Dynamical Behavior of Communicability Structures in Complex Networks

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Abstract—**We investigate the microscopic community structure of the Korean meteorological society in the author network. Through oscillator networks, we simulate and analyze the averaged communicability functions. After constructing networks triggered an equally contributed weight between the first author and other authors in one published paper, we mainly treat these structures of communicability after constructing networks triggered an equally contributed weight between the first author and other authors in an author network. Our results support the development of the adaptability and the stability of social organization in the social networks.**

Keywords-Communicability function; Oscillator network; Community structure; Author network.

I. Introduction

Network science has emerged and been utilized as one of the important frameworks when each researcher studies complex systems [1-4]. An important property of networks is the existence of modules or communities, and the communicability between a pair of nodes in a network is concerned with the shortest path connecting both nodes. Estrada et. al. [5] proposed a generalization of the communicability by elucidating both for the shortest paths communicating between two nodes and for all the other walks travelling between two distances. The communicability detection allows one to determine potentially the unaware and hidden relationships between nodes and also allows one to reduce a large complex network into smaller and smaller groups. Presently, the community detection within networks is an open subject of great interest.

Complex networks are also ubiquitous in many biological, ecological, technological, informational, and infrastructural systems [6–12]. It is clear that the atomic, oscillating, and social systems display network-like structures using the tools of statistical mechanics. These methods and techniques were contributed to shed light on the structure and dynamics of social, economic, biological, technological, and medical systems [13-15]. It is actually recognized that the analogy functions that describe the properties depend mainly on the structural properties of the system in networks as well.

In this paper, we study the community structure of the Korean meteorological society in the author network. The data we used are the published papers of 676 authors from the Korean meteorological society publications in the author network, from March 2008 to November 2013. We simulate and analyze four other kinds of averaged communicability.

II. Communicability in Networks

We mainly consider the theoretical methods of microscopic communicability in networks. First of all, let us introduce the concept of communicability in networks by describing a community structure. The communicability structure can invoke the concept of walks in networks. A walk of length k is a sequence of nodes $n_0, n_1, \ldots, n_{k-1}, n_k$ such there is a link from n_{i-1} to n_i for each $i = 1, 2, \ldots, k$ [16]. Using the concept of walk we can define the communicability between two nodes. The communicability function [4] is represented in terms of $G_{pq} = \sum_{k=0}^{\infty} c_k (A^k)_{pq}$.

Here, A is the adjacency matrix, which has unity if the nodes p and q are linked to each other, but has zero otherwise. The adjacency matrix $(A^k)_{pq}$ gives the number of walks of length k starting at the node p and ending at the node q [17,18]. The two novel communicability functions are calculated as

$$G_{pq}^{EA} = \sum_{k=0}^{\infty} e \frac{(A^k)_{pq}}{k!} = (e^A)_{pq} \quad (1)$$

where e^A is a matrix function that can be defined using the following Taylor series [19]. The communicability function G_{pq} is obtained by using the weighted adjacency matrix $W = (W_{ij})_{n \times n}$. Centrality measures were originally introduced in social sciences [20,21] and are now widely used in the whole field of complex network analysis [9]. We can derive the communicability function as

$$G_{pq}^{RA} = \beta K m \omega^2 G_{pq}(\beta) \quad (2)$$

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with the identification $\alpha = 1/K$.

From the fact that the Laplacian matrix of a connected network has a nondegenerate zero eigenvalue, we can calculate another correlation function as

$$G_{pq}^{D}(\beta) = \frac{1}{\beta K m \omega^2}(L^{+})_{pq}, \quad (3)$$

where L^{+} is the Moore–Penrose generalized inverse of the Laplacian.

In a network of quantum oscillators, we start by considering the quantum-mechanical counterpart of the Hamiltonian H_A. After arranging several equations, we can see that

$$G_{pq}^{EA} = \exp(\beta\hbar\Omega)G_{pq}^{A}(\beta). \quad (4)$$

The diagonal thermal Green's function is given in the framework of quantum mechanics, and we can compute the off-diagonal thermal Green's function as

$$G_{pq}^{A}(\beta) = \exp(-\beta\hbar\Omega)(\exp[\frac{\beta\hbar\omega^2}{2\Omega}A])_{pq}. \quad (5)$$

Note that when the temperature tends to infinity or $\beta \to 0$, there is absolutely no communicability between any pair of nodes. That is, $G_{pq}^{EA}(\beta \to 0) = 0$. If we consider the case when the temperature tends to zero or $\beta \to \infty$, then there is an infinite communicability between every pair of nodes, i.e., $G_{pq}^{A}(\beta \to \infty) = \infty$. Furthermore, the communicability function is represented in terms of

$$G_{pq}^{EL}(\beta) = G_{pq}^{L}(\beta) - 1, \quad (6)$$

where the same quantum-mechanical calculation by using the Hamiltonian H_L in Eq. (4) is calculated as

$$G_{pq}^{L}(\beta) = (\exp[-\frac{\beta\hbar\omega^2}{2\Omega}L])_{pq}. \quad (7)$$

From Eqs. (6) and (7), the communicability function G_{p}^{EL} gives $G_{pq}^{L}(\beta) - 1$ upon setting $\beta\ h\omega^2 = 2\Omega$ [4]. Lastly, we simulate and analyze the averaged communicability function for a given node defined as

$$G_p = \frac{1}{n-1}\sum_{p \neq q}^{n} G_{pq}. \quad (8)$$

Consequently, the communicability functions G_{pq}^{RA} and G_{pq}^{D} become the types of the thermal Green's function of classical harmonic oscillators in networks of the community structure. The communicability functions $G_{pq}^{EA}(\beta)$ and $G_{pq}^{EL}(\beta)$ also become the types of the thermal Green's function in quantum harmonic oscillators.

III. NUMERICAL CALCULATIONS AND RESULTS

In order to simulate and analyze the averaged communicability functions, the data are the published papers for 676 authors of the Korean meteorological society publications in the author network from March 2008 to November 2013. We assume that it only takes an equally contributed weight between all authors in one published paper.

We implement the computer-simulation of the four communicability functions. Figure 1 shows the color-map diagram of the communicability function matrices as G_{p}^{RA}, $1/G_{p}^{D}$, G_{p}^{EA}, and G_{p}^{EL} for 676 authors of the Korean meteorological society publications, among four averaged communicability functions [28]. If two members are highly correlated, the representation approaches the color red. If they are weakly correlated, the representation approaches dark blue.

We can simulate four averaged communicability functions constituting a number of published papers for 676 members of the author network. The weight of community means the value (that is, 1/the number of authors) that all authors are bestowed the same weight upon one published paper. Then, we assume that the weight of community and the weight of published papers for the 1-st author is one for the 1-st author. We now speculate that the phase transition among these functions may exist near 200-th authors. In next time, we will aim to find it through networks of other societies.

Table 1 summarizes the values of the averaged communicability functions, the weight of community, and a number of published papers for 100-th, 300-th, and 600-th authors, respectively. These values are normalized values divided by the maximum value of each factors. For the value of G_{p}^{EA} between two authors, the 600-th author approaches to zero. We find that the G_{p}^{EL} relatively correlates highly when this value is compared to other ones.

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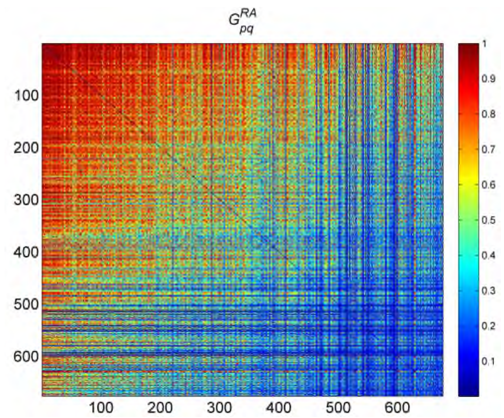

Figure 1. Color map diagram of relative communicability function matrices as G_p^{RA} from top to down for 676 members of the author network.

TABLE I. Values of the weight of published papers Pp, the weight of community W, and the averaged communicability functions.

Sequent order of authors	P_c	W_c	G_p^{EA}	G_p^{RA}	G_p^{EL}	$1/G_p^D$
100	0.247	0.059	0.057	0.065	0.998	0.149
300	0.082	0.015	0.016	0.017	0.946	0.042
600	0.030	0.005	0.0	0.004	0.554	0.026

IV. CONCLUSIONS

We have studied the community structure of Korean meteorology fields in the 676 author networks of all Korean meteorological society publications from March 2008 to November 2013. We mainly implemented the computer-simulation of the four communicability functions.

To compare the four averaged communicability functions, it was shown that the G_p^{EL} constructs a stronger community structure rather than the other three. The function G_p^{EA} finds the community structure weaker than the other three as well. We can make use of the four averaged communicability functions to compute the measures of a community structure, and it is hoped that our method and technique will lead us to more general results in the future.

It is not trustworthy now, but we anticipate that the phase transition among the averaged communicability functions may exist at one value near 200-th authors. Our results cannot yet be compared to that of other social networks, but we hope to compare to our results to other successful results in social networks that have been prominently produced and published. Next time, we hope to discuss the phase transition of the averaged communicability functions, with network systems of other societies. In the future, we will apply the community structure to the cases of different contributed weight between authors. Therefore, further work is needed for the case with societies of more than the author and citation networks. The formalism of our analysis can be extended to both the discrimination and the characterization of communicability functions in other various societies.

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