Geometry of Intentionality in Neurodynamics

Germano Resconi
Dept. of Mathematics and Physics
Catholic University
Brescia, Italy
resconi@speedyposta.it

Robert Kozma
Dept. of Mathematical Sciences
Computational Neurodynamics Laboratory
Memphis , TN 38152 USA
rkozma@memphis.edu

Abstract— **Brains are the most complex systems in the known Universe and they are composed of simple neural units producing electrical impulses. The key contribution of this work is the use of geometric concepts for neural states, in terms of currents and voltages. In the introduced adaptive neuromorphic electrical circuit, we relate voltage with current using the conductance matrix or alternatively using the impedance matrix. After a change of reference system and introducing a new distance measure, we transform the initially linear relationships into a geodetic over the phase space. Every change of variables can be reproduced by a similar change of voltages into currents and vice versa with the help of the conductance or impedance matrices, respectively. Our geometric approach to neurodynamics can be applied to integrate digital computer structure with neuromorphic computing principles, which produces an efficient new computational paradigm.**

Keywords- ***Neurodynamics; Intentionality; Geometry of Cognition; Neuromorphic Computing ; Geodesic; Memristor.***

I. INTRODUCTION

This work proposes a mathematical formulation of intentional dynamics following Freeman's half century-long dynamic systems approach [1-4]. We consider the electrical behavior of the brain at the microscopic level and derive principle leading to higher cognition at the mesoscopic and macroscopic levels. In the past decades, artificial neural networks have been successfully employed To model various brain functions such neural net models consist of a large number of high precision components, which leads scaling problems manifested in slow and often unstable learning, inefficient adaptation properties, and the need of external learning and control action. In this work, we aim at fast and stable learning and autonomous adaptation using low precision components. To achieve this goal we employ dynamical components, which are part of a closed feedback the loop with the outside world. The corresponding ordinary differential equations (ODEs) represent a stiff nonlinear system, which are very inefficient to solve on digital computers. An example is the IBM Blue Gene project with 4096 CPUs and 1000 Terabytes RAM [5]. To simulate the Mouse cortex with 8 x 10^6 neurons, 2 x 10^{10} synapses at 10^9 Hz, this digital implementation requires 40 KW. The human brain has 10^{10} neurons and 10^{14} synapses; its clock frequency is 10 Hz and uses 20 W with its analog circuitry. Apparently, analog is more efficient then digital by many orders of magnitude.

The use of analog electrical circuits for neuromorphic computing with memristors could solve the problem of neural computation in the future. It is conceivable that learning effects so eloquently displayed by memristors are in fact manifestations of memristive behavior in the neural tissue [6-7]. In this case memristors indeed could be the *Holy Grail* of building brain like computers by exploiting the same mechanisms in computer memories as the ones brains employ. This possibility has enormous long-term consequences, which is difficult even to imagine from our present limited vantage point. Recall that for Turing the physical device is not computable by a Turing machine, which is the theoretical version of the digital computer. If we use analog systems, we do not need algorithms to program the neurons. Rather, the computer algorithm is substituted by the network dynamics evolving in phase space. For example, we can program the CrossNet [8-9] electrical system, which has been used to compute the parameters describing the desired trajectories. Geometrical and physical descriptions of Freeman's intentional neurodynamics are beyond algorithmic and digital computation. To clarify better the new computation paradigm, we observe that animals and humans have a very efficient mechanism to use their finite brains to comprehend and adapt to infinitely complex environment [1-3]. We show that this adaptive system conforms to a geometric interpretation. This gives us the possibility to implement the required parameters in ODE to achieve the desired intentional neurodynamics. In this work, we consider two aspects of intentionality, namely cognitive and physiological,, both are reflections of nonlinear brain dynamics. At the cognitive level, intentionality corresponds to the change of the reference system, while physiological intention is defined by neuro-dynamical processes. First we describe the brain connectivity structure as a material support or hardware that makes it possible to achieve the required intentional transformation. The neural units are modeled by electrical circuits with capacitors and adaptive resistors and their dynamics is described by Ordinary Differential Equations (ODE). The solutions of these ODEs give the trajectory of the neural states as they evolve in time. ODEs are widely used in neural models, however they have shortcomings due to the difficulty of specifying the parameters and because their solutions cannot be found precisely in most practically relevant situations. In the introduced adaptive neuromorphic electrical circuit, we relate voltage with current using the conductance matrix or alternatively using the impedance matrix. Next, we introduce the change of reference system and a new distance measure, through which we transform the initially straight line into a geodesic over the phase space in the new reference. We show that every change of variables and reference can be reproduced by a similar change of the impedances and currents or voltages space coordinates and vice versa with the help of the conductance or impedance matrices, respectively, we can simulate by neural network (hardware) any change of reference or any change of variables (software). We conclude the work with discussion and outlining directions for future research.

II. INTENSION AND ELECTRICAL CIRCUIT

Because the brain is a complex electrical circuit with capacity and resistors, we think that it can express intentionality by suitable tune of the electrical parameters. The instrument to match intentionality with the dynamics neural mechanisms is the geometry in the space of the neural currents or voltages . We know that geometry is represented by the form of the distance in the given space. For example, in the Euclidean geometry the form is given by the straight line where the distance is the Euclidean distance ; on the sphere, the form of the distance is a circle and the distance is the non Euclidean distance on the sphere. The form of the geometry in the electro dynamics neural mechanism is the trajectory of a point in the space of the currents or voltages. The distance in the geometry of the neural system is given by the square root of the instantaneous electrical power W as follows:

$$W = \sum_j i_j(v_1, v_2, \dots v_p) v_j$$

and

$$i_j(v_1, v_2, \dots v_p) \approx \sum_k \frac{\partial i_j}{\partial v_k} v_k = \sum_k C_{j,k} v_k \qquad (1)$$

or

$$i = Cv, v = C^{-1} i = Zi, Z = C^{-1}$$

we have

$$W = \sum_j i_j(v_1, v_2, \dots v_p) v_j = \sum_{j,k} C_{j,k} v_k v_j = v^T C v$$

$$= (C^{-1} i)^T C (C^{-1} i) = i^T C^{-1} i = i^T Z i = \sum_{j,k} Z_{j,k} i_k i_j$$

In equations (1) the first equation is the ordinary computation of the electrical power by currents and voltages. The second equation is the computation of the matrix of the conductance C for which from the current we compute the voltages. To obtain the matrix of conductance C , we expand the relation between voltages and currents in Taylor form and we stop at the first derivative. The other equations in (1) are obtained substituting the voltage current relation by the matrix C. Z is the impedance matrix. Now the power gives us the *material* aspect of the intentionality. The other part of the intentionality is the *conceptual* that is given by the wanted transformation

$$\begin{cases} y_1 = y_1(x_1, x_2, \dots x_p) \\ y_2 = y_2(x_1, x_2, \dots x_p) \\ \quad \dots \\ y_q = y_q(x_1, x_2, \dots x_p) \end{cases} \qquad (2)$$

The transformation (2) can be compared with one statement in the digital computer where in input we have the x values and in output the y values. So, the (2) is a MIMO (many inputs many outputs) transformation or statement shown in Figure 1

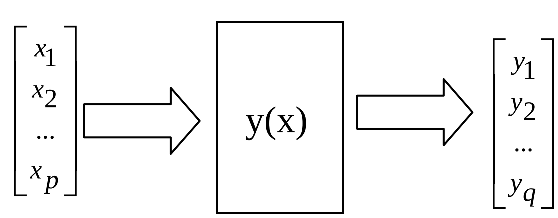

Figure 1 Transformation from input x into output y by MIMO y(x),

We can have different examples of the transformation as linear transformation

$$\begin{cases} y_1 = m_{1,1}x_1 + m_{1,2}x_2 + \dots + m_{1,p}x_p \\ y_2 = m_{2,1}x_1 + m_{2,2}x_2 + \dots + m_{2,p}x_p \\ \dots \\ y_q = m_{q,1}x_1 + m_{q,2}x_2 + \dots + m_{q,p}x_p \end{cases} \quad (3)$$

where m are the linear parameters of the transformation (3). We can have also the non linear transformation as follows

$$\begin{cases} y_1 = (1-x_1)x_2 + x_1(1-x_2) \\ y_2 = x_1x_2 + (1-x_1)(1-x_2) \end{cases} \quad (4)$$

Here the first equation is connected with XOR and the second is connected with EQ logic relation. Given the system in Eq.2, the total differential of any equation in (2) are

$$\begin{cases} dy_1 = \frac{\partial y_1}{\partial x_1}dx_1 + \frac{\partial y_1}{\partial x_2}dx_2 + \dots + \frac{\partial y_1}{\partial x_p}dx_p \\ dy_2 = \frac{\partial y_2}{\partial x_1}dx_1 + \frac{\partial y_2}{\partial x_2}dx_2 + \dots + \frac{\partial y_2}{\partial x_p}dx_p \\ \dots \\ dy_q = \frac{\partial y_q}{\partial x_1}dx_1 + \frac{\partial y_q}{\partial x_2}dx_2 + \dots + \frac{\partial y_q}{\partial x_p}dx_p \end{cases} \quad (5)$$

We know that when the reference system for variables y is Cartesian, the straight line is a geodesic and the distance between two points with infinitesimal distance is the classical quadratic form

$$ds = \sqrt{dy_1^2 + dy_2^2 + \dots + dy_q^2} \quad (6)$$

After changing the reference from y to x, the same distance can be written as a function of x as follows:

$$ds = \sqrt{\left(\frac{\partial y_1}{\partial x_1}dx_1 + \dots + \frac{\partial y_1}{\partial x_p}dx_p\right)^2 + \dots + \left(\frac{\partial y_q}{\partial x_1}dx_1 + \dots + \frac{\partial y_q}{\partial x_p}dx_p\right)^2}$$

$$= \sqrt{\sum_{j,k} G_{j,k}dx^j dx^k} \quad (7)$$

where the down indices are the covariant components and the indices up are the contravariant indices In the new reference x the distance is the same minimum value so is again a geodesic. The metric tensor is given as:

$$G_{j,k} = G_{k,j} = \sum_p \frac{\partial y_p}{\partial x_j}\frac{\partial y^p}{\partial x_k} \quad (8)$$

According to optimum property principle of differential geometry, the variation of the distance C must equal to zero:

$$\delta C = \delta \int ds = \delta \sqrt{\sum_{j,k} G_{j,k}(x_i)\frac{dx^j}{dt}\frac{dx^k}{dt}} = 0 \quad (9)$$

The conditions under which Eq. 8 gives the minimum distance is given by Euler's differential equations

$$\frac{d}{dt}\frac{\partial \sqrt{\sum_{j,k} G_{j,k}(x_i)\frac{dx^j}{dt}\frac{dx^k}{dt}}}{\partial \frac{dx^k}{dt}} + \quad (10)$$

$$-\frac{\partial \sqrt{\sum_{j,k} G_{j,k}(x_i)\frac{dx^j}{dt}\frac{dx^k}{dt}}}{\partial x^i} = 0$$

The solutions of Eq. 10 give a space with distance measure in Eq. 8. The trajectories with minimum distance in this geometry are called geodesic. The distance in Eq. 8 can be associated with a system of ordinary differential equations (ODEs), the solutions of which produce the geometry. The electrical power W defined in Eq. 1 can be written by the derivatives of charges q_j:

$$W = \sum_{j,k} Z_{j,k} i_j i_k = \sum_{j,k} Z_{j,k}\frac{dq_j}{dt}\frac{dq_k}{dt} \quad (11)$$

and

$$\sqrt{W} = \sqrt{\sum_{j,k} Z_{j,k}\frac{dq_j}{dt}\frac{dq_k}{dt}} \quad (12)$$

where q are the electrical charges.

Note that Eq. 9 and Eq. 11 have similar structures. Eq. 9 is the geometric expression of the conceptual intention, while Eq. 11 expresses the material intention given by electrical dynamics of the neural network. Our goal is to derive how the conceptual intention can be transformed into its material counterpart; therefore, we require that parameters Z are equal to G:

$$\sqrt{W} = \frac{ds}{dt} \text{ and } G_{i,j} = Z_{i,j} \qquad (13a)$$

Eq. 13a can be written in terms of the voltage V and conductance C as follows:

$$\sqrt{W} = \frac{ds}{dr} \text{ and } G_{i,j} = C_{i,j} \text{ ,W=} \sum_{j,k} C_{j,k} v_j v_k \quad (13b)$$

The geometric method to transform the abstract or conceptual part of the intention into the material part is similar to the transformation of the software into the hardware in a digital computer.

Now we illustrate the geodesic for a simple electrical circuit and its associated ODE. Given the trivial electrical circuit [10, 11],

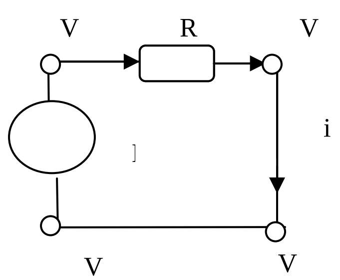

Figure 2 A simple electrical circuit with one generator E and one resistor R

we compute the power W that is dissipated by the resistance R and define the infinitesimal distance ds:

$$W = R\left(\frac{dq}{dt}\right)^2 = Ri^2$$

$$\left(\frac{ds}{dt}\right)^2 = W = R\left(\frac{dq}{dt}\right)^2 = Ri^2$$

$$ds = \sqrt{W}\,dt = \sqrt{R\left(\frac{dq}{dt}\right)^2}\,dt$$

In the electrical circuit, the currents flow in such a way as to dissipate the minimum power. The geodesic line in the one dimensional current space i is the trajectory in time. For the minimum dissipation of the power or cost C, we have

$$\delta C = \delta \int ds = \delta \int \sqrt{W}\,dt = \delta \int \sqrt{R\left(\frac{dq}{dt}\right)^2}\,dt = 0$$

We can compute the behavior of the charges for which we have the geodesic condition of the minimum cost. We know that this problem can be solved by the Euler differential equations [13]

$$\frac{d}{dt}\frac{\partial\left(\sqrt{R\left(\frac{dq}{dt}\right)^2}\right)}{\partial \frac{dq}{dt}} - \frac{\partial\left(\sqrt{R\left(\frac{dq}{dt}\right)^2}\right)}{\partial q} = 0$$

or

$$\frac{d}{dt}\frac{\partial R\left(\frac{dq}{dt}\right)^2}{\partial \frac{dq}{dt}} - \frac{\partial R\left(\frac{dq}{dt}\right)^2}{\partial q} = 0$$

When R is independent of the charges and constant in time, i.e., R has no memory, the previous equation can be written as follows:

$$\frac{d\left(\frac{dq}{dt}\right)}{dt} = \frac{d^2 q}{dt^2} = 0 \text{ , } q(t) = at + b \text{ , } i = \frac{dq}{dt} = a = \frac{E}{R}$$

The geodesic is a straight line in the space of the charges. When the *conceptual intention* moves on a sphere given by the equation

$$y_1^2 + y_2^2 + y_3^2 = r^2 \qquad (14)$$

 ISBN: 978-1-61208-218-9

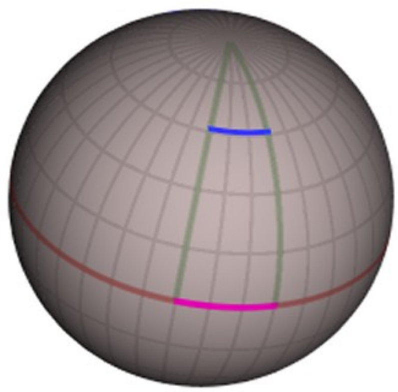

Figure 3: Illustration of the geodesic principle. In green we have the geodesic as line with minimum distance.

we have the following transformations for the *conceptual intention*:

$$\begin{cases} y_1 = r\sin(\alpha)\cos(\beta) \\ y_2 = r\sin(\alpha)\sin(\beta) \\ y_3 = r\cos(\alpha) \end{cases} \text{ or } \begin{cases} y_1 = x_1\sin(x_2)\cos(x_3) \\ y_2 = x_1\sin(x_2)\sin(x_3) \\ y_3 = x_1\cos(x_2) \end{cases} \quad (15)$$

Let's compute the geodesic in the space (x_1 , x_2 , x_3):

$$\begin{aligned}
(\frac{ds}{dt})^2 &= (\frac{dy_1}{dt})^2 + (\frac{dy_2}{dt})^2 + (\frac{dy_3}{dt})^2] \\
&= (\frac{\partial y_1}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial y_1}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial y_1}{\partial x_3}\frac{dx_3}{dt})^2 \\
&+ (\frac{\partial y_2}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial y_2}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial y_2}{\partial x_3}\frac{dx_3}{dt})^2 \\
&+ (\frac{\partial y_3}{\partial x_1}\frac{dx_1}{dt} + \frac{\partial y_3}{\partial x_2}\frac{dx_2}{dt} + \frac{\partial y_3}{\partial x_3}\frac{dx_3}{dt})^2 \\
&= (\frac{dx_1}{dt})^2 + x_1^2(\frac{dx_2}{dt})^2 + x_1^2\sin^2(x_2)(\frac{dx_3}{dt})^2 \qquad (16a)
\end{aligned}$$

We want to simulate the movement of the electrical circuit over the specified surface using Eq. 12a, where the impedance is given as:

$$G_{j,k} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & x_1^2 & 0 \\ 0 & 0 & x_1^2\sin^2(x_2) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & r_1^2 & 0 \\ 0 & 0 & r_1^2\sin^2(\alpha) \end{bmatrix} = Z_{j,k} \quad (16b)$$

Next we establish a connection between the derivative of the coordinates x and the currents *i*:

$$\text{for } \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} \frac{dq_1}{dt} \\ \frac{dq_2}{dt} \\ \frac{dq_3}{dt} \end{bmatrix} \equiv \begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} \frac{dr}{dt} \\ \frac{d\alpha}{dt} \\ \frac{d\beta}{dt} \end{bmatrix}$$

The power is

$$\begin{aligned}
W &= Z_{1,1}i_1^2 + Z_{2,2}i_2^2 + Z_{3,3}i_3^2 \\
&= i_1 + r^2 i_2^2 + r^2 sin^2(q_2)i_3^2 \\
&= \frac{dq_1}{dt} + q_1^2(\frac{dq_2}{dt})^2 + q_1^2 sin^2(q_2)(\frac{dq_3}{dt})^2 \quad (17)
\end{aligned}$$

Figure 4 illustrates the connection among the charges, the currents, the derivative of the distance *s* and the geodesic.

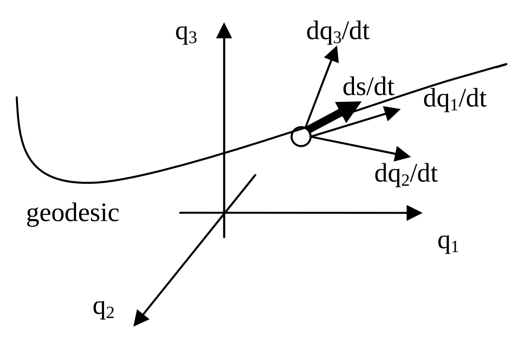

Figure 4 Geometric illustration of the geodesic with electrical charges, currents, and derivative of distance *s*.

Based on the conceptual transformation, we can compute the electrical parameters, impedances Z, by which the movement on the surface can be obtained by an electrical circuit whose parameters are in agreement with the conceptual intention.

 ISBN: 978-1-61208-218-9

III. Geodesic of the neural electrical circuit

The electrical activity of the synapse is given by the circuit

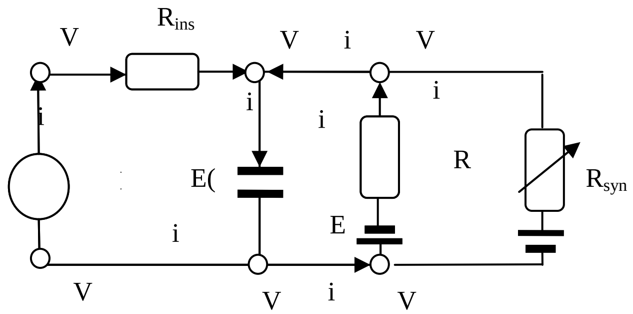

Figure 5 : Electrical circuit of synapses.

The impedance matrix is

$$Z = \begin{bmatrix} R_{ins} + 3 & 1 & 0 \\ 1 & R_m + 3 & R_m \\ 0 & R_m & R_{syn} + R_m + 2 \end{bmatrix}$$

The geodesic trajectory for a synapse is written as:

$$W = \left(\frac{ds}{dt}\right)^2 = (R_{ins} + 3)i_2^2 + (R_m + 3)i_5^2 +$$

$$+(R_m + R_{sys} + 2)i_8^2 + 2i_2 i_5 + 2R_m i_5 i_8$$

For more complex neural networks, we can derive the corresponding geodesics in a similar fashion. For example, we could consider have the electrical representation of a neural network as shown in Fig. 6. The axon circuitry is shown in Fig. 7. The geodesic can be derived for these networks as well, but we do not give the details in this brief paper.

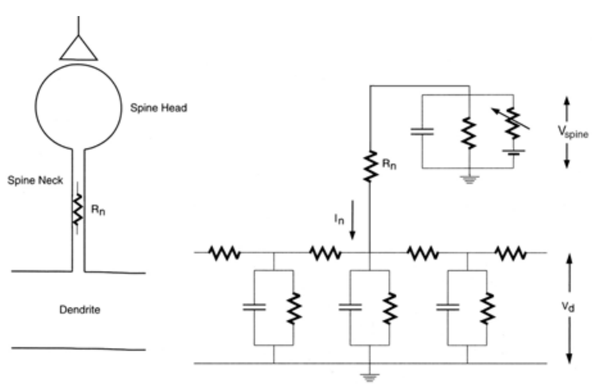

Figure 6 Complex electrical circuit of neural network system

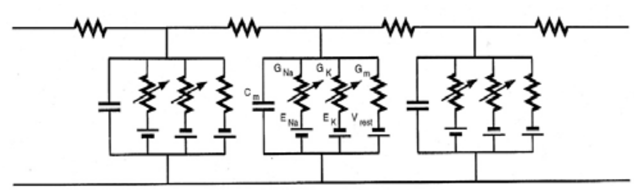

Figure 7. Example of axon and the electrical circuit

IV. Conclusion and future work

In this work, we employed geometric concepts for states of a neural network represented using an electrical circuitry. In the introduced neuromorphic electrical circuitries, we relate voltage with current using the conductance matrix or the impedance matrix. After a change of reference system and introducing a new distance measure, we transform the initially linear relationships into a geodesic over the phase space. With the introduced neural network, we can simulate the geodesic movement for any transformation of the reference system. For a given transformation of reference, we can build the associated geodesic, which allows implementing the transformation of reference in the neural network. The neural network as an analog computer gives the solution of the ODE of the geodesic in the wanted reference. Our work is closely related to Freeman's work [1] on intentional neurodynamics, according to which intentionality can be studied in its manifestations of goal-directed behaviors. The goal-directed behavior is the reflection of the cognitive transformation, which is manifested through the neural mechanisms that construct the mesoscopic amplitude modulation (AM) space-time patterns of cortical activity. AM patterns related to intentionality emerge through the neural interactions in the limbic system and neocortical regions, including the sensory and motor cortices. Intentionality in brain dynamics can be modeled by nonlinear dynamic systems, focused on the construction of meaning rather than representations [1- 4].

In this paper, we described the brain and its parts as complex electrical circuits. Intention has two different aspects, namely, the *conceptual* part and the *material* part. The *conceptual* part of intentionality is given by a nonlinear transformation of the brain states. We introduce a new reference system, in which the geodesic, the transformed image of a straight line satisfies the condition that the minimum distance between two points in the state space is realized along the geodesic. The other aspect of intentionality is its *material* part. We observe that any part of the brain can be modeled by an electrical circuit and that the transformations between the voltages and currents define the change of the reference. Therefore, we conclude that the

 ISBN: 978-1-61208-218-9

actual transformation in the brain states is the *material* part of the intention.

In a working intentional system, the *conceptual* and *material* parameters of intentionality should be equal. When the two parts are equal, we have the brain dynamic in agreement with the required transformation in the conceptual space. This means that brains realize the required transformation in the material world. If a task is defined in the conceptual domain, then the task can be realized in the material neural network using suitable parametric structures. In terms of the traditional digital computer paradigm, the conceptual part of intention can be viewed as the software, while the material part of intention corresponds to the hardware [12]. The difference between the geometric theory of intentionality and the traditional digital computer paradigm is in the way the software and hardware is described. In digital computers, there are logic statements for the software and logic gates for the hardware. In the geometric approach to intentionality, the "hardware" and "software" of brains are inseparable. Namely, we have geometric changes of the references in the multidimensional space as "software" and corresponding transformations in neuromorphic computing as "hardware." Further studies toward geometric approach to intentionality are in progress .

ACKNOWLEDGMENT

This work has been supported in part by a grant of FedEx Institute of Technology, University of Memphis.

REFERENCES

[1] W. J. Freeman (1975) *"Mass Action in The Nervous System"* Academin Press, New York.

[2] R.Kozma and W.J. Freeman (2008) "Intermittent spatial – temporal desynchronization and sequenced synchrony in ECoG signal " *Chaos* Vol. 18, 037131 pp. 1-8.

[3] B. Bollobas , R. Kozma and D. Miklos (2009) (Eds) "*Handbook of Large –Scale Random Networks* " Bolyai Society Mathematical Studies , Springer Verlag New York ISBN 978 -3 – 540 – 69394-9.

[4] W.J. Freeman (2008) "A pseudo-equilibrium thermodynamic model of information processing in nonlinear brain dynamics." *Neural Networks*, doi:10.1016/ j.neunet.2007.12.011.

[5] H. Makram (2006) "The blue brain project," *Nature Reviews Neuroscience*, Vol.7**,** pp. 153-160.

[6] M.D. Pickett, D.B. Strukov JL Borghetti, JJ Yang G.S. Snider GS, DR Stewart, and RS Williams (2009) "Switching dynamics in titanium dioxide memristive devices." *J Appl Phys* 106;074508R.

[7] R. Kozma, R. Pino and G. Pazienza (2012) "*Advances in Memristor Science and Technology,"* Dordrecht, Springer Verlag.

[8] T. Kohno and K. Aihara, "A Design Method for Analog and Digital Silicon Neurons Mathematical-Model-Based Method-," Proc. *AIP Conference*, Vol. 1028, pp. 113–128.

[9] J. Rinzel and B. Ermentrout (1998) "Analysis of Neural Excitability and Oscillations," in "*Methods in Neural Modeling*", ed. C. Koch and I. Segev, pp. 251–291, MIT Press.

[10] G. Resconi (2007) "Modelling Fuzzy Cognitive Map by Electrical and Chemical Equivalent Circuits," *Joint Conf. Inf. Science*, July8-24 2007 Salt lake City Center USA.

[11] G. Resconi and V.P.Srini (2009) "Electrical Circuit As A Morphogenetic System," GEST Int. Trans. Comp. Sci. Engng, Vol. 53(1), pp.47-92.

[12] A.A. Mandzel and T. Flach (1999) "Geometric methods in the study of human motor control," *Cognitive Studies*, 6 (3), pp. 309 – 321.

[13] S.V Fomin,. and I.M.: Gelfand, Calculus of Variations, Dover Publ., 2000