The Concept of Resonance: From Physics to Cognitive Psychology

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***Abstract*—There are very few connections between physics and cognitive psychology. But in this paper, we assume that recent models inspired by concepts issued from physics and problem-solving cognitive processes like the Model Human Processor with Real Time Constraints (MHP/RT) model (Kitajima and Toyota, 2012) [1] allow to better describe and predict human behaviors especially in complex and dynamical environments where interactions between several bands and space-time constraints exist. After presenting the importance of the concept of resonance in physics and in cognitive psychology, the deterministic chaos in human action and behavior will be described, by focusing on an innovative model directly inspired by models issued from physics and problem-solving cognitive processes, Model Human Processor With Real Time Constraints (MHP/RT). If nowadays, the distance between physics and psychology is very prominent, the main goal of this paper is to defend the necessity to (re-)create strong relationships between physics and psychology to better understand and predict human behaviors because these situations are the majority of situations where an individual takes actions (such as walks, reads, stops, watching the other pedestrians' behavior in complex buildings or in street, etc.).**

Keywords–Physics; Resonance; Deterministic chaos; Cognitive modelling

I. INTRODUCTION

The human mind is endowed with innate primordial perceptions such as space, distance, motion, change, flow of time, and matter. Nevertheless, nowadays, there is very few connection between physics and cognitive psychology. Can physics offer anything to the cognitive sciences? The primary goal of this paper is to demonstrate why the answer to this question is in positive.

Physics is science that deals with the structure of matter and the interactions between the fundamental constituents of the observable universe. Psychology is scientific discipline that studies mental processes and behavior in humans (and animals). Physics (the basic natural science) elucidates the simplest fundamental questions in nature. Psychology is intimately related to the humanitarian sciences. Physics is concerned with all aspects of nature. Psychology is the science of individual or group behavior. Despite fundamental differences, there are the same problems even in so completely different disciplines. Interdisciplinality is best seen as bringing together distinctive components of two or more disciplines [2].

From a historical point of view, physics and psychology were strongly related [3]. For instance, in Wilhelm Wundt's (1832–1920) Leipzig laboratory and at numerous other research sites, procedures, techniques and tools issued from physics were used to conduct reaction time experiments. In the same way, the purpose of Wheatstone (1802–1875), one of the most famous English physicist, was to test his theory of stereo vision and for investigations into what would now be called experimental psychology.

But nowadays, the distance between physics and psychology is important: first, very few courses of fundamental sciences (biology, physiology, etc.) exist in psychology curriculum; second, no psychology course is proposed in science curriculum except for some hours in specific curricula (e.g., curricula in cognitive sciences). From an epistemological point of view, one of the main determinant factors was the division of the human being into a soul and a body which are united but separated [4]. This division, which corresponded to the Aristotelian classification of substances (Metaphysics XII), posed the problem of the unity of the science of the soul and its place alongside other fields [5] and explain the absence of connection between physics and psychology in teaching. Yet, the experimental methods in cognitive sciences have physics as their basis.

In this paper, we assume that a majority of interactions, including psychological interactions between humans and and environment (social or physical environment), can be derived from physical processes and thus, be apprehended by using concepts issued from physics such as resonance.

II. RESONANCE IN PHYSICS AND IN COGNITION

This part is aiming to investigate the relationships between physics and psychology about the common concept of resonance.

A. *Resonance in Physics*

Initially, the term resonance originates from the field of acoustics, particularly observed in musical instruments, e.g., when strings started to vibrate and to produce sound without direct excitation by the player. Today, because this type of synchronization between two elements in a same environment (with the same constraints) is observed in other areas, resonance phenomena occur with all types of vibrations or wave (e.g., mechanical resonance, acoustic resonance, electromagnetic resonance, nuclear magnetic resonance, and so on).

The phenomenon of resonance is known and observed for a long time. For instance, In 1665, the Dutch physicist Christiaan Huygens, inventor of the pendulum clock, was lying in his bed with a minor illness and watching two of his clocks hanging on a wall. Huygens noticed something odd: No matter how the pendulums on these clocks began, in 10 minutes, they ended up swinging in exactly the opposite direction from each other. The cause of this effect, called “odd kind of sympathy” by Huygens, remained a mystery for centuries. Recently, two mathematicians Oliveira and Melo [6] calculated that, as pendulums move back and forth, sound pulses could travel through the wall from clock to clock. These pulses can interfere with the swings of the pendulums, eventually causing them to synchronize. Based on several experiments, Oliveira and Melo [6] have developed a model explaining the Huygens problem of synchronization between two clocks hanging from a wall. In their model, each element (i.e., clock) transmits once per cycle a sound pulse that is translated in a pendulum speed change. Today, because synchronization between two elements in a same environment (with the same constraints) is observed in a great number of areas, their model is used in several domains such as acoustics, cosmology, and mechanics. In other words, in physics, resonance occurs when the behaviors of two (or more) elements in a physical environment are synchronized by an autonomous (and unconscious) process of vibrating transmission between these elements.

B. Resonance in Psychology: Explanation of Imitation and Learning for Newborn

The young child’s ability to imitate the actions of other human beings is an important mechanism for social learning and for acquiring new knowledge. Since the first study conducted by Meltzoff and Moore [7] who reported evidence that two- to three-week-old infants imitate the behaviors of an adult model, a lot of reports of newborn imitation have been cited to support hypotheses about the origins and nature of imitation, including the hypothesis that imitation is a unitary competency inherited as a unit and may be shared as a unit by species with common ancestry (e.g., [8][9]).

Several studies about newborn [10]–[13] imitation have also shown that a specialized neurological mechanism underlies imitative behavior in human infants and adults, and that this neurological mechanism is inherited. According to these authors, this neurological mechanism is a “mirror system”. The idea of a mirror system specialized for imitation was initially suggested by the discovery of “mirror” neurons in monkeys [14][15]. Because mirror neurons appear to have both sensory and motor properties, nowadays researchers see in them the potential for a straightforward, automatic and heritable mechanism for imitation in humans [8]. The existence of this “mirror neurons” system for human has been demonstrated by using fMRI and was localized in the right inferior parietal lobe [16][17]. For human beings, the mirror system offers a way to make newborn imitation feasible: the “mirror-neuron” system appears to bypass the requirement for precocious knowledge and cognitive abilities in newborn infants who imitate because the system itself embodies that knowledge. The mirror system is thought to directly match visual input from an observed action with a stored motor program for the same behavior (e.g., [10][12][13]). If that motor program is then executed, the result is imitation.

Since ten years, the “mirror-neuron” system has received much interest in recent years because of its putative involvement in a range of important cognitive processes (for a synthesis, see [16]), from action understanding, observational learning, to socialization, theory of mind, and empathy. Moreover, dysfunction of the mirror system has been linked with some clinical disorders (e.g., in apraxia, autism,and schizophrenia, see [12]) and in social human behaviors, see [18].

The mirror-neuron system is the main biological support for resonance between individuals whatever the context (physical or social context): so, within natural social contexts, actions of individuals often imply interactions with others individuals by using the mirror-neuron system [19].

In other words, we assume that the mirror-neuron system is the physical/neurological support for the resonance in human behavior: newborn imitation is the result of a synchronization between behaviors issued from two elements in a same environment (i.e., the newborn and the adult). First, the adult initiates a behavior (e.g., tongue protrusion with mouth opening); Second, this external stimulus from the environment is processed by the newborn by perceptual/visual system to create internal representations, which “resonate with” memory to activate the relevant portion of memory automatically (Kitajima and Toyota (2012) [20]), and this initial activation triggers successive activation though its adjacent connected regions. The activated network at some time later defines what follows in the future, i.e., next behavior (i.e., tongue protrusion with mouth opening).

III. Resonance in Deterministic Chaos

If resonance is well-know in closed system from physicians and psychologists, the deterministic chaos is more relevant to understand it in our complex and open real systems.

A. The Deterministic Chaos: Generality

Classically, all physical laws are described by differential equations (e.g., [22]–[24]). Because initial state of elements present at the same time in a same physical environment, environmental constraint, and physical laws are known, it is possible to theoretically predict the future of the physical system (i.e., future states of elements) for all times. This is the deterministic view of nature. In other words, physics systems are deterministic because they obey deterministic differential equations. Because Wilhelm Wundt’s background was in physiology, and this was reflected in the topics with which the Institute was concerned (e.g., the study of reaction times, sensory processes, and attention), this deterministic principle was used in the Institute for Experimental Psychology at the University of Leipzig in Germany in 1879, the first laboratory dedicated to psychology. This deterministic principle is one of the main principles of modern psychology and always used in experimental design in psychology.

As observed for many systems in physics, acoustics and mechanics, knowledge about the future state is limited by the precision with which the initial state can be measured [25][26]. That is, knowing the laws of nature is not enough to predict the future. There are deterministic systems whose time evolution has a very strong dependence on initial conditions. In other words, the differential equations that govern the evolution of the system are very sensitive to initial conditions. Usually, physicians and mathematicians say that even a tiny effect, such as a butterfly flying nearby, may be enough to vary the conditions such that the future is entirely different than what it might have been, not just a tiny bit different. In this way, measurements made on the state of a system at a given time may not allow us to predict the future situation even moderately far ahead, despite the fact that the governing equations are exactly known. By definition, these equations are named chaotic and that they predict a deterministic chaos (for a recent review: [27]). This difficulty to have a global and

TABLE I. NEWELL'S TIME SCALE OF HUMAN ACTION (ADAPTED FROM [21, PAGE 122, FIG. 3-3]), LEFT PORTION OF THE TABLE, ADDED WITH ASSOCIATED ACTIVITIES.

Scale (sec)	Time Units	System	World (Theory) BAND	Activity: Internal	Activity: Bodily Habitual	Activity: Organic Habitual	Activity: Organic Interactive
				System 1	System 1 / System 2	System 2	System 2
10^7	months						
10^6	weeks		SOCIAL				✓
10^5	days						
10^4	hours	Task					
10^3	10min	Task	RATIONAL			✓	✓
10^2	minutes	Task					
10^1	10sec	Unit Task					
10^0	1sec	Operations	COGNITIVE		✓	✓	
10^{-1}	100ms	Deliberate Act					
10^{-2}	10ms	Neural Circuit					
10^{-3}	1ms	Neuron	BIOLOGICAL	✓	✓		
10^{-4}	100μsec	Organelle					

perfect knowledge of the initial state of a human being is also a common characteristics in human and social sciences. In other words, as said Lorentz in 1963 [28], deterministic chaos can be used:

> When the present determines the future, but the approximate present does not approximately determine the future.

A deterministic chaos system has three characteristics that can be applied to human behavior:

(A) **Long term behavior is difficult or impossible to predict**: Even very accurate measurements of the current state of a chaotic system become useless indicators of where the system will be;

(B) **Sensitive dependence on initial conditions (a property noted by Poincare, Birkhoff, and even Turing)**: Starting from very close initial conditions a chaotic system very rapidly moves to different states;

(C) **Local instability versus global stability**: In order to have amplification of small errors and noise, the behavior must be locally unstable: over short times nearby states move away from each other. But for the system to consistently produce stable behavior, over long times the set of behaviors must fall back into itself, i.e., resilience.

So, deterministic chaos focuses on the behavior of dynamical systems that are highly sensitive to initial conditions but sustainable in the environment, that could change within a limited range, with a resilience mechanism.

B. The Deterministic Chaos in Human Action: Time Scale of Human Action

Human behavior can be best viewed as a composite of non-linearly connected four bands that are associated with different characteristic times, ranging from 10^{-4} sec to 10^7 sec as suggested by Newell [21, page 122, Fig. 3-3]. As shown by Table I, it identifies non-linearly connected four bands; SOCIAL, RATIONAL, COGNITIVE, and BIOLOGICAL BANDs. The claim that human action should be structured in terms of the discrete four bands suggests that it should be possible to build predictive and explanatory models for activities that happen within a single band, and at the same time, it should be impossible, or inappropriate, to build predictive models that include activities that happen in multiple bands with inter-band interactions. Impossibility of constructing predictive models for inter-band activities comes from the existence of non-linear connections between bands. This should make the models non-linear and even if they are deterministic they suffer from SEnsitive Dependence on Initial Condition (SEDIC), the primary feature of such a non-linear system and make them unpredictable.

C. The Resonance in Deterministic Chaos for Organizing Human Behavior

The majority of situations where an individual takes actions, such as walks, reads, stops, watching the other pedestrians' behavior in complex buildings or in street, etc., is performed in space-time environment where different factors and physical constraints exist. For example, as Hoogendoorns and Bovy [29] said, one of the most interesting and challenging theoretical and practical problems in describing/predicting pedestrians behavior are route choice and activity scheduling. The main reason is that, compared to other modes of transport, a characteristic feature of pedestrian route choice is that routes are continuous trajectories in time and space: since a pedestrian chooses a route from an infinite set of alternatives, dedicated theories and models describing pedestrian route choice are required. There exists several models describing and predicting pedestrians behavior, these models being mainly issued from mathematics, computer science, psychology, and sociology. Recently, Kitajima and Toyota [1] conceived an innovative model (called Model Human Processor with Real Time constraints (MHP/RT)) to describe and predict human behavior in dynamic and complex environments.

MHP/RT is directly inspired by models issued from physics and problem-solving cognitive processes, and simulates people's action selection as interactions between System 1 (uncon-

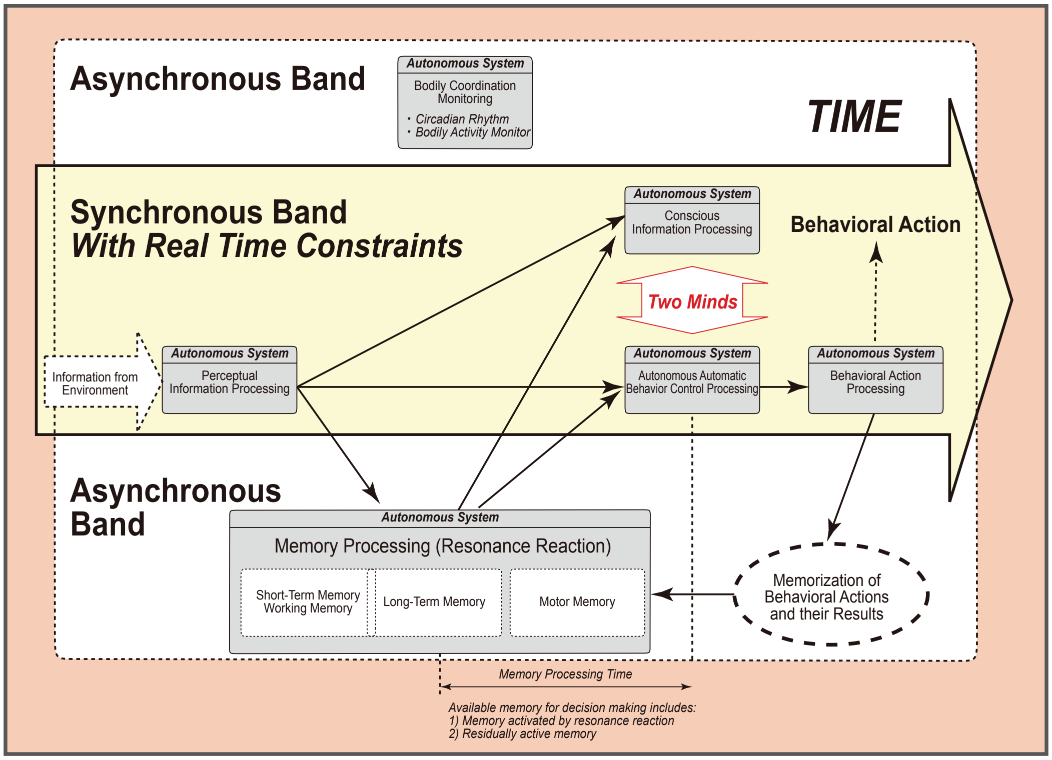

Figure 1. Outline of MHP/RT [1, Figure 4].

scious automatic fast processes carried out in the time range of $<$ 100 msec in the BIOLOGICAL BAND) and System 2 (conscious deliberate slow processes which take seconds, minutes, and even much longer in the COGNITIVE, RATIONAL and SOCIAL BANDs) that make use of memory through the resonance mechanism. System 1 and System 2, otherwise called "Two Minds", are the terms used by Kahneman [30][31]. They constitute the basis of behavioral economics, which deals with decision making in human beings' economic activities. Figure 1 illustrates MHP/RT schematically. It operates in two bands, the asynchronous band and the synchronous band. The Bodily Coordination Monitoring System and the Memory Processing System operate in the asynchronous band. The Perceptual Information Processing System, Conscious Information Processing System, Autonomous Automatic Behavior Control Processing System, and Behavioral Action Processing System operate in the synchronous band. These systems work autonomously. System 1 of the Two Minds corresponds to the Autonomous Automatic Behavior Control Processing System, and System 2 corresponds to the Conscious Information Processing System.

Even if the human brain is not literally divided in a dual system, the distinction between the two systems (System 1 and System 2) is a useful analogy. Figure 2 illustrates how MHP/RT outlined by Figure 1 works while a person behaves in the real environment:

1) When a sensory input is extracted from the physical and/or social environment (e.g., visually and auditory information), System 1, System 2 and Long Term Memory (LTM) are activated by neural activation based on matching and mapping processes;
2) System 1 (biological band) thinking is automatic, quick, intuitive, emotional and reactive while System 2 thinking is conscious, effortful, logical, and deliberate. System 1 is driven by emotion and snap judgement, especially if we are time-pressed, multitasking or tired. Most of the time, most people function using their System 1 as it requires little effort. System 1 operates automatically and quickly, with little or no effort and no sense of voluntary control;
3) System 2 allocates attention to the effortful mental activities that demand it, including complex computations. The operations of System 2 are often associated with the subjective experience of agency, choice, and concentration;
4) Finally, a motor output (i.e., a behavior, an effective action) is produced on the basis of the mental representation elaborated;
5) In reality System 1 and 2 work in tandem, they complement each other: System 1 feeds relevant input into our System 2.

This dual-process approach posit that a lot of cognitive process – decision-making, emotion processing, memory formation, or even the manifestation of thought itself – can arise from one of two "pathways" (System 1 and System 2), and that those two pathways can operate relatively independently from one another. In other words, the MHP/RT model processes through a combination of Systems 1 (intuition or pattern-recognition) and Systems 2 (analytic) thinking.

Kitajima and Toyota [1] provided detailed explanations for the results of field study at train stations which focused on the

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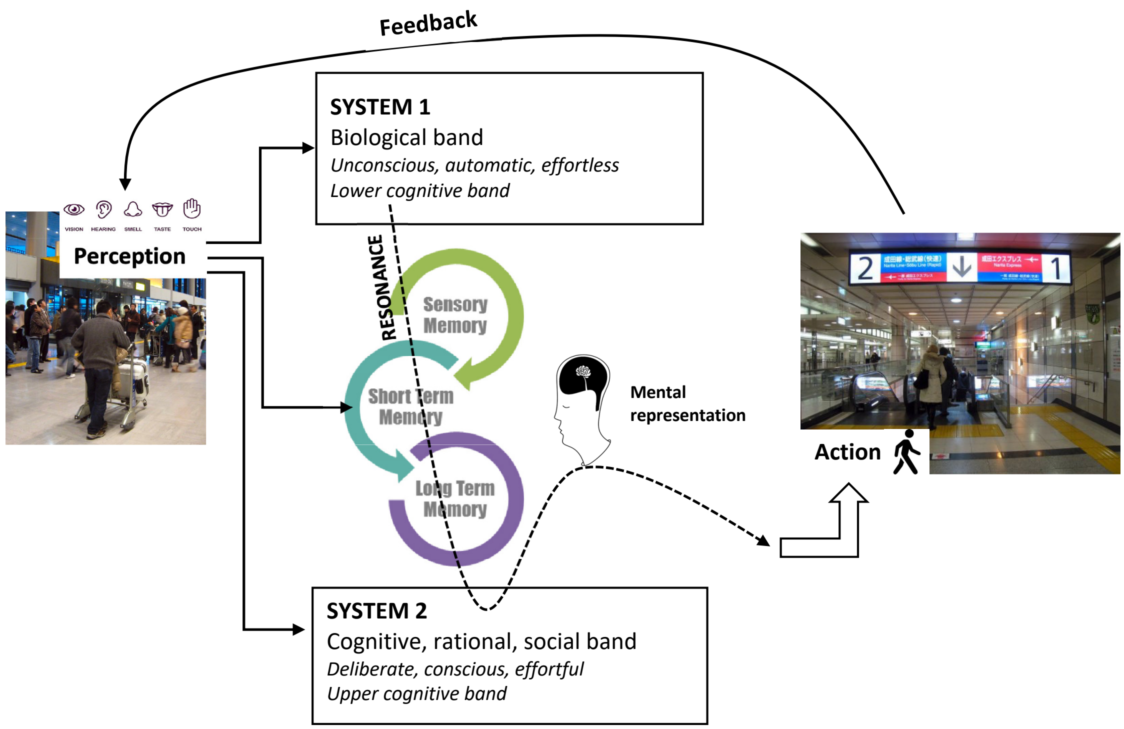

Figure 2. An illustration of MHP/RT's processes while a traveller tries to find his/her way at an airport.

characteristic behaviors of participants (or elderly passengers) who had deficit in one of three cognitive capabilities of attention, planning and working memory. The different operations can be described as follows:

- memory activation triggered by external/internal (bodily) stimuli starts in the lower BIOLOGICAL BAND, and it is kept activated in the lower BIOLOGICAL BAND;
- System 1 processes in the upper BIOLOGICAL BAND ($< 100 \sim 150$ msec) hook the part of activated memory to act in a feed-forward control, which we describe System 1 process uses the memory by resonance; because individual is arriving from Narita Airport by train and because s/he wants to go to visit the Imperial Palace in Tokyo, s/he has to go to the upper ground of the train station; so traffic signs about stairways and elevators are visually detected in surrounding environment;
- System 2 processes typically performed in COGNITIVE and RATIONAL BANDs hook the part of activated memory to act in a feed-back control, which we describe System 2 uses the memory by resonance; because many exits are possible from the upper ground of Tokyo train station, individual has to choose the most relevant; if s/he remembers that the Imperial Palace is located very closed to South exit, s/he has to choose the South exit; so s/he has to activate mental schema to find this relevant exit;
- System 1 and System 2 are synchronized occasionally to check if the feed-forward control works fine, – since System 1 and System 2 work in different BANDs, they may suffer from the features of deterministic chaos and cannot be connected linearly; if unfortunately, South exit is closed because works, individual has to choose another way; for instance, s/he can decide to go to the East exit then to take the way on the right side to access to the Imperial palace; or s/he can decide to follow other pedestrians who have map of the Imperial Palace in their hands because s/he can hypothesize that these other pedestrians want also to visit the same location;
- the memory activation that reflect directly the external and internal bodily situations which should represent "reality" is one part of resonance, and System 1 processes and System 2 processes for creating actions to interact with the environment and should cause changes that define the next situation for the behaving-self are the other part of resonance; several times during her/his movement, individual monitors and controls that s/he is on the good way;
- resonance is a mechanism for coordinating non-linearly connected BANDs to have the self behave in the ever-changing environment

IV. Conclusion

From a historical point of view, physics and psychology were strongly related. But nowadays, the distance between physics and psychology is very prominent. The main goal of this paper is to defend the necessity to (re-)create strong relationships between physics and psychology to better understand and predict human behaviors, especially in complex and dynamical environments where interactions between several bands and space-time constraints exist. And these situations are the majority of situations where an individual takes actions (such as walks, reads, stops, watching the other pedestrians' behavior in complex buildings or in street, etc.).

The modern MHP/RT elaborated by Kitajima and Toyota [1] is directly inspired by models issued from physics and problem-solving cognitive processes, and simulates people's action selection as interactions between System 1 (unconscious automatic fast processes carried out in the time range of $<$ 100 msec in the BIOLOGICAL BAND) and System 2 (conscious deliberate slow processes which take seconds, minutes, and even much longer in the COGNITIVE, RATIONAL and SOCIAL BANDs) that make use of memory through the resonance mechanism. Based on several notions issued from physics (deterministic chaos, resonance) and based on several concepts issued from psychology (long term memory, working memory, attention), we assume that this kind of model is a good example of models combining physics and psychology allowing to better describe human behaviors.

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