Validation of Simulation Model Based on Combined Consistency Analysis of Data and Feature

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Abstract—**Comparing the simulation output with reference output is an important measure to validate a simulation model. The classic methods either analyze the data consistency of simulation output and reference output or the consistency of their features. Considering the two types of consistencies simultaneously, a validation method of simulation model based on combined consistency analysis of data and feature is proposed. The measurement model of data consistency integrating the proximity of spatial position and similarity of geometrical shape is presented, and the measurement model of feature consistency based on relative error is given. Besides, the weight of each consistency index was determined by analyzing the correlation among the consistency indexes, and the consistency integration model is given. Finally, the validation method is applied to validate a simulation model of servo-control system.**

Keywords-model validation; data consistency; feature consistency; correlation analysis.

I. INTRODUCTION

Simulation has been widely used in many fields, such as military, economic and social areas, with the advantages of economy, security and efficiency. Because simulation is a scientific research based on models, the credibility of model attracts much attention. The simulation model validation is the primary means to research the credibility of model, and the consistency analysis of simulation output and the reference output is an important method for simulation model validation [1][2].

As early as 1962, Biggs made an assessment for the credibility of the "dogs" missile model [3]. Fishman and Kiviat used spectral analysis to assess the credibility of the queuing model [4]. Hermann made a consistency validation between the real system and simulation model, based on the intrinsic features or typical events [5]. Mckenny proposed some methods based on the analysis of variance, Kolmogorov-Smirnov test and χ^2 test [6]. Kheir and Holmes utilized Theil inequality coefficient (TIC) method to validate the effectiveness of a missile simulation system [7]. Through the analysis of features of grey correlation model, Wei and Li made an application to assess the credibility of a missile simulation system [8]. Damborg assessed the credibility of econometric models with the error analysis methods [9]. Liu, Liu and Zhang discussed the relation between credibility and similarity, and gave a quantitative method of simulation model credibility according to the similarity [10]. Moreover, Balci, Sargent and Kleijnen gave some summaries to the validation of simulation model [11][12][13].

The analysis of classical methods above shows that, there are mainly two research ideas, namely, 1) the features we concerned were extracted from the output data of simulation and reference, such as the rise time, overshoot and steady-state error in controlling response. Then, feature consistency can be analyzed via variance analysis and hypothesis test, etc., and 2) the data consistency of simulation output and reference output was analyzed directly, such as TIC method and gray correlation method, etc. The data consistency of simulation output and reference output reflects their panoramic consistency, but ignores the consistencies of some detailed features easily. For some simulation applications, the data consistency and feature consistency are focused on. So, the two types of consistencies should be considered simultaneously in the validation of simulation model.

To solve the above problem in terms of continuous dynamic simulation of multi-output, this paper presents a new method of simulation model validation, considering multiply features. The advantages of this new method are more comprehensive, convincing and reliable. The paper is organized as follows: In Section 2, the research problems will be described and analyzed. In Section 3, the measurement models of data consistency and feature consistency will be given. In Section 4, the integrated model of consistency will be given. In Section 5, the effectiveness of this method will be shown through application examples. In the last Section, summary is drawn and future research is discussed.

II. PROBLEM DESCRIPTION AND ANALYSIS

Assuming that $U_s=[u_{s1},u_{s2},\cdots,u_{sk}]^T$ and $U_r=[u_{r1},u_{r2},\cdots,u_{rk}]^T$ denote the inputs of simulation model and reference system respectively. Their outputs are represented as $Y_s=[y_{s1},y_{s2},\cdots,y_{sm}]^T$ and $Y_r=[y_{r1},y_{r2},\cdots,y_{rm}]^T$. The simulation model researched in the paper is described as follows:

$$\begin{cases}\dot{X}_s=F(X_s,U_s,T)\\ Y_s=G(X_s,U_s,T)\end{cases} \tag{1}$$

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$$F(X_s,U_s,T)=\begin{bmatrix} f_1(X_s,U_s,T) \\ \vdots \\ f_n(X_s,U_s,T) \end{bmatrix} \quad (2)$$

$$G(X_s,U_s,T)=\begin{bmatrix} g_1(X_s,U_s,T) \\ \vdots \\ g_m(X_s,U_s,T) \end{bmatrix} \quad (3)$$

where $X_s=[x_{s1},x_{s2},\cdots,x_{sn}]^T$ is state variable, $F(X_s,U_s,T)$ is state equation and $G(X_s,U_s,T)$ is output equation, T is time series.

Assuming that $C(Y_s,Y_r)$ denotes the consistency between Y_s and Y_r when $U_s=U_r$ (the consistency of simulation output for short), and $C(Y_s,Y_r)\in(0,1]$. When $C(Y_s,Y_r)=1$, it indicates that Y_s is the same with Y_r exactly, i.e., the simulation model is quite credible. If the consistency between Y_s and Y_r becomes more and more bad, i.e., the simulation model is more and more incredible, then $C(Y_s,Y_r)\to 0$.

Assuming that $D(y_{si},y_{ri})$, $i=1,2,\cdots,m$ denotes the data consistency between y_{si} and y_{ri} when $U_s=U_r$ (the consistency of y_{si} for short), and $D(y_{si},y_{ri})\in(0,1]$. When $D(y_{si},y_{ri})=1$, it indicates that y_{si} is the same with y_{ri} exactly. If the data consistency between y_{si} and y_{ri} becomes more and more bad, then $D(y_{si},y_{ri})\to 0$.

Assuming that $d(y_{si},y_{ri})$ and $s(y_{si},y_{ri})$ denote "proximity" of position (i.e., proximity of spatial position) and "similarity" of shape (i.e., similarity of geometrical shape) between y_{si} and y_{ri} when $U_s=U_r$. The definition is as follows:

$$D(y_{si},y_{ri})=G(d(y_{si},y_{ri}),s(y_{si},y_{ri})),\ i=1,2,\cdots,m \quad (4)$$

where $G(\bullet)$ is the integrated model of $d(y_{si},y_{ri})$ and $s(y_{si},y_{ri})$.

Assuming that $C_s=[c_{s1},c_{s2},\cdots,c_{sl}]^T$ and $C_r=[c_{r1},c_{r2},\cdots,c_{rl}]^T$ denote features extracted from Y_s and Y_r respectively. A definition is as follows:

$$C_s=H(Y_s),\ C_r=H(Y_r) \quad (5)$$

where $H(\bullet)$ is the feature extraction model.

Assuming that $V(c_{sj},c_{rj})$, $j=1,2,\cdots,l$ denotes the feature consistency between c_{sj} and c_{rj} when $U_s=U_r$, and $V(c_{sj},c_{rj})\in(0,1]$. When $V(c_{sj},c_{rj})=1$, it indicates that c_{sj} is the same with c_{rj} exactly. If the feature consistency between c_{sj} and c_{rj} becomes more and more bad, then $V(c_{sj},c_{rj})\to 0$.

To gain $C(Y_s,Y_r)$, the classic methods only consider $D(y_{si},y_{ri})$, $i=1,2,\cdots,m$ or $V(c_{sj},c_{rj})$, $j=1,2,\cdots,l$. For some simulation application, the two types of consistencies should be considered simultaneously. As shown in Fig. 1, the index system of simulation output consistency is given.

From the above, we can notice that the application of method proposed in this paper has two difficulties: 1) how to measure the data consistency and feature consistency; 2) how to deal with the correlation among the consistencies of data and features and integrate them to gain the consistencies of simulation output.

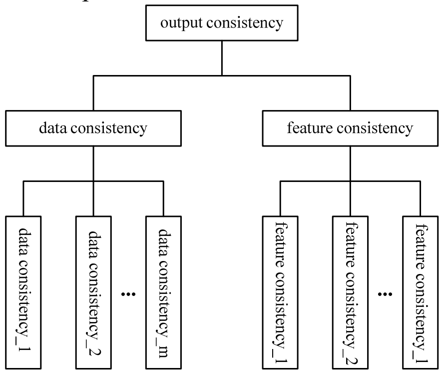

Figure 1. Index system of simulation output consistency

III. CONSISTENCY MEASUREMENT MODEL

In this section, two types of consistency measurement model are presented.

A. Data consistency measurement model

The reference output and the simulation output of a component are denoted by $y_r=<y_r(1),y_r(2),\cdots y_r(p)>$ and $y_s=<y_s(1),y_s(2),\cdots y_s(p)>$, respectively. The data consistency can be obtained by considering the "proximity" of position and the "similarity" of shape comprehensively.

The simulation model is validated by the difference of position between y_r and y_s, which is depicted by the coefficient of TIC in [7]. The formula is as follows:

$$U_{TIC}(y_r,y_s)=\frac{\sqrt{\frac{1}{p}\sum_{i=1}^{p}(y_r(i)-y_s(i))^2}}{\sqrt{\frac{1}{p}\sum_{i=1}^{p}y_r(i)^2}+\sqrt{\frac{1}{p}\sum_{i=1}^{p}y_s(i)^2}} \quad (6)$$

where $U_{TIC}(y_r,y_s)$ is the coefficient of TIC.

From (6), it results that $U_{TIC}(y_r,y_s)\in[0,1]$. It describes a kind of relative error which is very convenient to understand and use. But, when applied to the validation of simulation model directly, the following problems will be appeared.

As shown in Fig. 2(a), $y_r(t)\equiv c_r$, $y_s(t)\equiv c_s$, $t=1,2,\cdots,p$. Meanwhile, $C_r\le 0$ and $C_s>0$ are constant. From (6) results that, $U_{TIC}(y_r,y_s)\equiv 1$. It indicates that the "proximity" of position of y_r and y_s is the worst, which is obviously unreasonable. As shown in Fig. 2(b), $y_r(t)=f(t)$,

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$y_{s1}(t) = f(t) + c$, $y_{s2}(t) = f(t) - c$, $t = 1, 2, \cdots, p$, where $c > 0$ is constant. It is easily obtained that $U_{TIC}(y_r, y_{s1}) = U_{TIC}(y_r, y_{s2})$ by the intuitive judgment. However, from (6) results that $U_{TIC}(y_r, y_{s1}) > U_{TIC}(y_r, y_{s2})$.

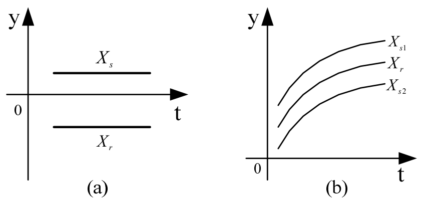

Figure 2. Outputs of reference and simulation in special conditions

The reason why the results above are not consistency is that the relative error of y_r and y_s is calculated by the benchmark of

$$\sqrt{\sum_{i=1}^{p} y_r(i)^2} + \sqrt{\sum_{i=1}^{p} y_s(i)^2} \ , \tag{7}$$

not by the benchmark of reference outputs.

The measurement model of the "proximity" of position of y_r and y_s is given based on the coefficient of TIC in this paper. The measurement model is as follows.

$$U_{TIC}(y_r, y_s) = \begin{cases} \dfrac{\sqrt{\sum_{i=1}^{p}(y_r(i) - y_s(i))^2}}{\sqrt{\sum_{i=1}^{p} y_r(i)^2}}, & \sqrt{\sum_{i=1}^{p} y_r(i)^2} \neq 0 \\ \sqrt{\sum_{i=1}^{p}(y_r(i) - y_s(i))^2}, & \sqrt{\sum_{i=1}^{p} y_r(i)^2} = 0 \end{cases} \tag{8}$$

$$d(y_r, y_s) = e^{-\lambda_d T(y_r, y_s)} \tag{9}$$

where $\lambda_d > 0$ is the parameter of measurement model for the "proximity" of position. λ_d is given by the area of specific application.

The simulation model is validated by the "similarity" of shape between y_r and y_s , which is depicted by the method of grey relevance coefficient in [8]. The formula is as follows:

$$r(t) = \frac{\min_t \Delta(t) + \lambda_s \max_t \Delta(t)}{\Delta(k) + \lambda_s \max_t \Delta(t)} \ , \ t = 1, 2, \cdots, p \tag{10}$$

$$s(y_r, y_s) = \frac{1}{p} \sum_{t=1}^{p} r(t) \tag{11}$$

where $\Delta(t) = |y_r(t) - y_s(t)|$, $\lambda_s \in [0,1]$ is resolution coefficient. The range of λ_s is $[0, 0.5]$.

Considering the "proximity" of position and the "similarity" of shape for y_r and y_s based on (9) and (11), the data consistency measurement model of y_r and y_s is given as follows:

$$D(y_r, y_s) = \sqrt{d(y_r, y_s) \times s(y_r, y_s)} \tag{12}$$

B. Feature consistency measurement model

Some features extracted from the reference output and the simulation output, are denoted by c_r and c_s respectively. The relative error is used to describe the difference between them, as shown in (13) [14]. The degree of consistency is obtained by mapping the relative error to interval (0,1] through negative exponential function, as shown in (14) [15].

$$\eta_c = \begin{cases} \dfrac{|c_s - c_r|}{|c_r|}, & c_r \neq 0 \\ |c_s - c_r|, & c_r = 0 \end{cases} \tag{13}$$

$$V(c_s, c_r) = e^{-\lambda_c \eta_c} \tag{14}$$

where $\lambda_c > 0$ is the parameter of consistency measurement model, given by the area of specific application.

IV. INTEGRATED MODEL OF CONSISTENCY

$D(y_{si}, y_{ri})$, $i = 1, 2, \cdots, m$ and $V(c_{sj}, c_{rj})$, $j = 1, 2, \cdots, l$ can be got by using the consistency measurement model above. Furthermore, the consistency of simulation output can be obtained by integrating $D(y_{si}, y_{ri})$ and $V(c_{sj}, c_{rj})$ as:

$$C(Y_s, Y_r) = \sum_{i=1}^{m} \omega_{di} \times D(y_{si}, y_{ri}) + \sum_{j=1}^{l} \omega_{vj} \times V(c_{sj}, c_{rj}) \tag{15}$$

where ω_{di} and ω_{vj} are the weights of data consistency and feature consistency, respectively.

From all above, in order to get integrated model of consistency, the important thing is to determine the weights of $D(y_{si}, y_{ri})$, $i = 1, 2, \cdots, m$ and $V(c_{sj}, c_{rj})$, $j = 1, 2, \cdots, l$. However, the correlation exist between $D(y_{s1}, y_{r1}), \cdots, D(y_{sm}, y_{rm})$, $V(c_{s1}, c_{r1}), \cdots, V(c_{sl}, c_{rl})$. Thus, the weight of each index is determined based on the correlation analysis in [16].

Due to the reference output samples are usually less, it assumes that there is only a single sample here. Each component of the output time series is expressed as $y_{ri} = < y_{ri}(1), y_{ri}(2), \cdots y_{ri}(p) >$, $i = 1, 2, \cdots, m$. Even when a few samples of the reference output exist, they should be averaged into a single time series. Simulation output samples are usually easy to obtain. It is assumed that q samples of each output component are $y_{si}^j = < y_{si}^j(1), y_{si}^j(2), \cdots y_{si}^j(p) >$, $i = 1, 2, \cdots, m$, $j = 1, 2, \cdots, q$.

Furthermore, z_i , $i = 1, 2, \cdots, m + l$ denotes the data consistency and feature consistency between simulation output and the reference output. z_i^j , $i = 1, 2, \cdots, m + l$, $j = 1, 2, \cdots, q$ denotes the data consistency and feature consistency between the j th sample of simulation output and the reference output. To determine the weight of z_i ,

$$\overline{z_i} = \frac{1}{q} \sum_{j=1}^{q} z_i^j \ , \ i = 1, 2, \cdots, m + l \tag{16}$$

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$$s_{ii} = \frac{1}{q}\sum_{j=1}^{q}(z_i^j - \bar{z}_i)^2 \text{ , } i = 1,2,\cdots,m+l \tag{17}$$

$$s_{ij} = \frac{1}{q}\sum_{h=1}^{q}(z_i^h - \bar{z}_i)(z_j^h - \bar{z}_j) \text{ , } i = 1,2,\cdots,m+l \tag{18}$$

Furthermore,

$$r_{ij} = \frac{s_{ij}}{\sqrt{s_{ii}s_{jj}}} \text{ , } i,j = 1,2,\cdots,m+l \tag{19}$$

z_i and other indexes of the multiple correlation coefficient can be obtained as follows:

$$R_i = \frac{1}{m+l-1}(\sum_{j=1}^{m+l} r_{ij}^2 - 1) \text{ , } i = 1,2,\cdots,m+l \tag{20}$$

The degree z_i "included" by other indexes can be reflected by the multiple correlation coefficient. If the larger R_i became the worse the independence of z_i, whereas the better. So the weight of z_i can be determined as follows:

$$a_i = \frac{\max_j R_j}{R_i} \text{ , } i,j = 1,2,\cdots,m+l \tag{21}$$

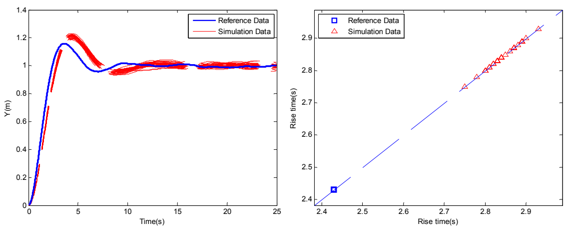
$$\omega_i = \frac{a_i}{\sum_{j=1}^{5} a_i} \text{ , } i,j = 1,2,\cdots,m+l \tag{22}$$

Thus the integration model of consistency can be got as:

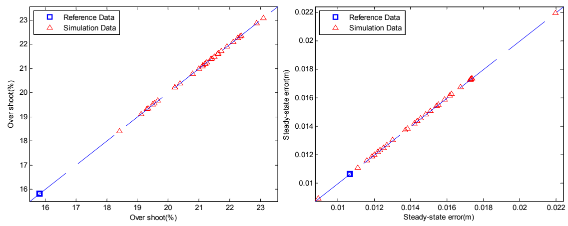
$$C(Y_s, Y_r) = \sum_{i=1}^{m+l} \omega_i \times \bar{z}_i \tag{23}$$

V. APPLICATION

The object of application studied in this paper is a model of servo-control system. The reference output is the step response of the actual system; the simulation output is 30 times the running output of the simulation model, as shown in Fig. 3(a). There are two ways to validate the credibility of simulation model using the classical methods: 1) Analyze the consistency of step response between the actual system and simulation model directly; 2) Extract rise time, overshoot and steady-state error from step response of the actual system and simulation model respectively, then analyze the features consistency between them. However, no matter which way is used above, the consistency between the simulation output and the reference output is portrayed from one-sided. In addition, the correlation between multiple data consistency and features consistency was not considered by the classic methods.



(a) step response (b) rise time



(c) overshoot (d) steady-state error

Figure 3. Outputs of reference and simulation in special conditions

TABLE I. THE ANALYSIS RESULTS OF DATA CONSISTENCY AND FEATURES CONSISTENCY

N	C_y	C_t	C_p	C_e
01	0.852	0.913	0.894	0.822
02	0.838	0.919	0.824	0.730
03	0.829	0.910	0.865	0.733
04	0.852	0.910	0.869	0.769
05	0.851	0.902	0.885	0.840
06	0.841	0.923	0.854	0.799
07	0.860	0.931	0.846	0.922
08	0.845	0.910	0.900	0.909
09	0.859	0.921	0.829	0.926
10	0.855	0.915	0.832	0.772
11	0.843	0.936	0.815	0.783
12	0.838	0.927	0.838	0.846
13	0.831	0.927	0.819	0.939
14	0.837	0.919	0.845	0.730
15	0.867	0.910	0.890	0.833
16	0.848	0.923	0.837	0.895
17	0.834	0.919	0.800	0.732
18	0.858	0.920	0.835	0.796
19	0.845	0.913	0.895	0.840
20	0.855	0.919	0.814	0.931
21	0.837	0.921	0.870	0.751
22	0.840	0.921	0.832	0.980
23	0.866	0.925	0.849	0.917
24	0.839	0.912	0.812	0.733
25	0.857	0.908	0.921	0.862
26	0.840	0.913	0.843	0.957
27	0.852	0.925	0.841	0.812
28	0.873	0.927	0.794	0.866
29	0.848	0.917	0.843	0.945
30	0.821	0.921	0.888	0.588

For the problems above, this paper not only considers the data consistency of step response, but also considers the consistency of rise time, overshoot and steady-state error. By analyzing the correlation between them, the respective weights are determined. The consistency analysis results based on (7) to (13) are shown in Table 1, where λ_d, λ_s, and λ_c are taken as 0.5; N, C_y, C_t, C_p and C_e in these equations represent the number of simulation runs, the consistency degree of step response, the consistency degree of rise time, the consistency degree of overshoot, and the consistency degree of steady-state error respectively. Furthermore, the weight of each index obtained by (15) to (21) shows as follows:

$$\omega = [\omega_y, \omega_t, \omega_p, \omega_e] = [0.1523, 0.1110, 0.2126, 0.5241] \quad (24)$$

For the 30 samples of C_y, C_t, C_p and C_e, the average value can be obtained as follows:

$$[\bar{C}_y, \bar{C}_t, \bar{C}_p, \bar{C}_e] = [0.7607, 0.7459, 0.7367, 0.8462] \quad (25)$$

Finally, according to (22), the consistency analysis result of simulation output can be obtained as follows:

$$C(Y_s, Y_r) = [\omega_y, \omega_t, \omega_p, \omega_e] \times [\bar{C}_y, \bar{C}_t, \bar{C}_p, \bar{C}_e]^T = 0.7988 \quad (26)$$

The analysis result shows that compared to the consistency of the reference output in the instance, the consistency of the simulation output consistency is better. It indicates that the simulation model is credible. By observing the simulation output and the reference output from Fig. 3(a) to 3(d), it can be concluded that they have good consistency. The results of this paper are identical with this conclusion. Thus, the effectiveness of the method in this paper is verified.

VI. CONCLUSION AND FUTURE WORK

This paper focused on the study of the validation of simulation model. Our goal is to obtain the consistency degree of simulation output and reference output by considering the data consistency and feature consistency simultaneously. In this paper, three conclusions are obtained as follows: 1) The new research thought of simulation model validation was given. 2) The measurement model of data consistency integrating the "proximity" of position and "similarity" of shape and the measurement model of feature consistency based on relative error are presented. 3) The method of how to integrate two types of consistency measurement model was proposed. However, the scarcity of the reference output may exist in multi-output dynamic simulation. So, in future research, it is planned to concentrate on how to integrate with the expert knowledge to validate simulation model.

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