset at the first step. Although the raw data collected corresponds to very detailed usage information, not all the recorded events directly make contribution to energy consumption of the handset, for instance, the location information and mobility sensors information are not necessary useful at first steps. Meanwhile, it should be reminded that our energy modeling approach is based on user behavior data. Hence, some hardware information will not be used neither, even they can reflect the energy drain condition, e.g., battery voltage and CPU utilization.

The principle of selecting the features is that the chosen features not only imply the significant information of user context but also are essential to profile energy consumption in smartphone handsets. For instance, when the screen is turned on, and several minutes later, it is turned off. During this time period, the user might watch a video using the smartphone and as a result, the battery level decreased. Apparently, this screen-on event caused significant energy consumption, meanwhile, it also represents essential information of the user-smartphone interaction. Similarly, according to previous works on investigating the energy-consuming effects caused by energy-

TABLE II SELECTED ENERGY-RELATED EVENTS

Category	Event Type
Battery	Battery Level
Screen	Screen On/Off Brightness
User Context	APP Foreground APP Background Phone Call Plugged In/Out
Data Transfer	Received Bytes Received Packets Send Bytes Send Packets
WIFI	WIFI On/Off

TABLE III VARIABLES DESCRIPTION

Event	Value Type	Unit
Battery Level	Variation	Percentage
Screen-on Time	Duration	Second
Brightness Level	Average	Scale(0˜255)
APP Foreground	Aggregation	-
APP Background	Aggregation	-
Phone-call Time	Duration	Second
Plugged State	State	1/0
Received Bytes	Aggregation	Byte
Received Packets	Aggregation	-
Send Bytes	Aggregation	Byte
Send Packets	Aggregation	-
WiFi-on Time	Duration	Second

related events [4] [6] [13], other energy-related events, such as network data traffic, phone-call, application usage and WiFi status are also important to describe smartphone usage and reflect the energy drain rate change. And they function in the similar way as the screen-on events in our proposed energy models. Moreover, the charging state of the smartphone battery apparently affects the battery level, therefore it is utilized in our models as well. Eventually, the events that we choose to build our energy model are screen-on time, the level of screen brightness, received data bytes, received data packets, sent data bytes, sent data packets, phone-call time, application foreground, application background, WiFi state and charging state.

In our approach, the energy consumption is measured by the smartphone battery level change. As mentioned in Section II-B, the battery level is sampled every 5 minutes by Device Analyzer, thus our task is to predict the energy consumption over time period of 5 minutes. Since the battery level is obtained by the smartphone battery indicator, the unit of daily energy drain is percentage of whole battery capacity. For example, if the value is 7, it means that the energy consumed by the smartphone device over this time period is 7% of the battery capacity. All the collected events utilized in our approach are listed in Table II.

B. Feature Generation

The measurement of energy drain is the level of the smartphone battery which is sampled every 5 minutes by Device Analyzer. Therefore, the values of each event is quantized within the time window between two battery level sampled time points. Furthermore, to make the events' values feed the model appropriately, i.e., according to their usage properties, we classify the possible values of the energy-related events into different categories: the variation, the duration, the average, the aggregation and the state as it is demonstrated in Section III. The 5 categories are described as follow:

- **Variation**: It should be noticed that all the events' contribution to the smartphone energy consumption over a time period is in fact the variation of the battery capacity over this time period. Thus, instead of using the battery percentage at the sampling time point as modeling target directly, it is more reasonable to use the variation of battery level during the sampling interval.
- **Duration**: If an event is the human-smartphone interaction that lasts for a certain time period, then we use the range of this time period to measure its influence on smartphone energy drain. For instance, the screen of a smartphone at time point T_1 is turned on and the screen is turned off at time point T_2, naturally, the total screen-on time will be T_2-T_1. This category includes screen-on time, phone-call time and WiFi-on time.
- **Average**: For the brightness level of screen, we use the average value over the sample interval.
- **Aggregation**: Some of the event may occur more than once, such as application foreground and application background, so we use sum of the occurrences to represent its energy contribution. As for the network transfers, received Bytes, received packets, send Bytes and send packets are aggregated values as well.
- **State**: The charging state of the battery can be whether plugged-in or plugged-out, so it is a boolean value.

Based on the categories we introduced, the energy-related events is quantized to a series of input vectors with 11 items at different sampling time points. Each item of a vector represents its related event's quantity. The target value is the energy consumption at the same sampling time point. Moreover, due to the malfunction of the data-collecting process, some values are missing. In this case, the average values are used to fill in the blanks. Before the input vectors are fed to the model, they are normalized to eliminate the effect of the different events' scales.

Once the input variables and the output target are well defined, the following step is to find an appropriate model to describe the mathematical relationship between the inputs and the outputs.

V. Experiments and Results

A. Experiments

To investigate the smartphone energy consumption under different usage scenarios, we select different variables combinations as inputs of the energy models. The 6 different experimental scenarios is demonstrated in Table IV. In the first scenario, we put all the energy-related events together. Then, in scenario 2, scenario 3, scenario 4, scenario 5 and scenario 6, we emphasize each energy-related events individually to study

TABLE IV EXPERIMENTAL SCENARIOS

Scenario	Context
Scenario 1	All contexts
Scenario 2	Screen-on
Scenario 3	Phone-call
Scenario 4	WiFi-on
Scenario 5	Plugged-in
Scenario 6	Plugged-out

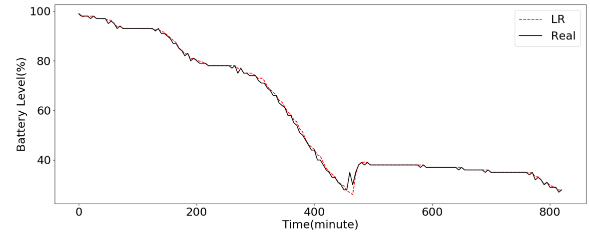
each event's contribution to energy consumption. It means that the data collected only in the scenario where screen-on events, phone-call events, WiFi-on events, plugged-in events or plugged-out events occur is utilized, respectively.

In order to evaluate the energy models, we resort to cross-validation method, the total dataset is split into two sub datasets, the training dataset and the testing data, with the Bootstrap method. The training subset data is to generate the energy models and the testing subset data is to examine generalization of the model to measure over-fitting. The criterion of a good energy mode is that it should not only fit the training dataset well but also have good performance on testing data. 70% of the whole dataset is used to train the energy models and the 30% rest is used to test the energy models. The proposed energy models are trained and tested on the dataset for 20 times for each scenario. The metrics to evaluate the energy models is the Root Mean Square Error (RMSE), and the mean value and Standard Deviation (STD) of RMSE are calculated.

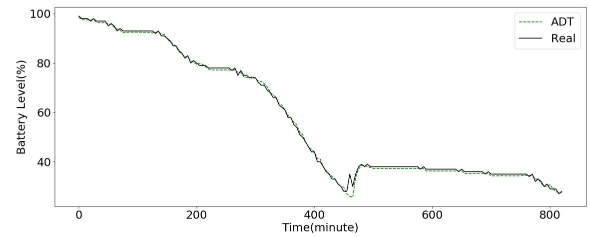
B. Results

In Figure 2, the battery level curves are reconstructed by predicted results of our devised models, and it shows that each energy model is capable of predicting battery level in the smartphone devices, either when the battery is discharging or charging. From the more detailed results shown in Table V and Table VI, in Scenario 1, when putting all contextual information together, the Random Forest regression model has the smallest training error and testing error. In Scenario 2, where the display is turned on, once again the RF energy model has the best performance both on training dataset and testing dataset among all the models. In Scenario 3, where calling events occur, the ADT model has the least training error and surprisingly, the Linear Regression model has the most accurate results on testing data. And in Scenario 4, where the WiFi module is turned on, the ADT model perform best on training dataset and the Random Forest model performs best on testing data. In Scenario 5, where the battery is plugged in, the GBDT model and the RF model has the smallest training error and has the smallest testing error, respectively. In Scenario 6, where the smartphone is not charging, the ADT model outperforms other models on training dataset and the RF model outperforms other models on testing dataset.

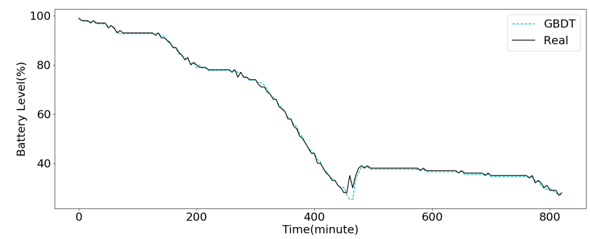
It also can be seen that, among all the scenarios, all the models' accuracy decreases in Scenario 5. As opposed to this, in Scenario 6, all the models have their own best performance on testing dataset. The reason could be that the



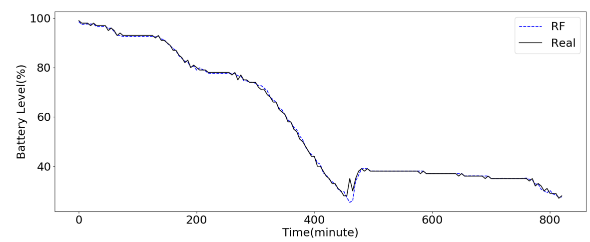
(a) LR Predicted Results



(b) ADT Predicted Results



(c) GBDT Predicted Results



(d) RF Predicted Results

Figure. 2 Predicting smartphone battery level over a time period

smartphone energy consumption is used to measured change of the battery level. However, it will not be accurate when the smartphone reaches its maximum energy volume (the battery level is 100%) and remains at the same battery level even energy-consuming events occur. Overall, in all scenario, except Scenario 3, all the tree-based models perform better than the Linear Regression model on the training dataset and testing dataset and the Random Forest energy model has the best performance on testing sub dataset. Thus, we can say that the RF is the most appropriate energy model in our research.

VI. CONCLUSION

In this paper, we presented a novel smartphone energy consumption modeling methodology based on user contextual data and machine learning algorithms. First, we extract the 11

TABLE V RMSE OF TRAINING RESULTS

Scenario	LR		ADT		GBDT		RF	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Scenario 1	0.8550	0.0063	0.4703	0.0107	0.5250	0.0072	**0.4181**	0.0058
Scenario 2	0.9251	0.0101	0.4197	0.0526	0.5115	0.0112	**0.3966**	0.0096
Scenario 3	0.6457	0.0372	**0.2227**	0.0248	0.3517	0.0450	0.3068	0.0142
Scenario 4	0.6999	0.0688	**0.2153**	0.0204	0.3453	0.0381	0.2963	0.0272
Scenario 5	1.5406	0.0110	0.8763	0.0242	**0.6525**	0.0173	0.8630	0.0181
Scenario 6	0.6179	0.0053	**0.3147**	0.0082	0.3605	0.0074	0.4600	0.0052

TABLE VI RMSE OF TESTING RESULTS

Scenario	LR		ADT		GBDT		RF	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
Scenario 1	0.8612	0.0147	0.7230	0.0131	0.7052	0.0126	**0.6975**	0.0145
Scenario 2	0.9327	0.0235	0.8628	0.0326	0.8538	0.0306	**0.8411**	0.0325
Scenario 3	**0.8206**	0.0743	0.8554	0.1132	0.8944	0.1409	0.8421	0.1182
Scenario 4	0.9213	0.1772	0.8217	0.1732	0.8375	0.1205	**0.7814**	0.1430
Scenario 5	1.5584	0.0269	1.2881	0.0276	1.2352	0.0346	**1.2327**	0.0326
Scenario 6	0.6170	0.0123	0.5825	0.0154	0.5638	0.0146	**0.5624**	0.0137

most important energy-related contexts from the comprehensive and detailed raw user data. Then, so as to feed the machine learning model properly, a series of features are generated from the properties of the extracted events. Afterwards, the energy consumption is used as the output target and the generated features are used as input variable for the energy model. Four different machine learning regression models are trained, tested and compared on the dataset. The final results indicate the feasibility of our proposed methodology.

For future work, we plan to take into account more energy-related events, for instance, the GSM signal strength and its types. Besides, we also plan to improve the accuracy of the existing models and to make a comparison with an agent based method developed in our previous work [7].

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