The MAP in LC Decoding of MTR Codes in Two-Track Magnetic Recording Systems

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Abstract **– The maximum *a posteriori* probability (MAP) algorithm has recently been implemented in Boolean logic circuits (MAP in LC) and presented as an additional method for soft-decision decoding of maximum-transition-run (MTR) codes. Substantial benefits were noticed when the MAP in LC method was used for decoding in an MTR encoded one-track one-head E^2PR4 magnetic recording channel. Those benefits reveal the great potential of possible employment of MTR codes in iterative decoding schemes. Having in mind that MTR codes are able to reduce the overall complexity of trellis channel detection, their utilization in magnetic recording systems with multiple-tracks multiple-heads could be valuable. In this paper, the performance of the MAP in LC decoding was considered in the in-track MTR encoded two-track two-head E^2PR4 channel. The subversion named as *max-log* MAP in LC is presented and compared with *min-max* in LC and *max-log* MAP approach for MTR decoding. In case of the low-level inter-track interference over the recording channel, the *max-log* MAP in LC subversion shows nearly 1 dB of decoding gain, for BER = 10^{-5}, when rate 4/5 (2, 8) MTR was used.**

Keywords-constrained coding; MTR codes; soft-decision decoding; multiple-head recording.

I. INTRODUCTION

Utilization of maximum transition run (MTR) codes [1] as a constrained code for magnetic recording channels, especially in the case of the extended class four partial response channel (E^2PR4) [2], has shown to be quite beneficial [3]. The MTR codes increases the minimum squared Euclidean distance by preventing the ±[+1 –1 +1] error-event, which is dominant at some densities [4]. Consequently, in the case of the two-track two-head E^2PR4 channel model, the MTR employment resulted in 23% of reduction in the number of detection trellis states, and nearly 42% of reduction in the overall channel detection complexity [3].

Even though MTR codes appear to be valuable as constrained codes, it should be emphasized that they possess just a modest error correcting ability. Thus, they have to be used in combination with some proven and powerful error correcting codes, such as the low-density parity-check (LDPC) [5].

Several different approaches had been presented, ranging from the enforcement of MTR constraint into LDPC encoding [6], to the straightforward serial concatenation of LDPC and MTR codes [7], [8]. In all of these instances, the MTR decoding has to be based on a soft-decision approach, so that the corresponding decoder is able to handle soft-values [9], and to produce decision confidence.

In order for them to be implemented in a modern framework of iterative decoding, MTR codes require soft-decision decoding techniques. As a result, this fact encouraged development of several different techniques, such as *min-max* in LC method [7], [8], and the MAP approach [10], [11].

Recently, an additional approach was presented, entitled MAP in LC method, which implements the MAP algorithm into Boolean logic circuits [12], [13]. Utilization of the MAP in LC method offers considerable decoding gain and overall decoding complexity reduction, in case of the MTR encoding over a one-track E^2PR4 magnetic recording channel [7], [8].

In this paper, an extension was made with MAP in LC utilization in magnetic recording systems that use the multiple-track multiple-head approach for data storage [3], [14].

This paper is organized in such a manner that Section II presents an overview of *min-max* in LC and *max-log* MAP algorithm for soft-decision decoding of MTR codes. Section III demonstrates complexity analyses of the MAP decoder, while Section IV explains the MTR encoding over the E^2PR4 two-track channel. Section V offers results of the simulations and performance comparisons, while Section VI gives the conclusion to this paper.

II. MTR SOFT-DECISION DECODING METHODS

This section presents a brief and simple overview of two previously presented methods, the *min-max* in LC [7], [8], and *max-log* MAP approach [10], [11]. In addition, this section is intended to reintroduce notation that will be used in further analyses and explanation of the MAP in LC.

A. Soft-values and Soft-decision Concept

The soft-value of binary variable x is defined as

$$L(x) = \log\big(P(x=1)/P(x=0)\big), \tag{1}$$

where *log*() function is a natural logarithm. The sign of $L(x)$ is the binary decision, the so called hard-decision, while the magnitude of represents the confidence of this decision [9].

The MTR decoder should be able to handle input soft-values and to produce subsequent soft-values on its outputs.

B. min-max in LC Approach

MTR codes can be easily realized using integrated circuit technology and Boolean logic circuits, as it was originally presented by Moon and Brickner [1].

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Straightforward and low-cost implementation of the simple and well know rate 4/5 (2, 8) MTR code can be realized, as shown in Fig. 1 [1], [7], [8].

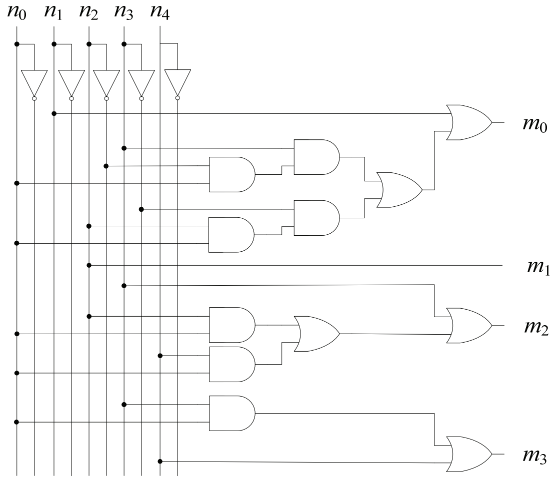

Figure 1. Rate 4/5 (2, 8) MTR decoder.

The idea with the *min-max* in LC method was to use redesigned Boolean logic circuits, which can produce output soft-values according to the following expressions [7], [8]

$$\begin{aligned} L_{out}^{NOT}(x) &= -L_{in}(x), \\ L_{out}^{AND}(x_1,x_2) &= \max[L_{in}(x_1), L_{in}(x_2)], \\ L_{out}^{OR}(x_1,x_2) &= \min[L_{in}(x_1), L_{in}(x_2)]. \end{aligned} \tag{2}$$

where *min*() and *max*() functions return minimal and maximal soft-values, of the input variables.

By implementing such an approach, simple propagation of input soft-values can be realized through the newly created circuits, enabling the *min-max* in LC soft-decision MTR decoding [7], [8].

The *min-max* in LC approach demonstrated excellent performance, when it was used for soft-decision decoding of the MTR code that is combined with LDPC code, over a one-track one-head [7], [8], as well as a multiple-track multiple-head E^2PR4 magnetic recording channel [3].

The *min-max* in LC decoding approach is primarily intended for hardware realization of the MTR decoder, using integrated circuits technology.

C. The MAP Approach

MTR codes are simple block codes that basically perform mapping between two sets of sequences. With the defining set

$$N_{set} = \{\underline{n} = (n_0 n_1 \cdots n_{N-1}) \in Z_2^N \mid n_i \in (0,1)\}, \tag{3}$$

$i \in \{1, 2, \ldots N-1\}$, containing MTR codewords, and set

$$M_{set} = \{\underline{m} = (m_0 m_1 \cdots m_{M-1}) \in Z_2^M \mid m_k \in (0,1)\}, \tag{4}$$

$k \in \{1, 2, \ldots M-1\}$, representing output sequences of the MTR decoder, the process of the MTR decoding can be described as "1–1" mapping

$$MTR^{-1} : \underline{n} \in N_{set} \to \underline{m} \in M_{set}, \tag{5}$$

transforming one sequence from N_{set} to corresponding sequence from M_{set}.

The MAP algorithm is based on subsets

$$N_{bk\,subset} = \{\underline{n} \in N_{set} \mid \underline{m} = MTR^{-1}(\underline{n}) \wedge m_k = b\}, \tag{6}$$

containing those codewords for which MTR^{-1} mapping produces that in a decoder output, at a particular position k, the bit m_k is equal to b, where $b \in (0, 1)$.

These subsets are shown in Table I, for 4/5 (2, 8) MTR code, and for output bit at position $k = 2$ [10], [11], [15],[16].

TABLE I. THE MAP SUBSETS FOR MTR DECODING

$N_{1\,(k=2)\ subset}$		$N_{0\,(k=2)\ subset}$	
$n_0n_1n_2n_3n_4$	$m_0m_1m_2m_3$	$n_0n_1n_2n_3n_4$	$m_0m_1m_2m_3$
0 0 0 1 0	0 0 **1** 0	1 0 0 0 0	0 0 **0** 0
1 0 0 0 1	0 0 **1** 1	0 0 0 0 1	0 0 **0** 1
0 0 1 1 0	0 1 **1** 0	0 0 1 0 0	0 1 **0** 0
1 0 1 1 0	0 1 **1** 1	0 0 1 0 1	0 1 **0** 1
0 1 0 1 0	1 0 **1** 0	0 1 0 0 0	1 0 **0** 0
1 0 0 1 0	1 0 **1** 1	0 1 0 0 1	1 0 **0** 1
1 0 1 0 0	1 1 **1** 0	0 1 1 0 0	1 1 **0** 0
1 0 1 0 1	1 1 **1** 1	0 1 1 0 1	1 1 **0** 1

The $N_{bk\ subsetst}$ subsets are pre-requested for the MAP algorithm and they can be prepared in advance, so that the decoding process can be accelerated.

a) The max-log MAP Subversion

The main subversion of the MAP algorithm operates with probabilities $q_{in\,0}(i)$ and $q_{in\,1}(i)$ of input bit, at sequence position i, which are obtained from (1) as

$$q_{in\,b}(i) = \frac{\exp((-1)^{b+1} L_{in}(i))}{F(i)}, \tag{7}$$

where $F(i) = \exp(L_{in}(i)) + \exp(-L_{in}(i))$.

The output probabilities of bit, at position k, in the decoder output sequence, are calculated as

$$\begin{aligned} q_{out\,b}(k) &= P(m_k = b) = \\ &= \sum_{\underline{n} \in N_{bk\ subset}} \prod_{i=0}^{N-1} P(n_i = b) = \sum_{\underline{n} \in N_{bk\ subset}} \prod_{i=0}^{N-1} q_{in\,b}(i). \end{aligned} \tag{8}$$

It can be observed that expression $F(i)$ depends solely on input soft-values, and does not depend on codewords in subsets $N_{bk\ subset}$. Thus, it can be extracted from the sum, when expression (7) is substituted in (8) [10], [11], [15], [16].

Moreover, the probabilities $q_{out\,0}(k)$ and $q_{out\,1}(k)$ will appear in output soft-value (1), the same as the difference of two *log*() functions, and, thus, it is possible to work with

$$\begin{aligned} r_b(k) &= \sum_{\underline{n} \in N_{bk\,subset}} \prod_{i=0}^{N-1} \exp((-1)^{b+1} L_{in}(i)) \\ &= \sum_{\underline{n} \in N_{bk\,subset}} \exp\left(\sum_{i=0}^{N-1} (-1)^{b+1} L_{in}(i)\right), \end{aligned} \tag{9}$$

without affecting or changing the way in which the output soft-value $L_{out}(k)$ can be calculated [10], [11], [15], [16].

Furthermore, decoder probabilities from (9), can be expressed in logarithmic form as

$$R_b(k) \equiv \log r_b(k) \approx \max_{\underline{n} \in N_{bk\,subset}} \left(\sum_{i=0}^{N-1} (-1)^{b+1} L_{in}(i) \right), \quad (10)$$

using the following approximation [9], [10], [12], [13]

$$\log\left(\sum_{i=1}^{P} \exp(a_i) \right) \approx \max(a_1, a_2, \ldots, a_P), \quad (11)$$

where *max*() returns maximal value among variables.

Using this logarithmic form, the output soft-value $L_{out}(k)$ can be computed as

$$L_{out}(k) = R_1(k) - R_0(k), \quad (12)$$

leading to the new subversion for soft-decision decoding of MTR codes, named as *max-log* MAP subversion.

D. The MAP in LC Approach

The conventional MAP approach considers MTR decoding as simple sequence mapping, computing $L_{out}(k)$, of particular bit k, using corresponding $N_{bk\,subsets}$.

The Boolean logic circuits can be seen, also, as sequence translators and thus implementation of the MAP approach imposed in logical circuits seems rational. The idea is to try to embed the *max-log* MAP approach into decision logic of new circuits, and later to design a new MTR decoder with such redesigned logic circuits.

a) The max-log MAP in LC for new AND circuit

In case of the AND circuit, the mapping and the corresponding MAP subsets are shown in Table II [12], [13].

TABLE II. MAP SUBSETS FOR AND LOGIC CIRCUIT

AND			$N_{1\,subset}$		$N_{0\,subset}$	
n_0n_1	m_0		n_0n_1	m_0	n_0n_1	m_0
0 0	0				0 0	0
1 0	0				1 0	0
0 1	0				0 1	0
1 1	1		1 1	1		

It can be observed that the length of the input codeword is $N = 2$, while of the output word $M = 1$. In that sense, the *max-log* MAP in LC works with just a few elements in the corresponding subsets, and, most importantly, with short-length codewords.

According to (10), the *max-log* MAP in LC subversion works with values

$$\begin{aligned} R_0 &= \max[-L_{in}(0) - L_{in}(1), -L_{in}(0) + L_{in}(1), \\ &\qquad L_{in}(0) - L_{in}(1)], \\ R_1 &= L_{in}(0) + L_{in}(1), \end{aligned} \quad (13)$$

while, the output soft-value is calculated as

$$L_{out}^{\max-\log MAP\,in\,LC} = R_1 - R_0, \quad (14)$$

allowing for simple creation of the output soft-values of the new redesigned AND circuit.

b) The max-log MAP in LC for new OR circuit

In case of the OR circuit, the corresponding MAP subsets are shown in Table III.

TABLE III. MAP SUBSETS FOR OR LOGIC CIRCUIT

OR			$N_{1\,subset}$		$N_{0\,subset}$	
n_0n_1	m_0		n_0n_1	m_0	n_0n_1	m_0
0 0	0				0 0	0
1 0	1		1 0	1		
0 1	1		0 1	1		
1 1	0				1 1	0

According to (10), the *max-log* MAP in LC for new OR circuits works with values

$$\begin{aligned} R_0 &= \max[-L_{in}(0) - L_{in}(1), L_{in}(0) + L_{in}(1)], \\ R_1 &= \max[-L_{in}(0) + L_{in}(1), L_{in}(0) - L_{in}(1)], \end{aligned} \quad (15)$$

while the output soft-value is obtained similarly to (14).

Via utilization of such redesigned circuits into hardware realization, as depicted in Fig. 1, the new soft-decision decoder for MTR decoding can be realized and, what is more, implemented in iterative decoding schemes.

III. COMPLEXITY OF THE MAP DECODER

The main problem with the MAP algorithm implementation lies in the potentially high number of codewords in the corresponding $N_{bk\,subsets}$ [10], [11], [15], [16].

The MAP decoding approach primarily leans towards implementation at the software level. Unfortunately, depending on the code rate of the used MTR code, the $N_{bk\,subsets}$ can contain a considerable number of codewords that can unnecessarily slow down the decoder and the decoding process.

A. Complexity of the max-log MAP Approach

Considering general MTR code, with code rate $R = M/N$, it can be easily shown that *max-log* MAP subversion requires

$$\begin{aligned} N_{oper}^{b}(k) &= N \cdot (1add \cdot 1multip) \cdot (N-1)add \cdot \frac{2^M}{2}\,words \\ &= 2^{M-1} \cdot N \cdot (N-1), \end{aligned} \quad (16)$$

operations to produce the $R_b(k)$, according to (10), leading to the total number of

$$N_{oper}^{total}(k) = 2 \cdot N_{oper}^{b}(k) = 2^M \cdot N \cdot (N-1) \quad (17)$$

operations, necessary to produce the output soft-value, for bit at position k, according to (12). During this analysis it was assumed that addition has the same complexity as multiplication.

Given that the both $N_{bk\,subsets}$ contain the same number of codewords, each output of the *max-log* MAP decoder requires the same number of operations. In case of the rate R = 4/5 MTR code this number is

$$N_{oper}^{total} = 2^4 \cdot 5 \cdot (5-1) = 320, \quad (18)$$

operations for each decoder output.

B. Complexity of the max-log MAP in LC Approach

Considering the *max-log* MAP in LC implementation for AND and OR logic circuit, it can be seen that the required number of operations, in order to produce probabilities R_0 and R_1, according to (13) and (15), is

$$N_{oper}^{AND\ or\ OR} = 4 \cdot (2multip + 1add) = 4 \cdot 3 = 12. \quad (19)$$

This is the number of operations for the output of one redesigned circuit (AND or OR circuit). Although, it should be kept in mind that if an MTR decoder is realized, as presented in Fig. 1, then the actual number of required operations for the decoder outputs, in case of the *max-log* MAP in LC, is

$$\begin{aligned} N_{oper}^{m_0} &= 4AND \cdot 12 + 2OR \cdot 12 = 4 \cdot 12 + 2 \cdot 12 = 72. \\ N_{oper}^{m_1} &= 0, \\ N_{oper}^{m_2} &= 2AND \cdot 12 + 2OR \cdot 12 = 2 \cdot 12 + 2 \cdot 12 = 48, \\ N_{oper}^{m_3} &= 1AND \cdot 12 + 1OR \cdot 12 = 1 \cdot 12 + 1 \cdot 12 = 24. \end{aligned} \quad (20)$$

It can be observed that the *max-log* MAP in LC approach considerably decreases the required number of operations per output, comparing with conventional *max-log* MAP, as presented in Fig. 2.

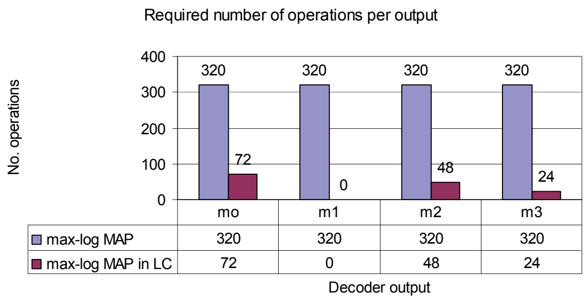

Figure 2. *max-log* MAP in LC versus *max-log* MAP.

Such a result is quite valuable in the case of extensive decoder utilization and demanding signal processing. However, as the realization of the MTR decoder using logic circuits implies that some optimization techniques will be used to decrease the overall number of logic circuits, then the real percent of reduction in a number of operations can differ between realizations.

However, an important result of analyses is that merging the MAP algorithm into Boolean logic circuits and working with the *max-log* MAP in LC will overcome the complexity of the original *max-log* MAP method.

IV. MTR Encoding Over a Two-track Channel

Multiple-head arrays had been proposed to enable reading and writing data at the same time on multiple tracks [14]. Such heads provide both high density and high speed [17], but they suffer from inter-track interference (ITI) [18]. The ITI is a result of a signal induced in reading heads as a superposition of magnetic transitions in neighboring tracks.

A. Channel Model

This paper considers a simple two-track two-head E^2PR4 recording channel, where two independent tracks exist and the reading heads simultaneously detect signals from both tracks [18]. It is assumed that linear and symmetrical ITI is present and modeled with the following matrix

$$A = \begin{bmatrix} 1 & \varepsilon \\ \varepsilon & 1 \end{bmatrix}, \quad (21)$$

where $\varepsilon \in [0,1]$ represents the ITI level between tracks [18].

A coding scheme employs rate 4/5 (2, 8) MTR code [1], as an in-track constrained code, so that each track is independently encoded, as shown in Fig. 3.

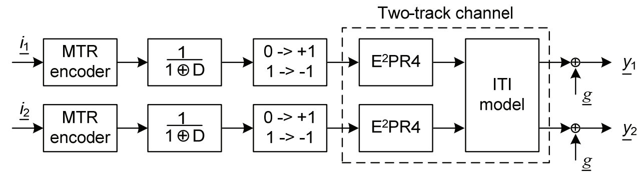

Figure 3. In-track MTR encoding over two-track channel.

where $\underline{i}_1$, $\underline{i}_2$, $\underline{y}_1$ and $\underline{y}_2$ are in-track input and output sequences.

Furthermore, it is assumed that read-back signals are distorted with additive, white and zero-mean, Gaussian noise g and that signal-to-noise ratio (SNR) is defined as

$$SNR = 10\log\left(\frac{E_b}{N_o}\right) = 10\log\left(\frac{E_b}{2\sigma^2}\right) = 10\log\left(\frac{E_c}{2R\sigma^2}\right), \quad (22)$$

where $E_c = RE_b$ is symbol bit energy at channel output, N_o is one-sided power spectral density and σ^2 is noise variance.

Channel detection was performed with the optimum two-head soft-output Viterbi detector (2H-SOVA) that uses ideal ITI estimation and the twenty symbols detection window [3], as shown in Fig. 4.

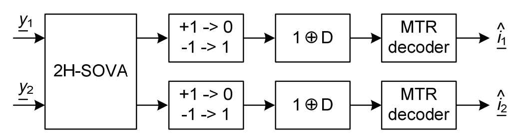

Figure 4. MTR decoding over two-track two-head.

Assuming that y_{ki} is a received symbol at instant i, in track k, the 2H-SOVA calculates branch distance between (y_{1i}, y_{2i}) and noiseless trellis transition label (v_{1i}, v_{2i}), as

$$[y_{1i} - (v_{1i} + \varepsilon v_{2i})]^2 + [y_{2i} - (\varepsilon v_{1i} + v_{2i})]^2, \quad (23)$$

where (u_{1i}, u_{2i}) is corresponding information bits label for the ITI-based trellis and ε represents ITI level [3].

B. Two-track Squared Euclidian Distance

The ITI presence in the two-track channel model can be used to partially improve channel detector performance [18].

It can be shown that knowing and incorporating ITI level into 2H-SOVA branch metric (23) considerably enhances the square Euclidian distance of two-track detector, regarding to independent in-track detection approach, as shown in Fig. 5 [4], [18], [19], [20].

Utilization of the two-head detector that simultaneously reads data from both tracks can mitigate detector performance degradation encountered by ITI presence.

The depicted square Euclidian distance demonstrates the advantage of two-head detector employment over an interfering channel. Over the range of ITI values

$$0 < \varepsilon < \varepsilon_d = 0.293 \quad (24)$$

Euclidian distance gradually increases and a growth of about 7.18% can be noticed, even though track interference is present [18], [19], [20].

This feature will help the two-head detector to successfully combat the low-level ITI in the interfering channel.

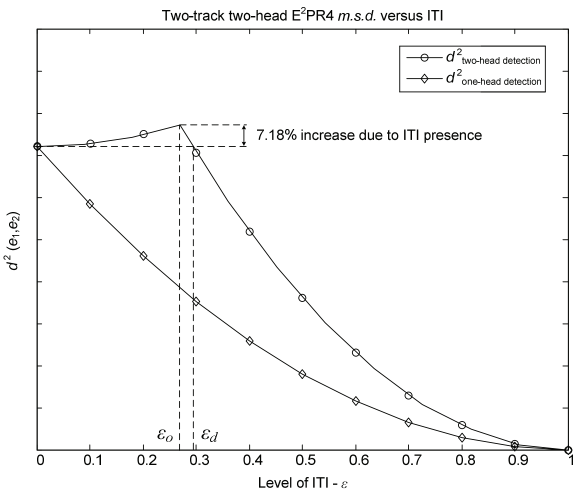

Figure 5. Two-track two-head *m.s.d.* versus ITI.

Regrettably, independent in-track detection is hindered by the ITI presence, degrading the performance of the one-head detector [19], [20]. In such a detection approach, the one-head detector is unable to combat against ITI. This fact additionally suggests that a two-head detector is highly desirable within interfering magnetic recording systems.

V. SIMULATION RESULTS

This paper is intended to analyze only the performance of the proposed algorithms. Thus, the simulation scheme implements only the MTR code, even though they possess a modest error correcting ability [1]. The paper focus is soft-value propagation through the MTR decoder and the complexity of the proposed algorithms.

A. MTR Decoding Using min-max in LC Approach

The *min-max* in LC was the first presented method for the MTR soft-decision decoding [7], [8]. In order to maintain consistency and to easily evaluate the MAP in LC method, the performances of soft-decision decoding using the *min* and *max* functions in logic circuits, are repeated in Fig. 6 [7], [8].

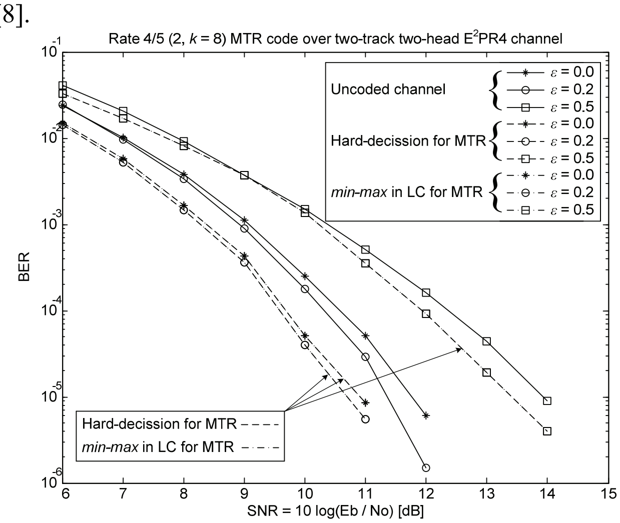

Figure 6. MTR decoding using *min-max* in LC.

The MTR is a single code implemented in an analyzed simulation scheme. Therefore, when the appropriate calculations are finished, the final decision is made in a binary way, even the MTR decoder internally operates with soft-values.

It can be observed that utilizing the *min-max* in LC, the decoding gain is about 0.5 dB for BER = 10^{-5} and ITI level ε = 0.0. Moreover, an additional gain is present for ITI level of ε = 0.2, because of the enhancement of the squared Euclidian distance [19], [20].

The *min-max* in LC method application outcome is identical to the performance achieved with the classical hard-decision approach, but the reason behind this is the solitary role of the MTR decoder, and its inability to fully exploit the soft-values and the corresponding confidences.

However, its advantage is that the MTR decoder is now able to handle soft-values, having performances not weaker than those obtained with conventional hard-decision [1].

The capability of the MTR decoder to manage the soft-values will become overt in an encoding scheme that uses combination of MTR and error-correcting codes [15], [16], e.g. in some of the iterative decoding schemes.

B. MTR Decoding Using the Conventional MAP Approach

The performance of the *max-log* MAP soft-decision decoding of MTR codes is presented in Fig. 7.

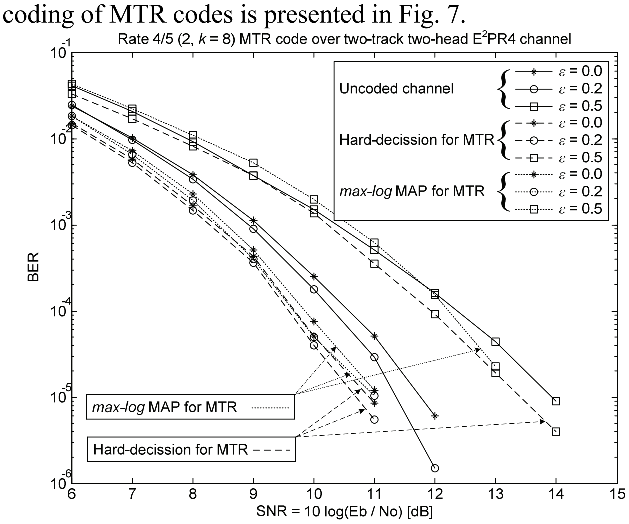

Figure 7. MTR soft-decision decoding using *max-log* MAP.

The *max-log* MAP approach shows a slightly inferior performance to that of hard-decision, but this slight difference is overshadowed by the fact that the hard-decision approach cannot produce output soft-values.

The decoding gain is again around 0.5 dB for BER = 10^{-5} and ε = 0.0, but for ITI level of ε = 0.2, the gain of nearly 1 dB is obtained, for the same BER level.

Such a result indicates that the approximation applied for output probabilities $R_b(k)$ (10), does not hinder the performance of the *max-log* MAP method [10], [11], [15], [16].

C. Comparison of the MTR Decoding Approaches

Finally, a comparison was made between the approaches for MTR soft-decision decoding over the E^2PR4 two-track system. The performances are summarized in Fig. 8.

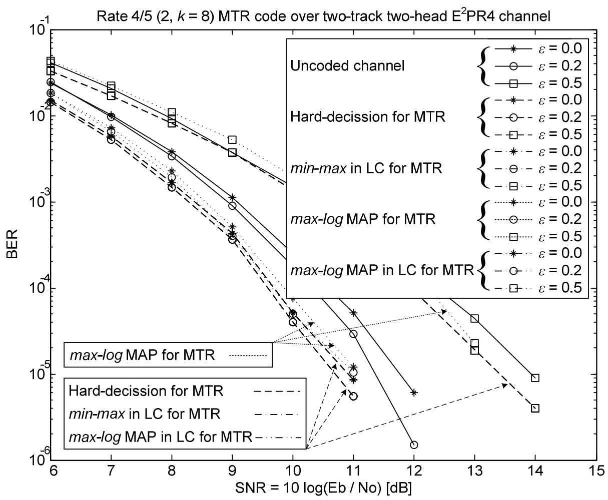

Figure 8. Approaches for MTR soft-decision decoding.

It can be observed that logic circuits utilization, both with *min-max* functions or *log-max* MAP in LC, resulted in a decoding gain of around 0.5 dB for BER = 10^{-5} and $\varepsilon = 0.0$ and nearly 1 dB for of $\varepsilon = 0.2$ and the same BER level.

Considering all presented simulation results, it can be observed that the soft-decision approach has performances that are similar to or somewhat worse than the classical MTR hard-decision [1]. Unfortunately, the solitary position of the MTR decoder, in this simulation scheme, fails to offer exploitation of both decision confidence, and, consequently, the full benefits of soft-decision decoding. Therefore, it is to be expected that the real power of the analyzed approaches will be in some iterative simulation scheme and in combination with some of the error-correcting codes [15], [16].

VI. CONCLUSION

This paper considers the concept of the *max-log* MAP in LC, as an additional method for MTR soft-decision decoding, over the two-track E^2PR4 magnetic recording channel.

This method was compared to already presented MTR soft-decision approaches, suggesting that *min-max* in LC and *max-log* MAP in LC are simpler and more effective than regular *max-log* MAP, which works on the set of MTR codewords.

The low complexity of *max-log* MAP in LC decoder, as well as the fact that MTR codes can reduce the overall two-track channel detection, suggests that their utilization and its soft-decision approaches would be quite valuable, especially in the case of interfering channels.

In addition, simulation results suggest that the *max-log* MAP in LC method will additionally enable MTR code utilization in the iterative decoding framework, and that will result in considerable gain when MTR is to be used in combination with some powerful error-correcting codes over channels for high magnetic recording densities.

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