New Spectrum Sensing based on Goodness of Fit Testing for Cognitive Radio

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***Abstract*—Recently, the Goodness of Fit Test (GoF) has been applied for hypothesis testing in the case of spectrum sensing for Cognitive Radio (CR). GoF sensing has the desirable feature of needing only a few samples to perform sensing. In this paper, we first compare the existing GoF sensing methods in the literature. Secondly, we study some typical impairment for spectrum sensing, i.e., the effect of a non Gaussian noise, noise uncertainty and Rayleigh fading channel on the performance of GoF based sensing. Thirdly, we propose two GoF sensing methods and compare them against the conventional Anderson Darling (AD) sensing. The first proposed method is the IQ (In-phase and Quadrature components) GoF sensing method, which consists in testing the real and the imaginary part of the received samples against the Gaussian distribution to make a decision. In the second method, we propose a new GoF test statistic by taking into account the physical characteristic of spectrum sensing. The derived GoF sensing method results in significant improvement in terms of sensing performance.**

Keywords*—*Cognitive Radio; Spectrum Sensing; Goodness of Fit test; Test Statistic.

I. Introduction

Due to the rapid development of wireless communications services, the requirement of spectrum is growing dramatically. The Federation Communications Commission (FCC) has stated that some allocated frequency bands are largely unoccupied (under-utilized) most of the time [1]. Cognitive Radio has emerged as a novel approach to enable Dynamic Spectrum Access (DSA) by allowing unlicensed users to access the under-utilized licensed spectra when/where licensed Primary Users (PU) are absent and to vacate the spectrum immediately once a PU becomes active without causing harmful interference [2] [3]. This ability is dependent upon Spectrum Sensing. Spectrum Sensing is a key component of dynamic spectrum sensing paradigm to find spectrum opportunities [4]. For practical dynamic spectrum sensing and access, power detectors are required. Generally, in CR environments, sensing algorithms are expected to be able to detect the presence of signals at very low Signal to Noise Ratio (SNR) levels within a limited observation time. Moreover, it is necessary that they are robust to practical impairments and parameter uncertainties. Therefore, spectrum sensing is a difficult task in CR and to design detection algorithms that are capable to work under very harsh conditions is of fundamental importance.

Many studies have focused on spectrum sensing algorithms in literature. The Matched Filter (MF) is considered as the optimum detector based on the classical detection theory but it has the disadvantage that it requires the knowledge of the signal to be detected [5], condition that in general is not satisfied in cognitive radio applications. The Energy Detector (ED) is the most used detector when the signal is unknown [6]. The ED exhibits a low computational complexity and is widely used because it has a simple implementation. The main disadvantage of the ED is that it requires knowledge of the noise power to properly set the threshold. This requirement is often critical, in particular in low SNR environments, in which an imperfect knowledge of the noise power can cause severe performance losses. Moreover, the ED cannot distinguish between interference and signal [7].

When the signal to be detected has some known characteristics, the detection of such features is an effective method to identify such kind of signal. The cyclostationary method can be an appropriate sensing technique to recognize a particular transmission and/or extract its parameters [8]. This technique enables separation between signal and noise components and it can be adopted for signal classification. This spectrum sensing method has high computational and implementation requirements. It is worth to mention that the cyclostationary method outperforms the ED method if the noise power is wrongly estimated [9].

To the above mentioned spectrum sensing algorithms, we can also add other algorithms derived from spectral analysis, such as: multi-taper spectral analysis [10], wavelet transforms [11] and filter banks receivers based sensing methods [12].

There are several important characteristics to be considered in order to decide on a specific sensing method such as : prior knowledge, sensing time, computational complexity and noise rejection. To make trade-offs between these different characteristics, we propose in this paper the study of a spectrum sensing method based on statistic test ((GoF) test). In literature, many GoF sensing methods are proposed. The most important ones are the Anderson-Darling based sensing [13], Kolmogorov-Smirnov based sensing [14], the Cramer-Von Mises based sensing [15] and Order Statistics [16]. All these GoF sensing methods are based on the same hypothesis test, but differ in the way the distance between the empirical cumulative distribution of the observations made locally at the CR user and the noise distribution is calculated. The calculated distance is compared with a threshold to decide whether the signal is present or not, given a certain probability of false alarm. The first GoF sensing was presented in [13]. It is based on the Anderson-Darling GoF test to decide whether the received samples are drawn from the noise distribution F_0 (Gaussian distribution) or a different distribution. In [17], the authors reformulate the AD sensing to a Students t-distribution testing problem and propose a method which does not require any knowledge of the transmitted signal. The performance of the proposed method is better than ED sensing but less than AD sensing proposed in [13]. Kurtosis GoF sensing is proposed in [18] in which the kurtosis is calculated from the absolute values of the Fast Fourier Transform (FFT) of the received samples. The value of the kurtosis statistic is then compared to a predefined threshold to decide about the presence of the signal. Skewness and Kurtosis GoF sensing, Goodness of fit High Order Statistic Testing (GHOST) is proposed in [19]. The method is based on the kurtosis and skewness computed from the received signal. Jarque-Bera (JB) GoF sensing is presented in [20]. Moreover, detection methods based on Tietjen-Moore (TM) and Shapiro-Wilk (SW) tests are proposed to detect and suppress Spectrum Sensing Data Falsification (SSDF) attacks by malicious user in cooperative spectrum sensing [21]. Most of the above methods take a normal noise distribution for the GoF test, and they all assume that the samples of the received signal are real valued. As CR is based on the Software Defined Radio (SDR) technology, the received base-band samples in the digital domain are complex in nature. Therefore, the most practical approach to apply the GoF test for spectrum sensing is to consider the squared magnitude of the complex samples (i.e., energy of the samples) and to test their empirical distribution against the hypothetical noise energy distribution [22]. In [23], and based on our new model in [22], we have proposed a blind spectrum sensing method based on GoF test using Likelihood Ratio (LR). Motivated by its desirable feature of needing only a few samples to perform sensing, in [24], the narrowband spectrum sensing based on GoF is used for a Nyquist wide-band sensing also known as a conventional wide-band sensing. Besides, we have studied in [25] the GoF sensing methods under noise uncertainty.

In this paper, we propose a new GoF based spectrum for cognitive radio. The first proposed method is the IQ GoF sensing method, which consists in testing the real and the imaginary part of the received samples against the Gaussian distribution to make a decision. In the second method, we propose a new GoF test statistic by taking into account the physical characteristics of spectrum sensing. Besides, we evaluate the GoF based sensing methods under some typical impairment such as the effect of a non Gaussian noise, noise uncertainty and Rayleigh channel.

The paper is organized as follows. In Section II, we explain the Goodness of Fit tests and we mention the most important among them. We present some existing GoF sensing methods and compare their detection performances in Section III. In Section IV, the GoF based spectrum sensing is investigated under non Gaussian noise, noise uncertainty and Rayleigh channel. In Section V, two new spectrum sensing methods are proposed and evaluated. We conclude this paper in Section VI.

II. Goodness of Fit Tests

GoF tests were proposed in mathematical statistics by measuring a distance between the empirical distribution of the observation made and the assumption distribution. In CR, GoF sensing is used to solve a binary detection problem and to decide whether the received samples are drawn from a distribution with a Cumulative Distribution Function (CDF) F_0, representing the noise distribution, or they are drawn from some distribution different from the noise distribution. The hypothesis to be tested can be formulated as follows:

$$\begin{aligned} H_0 &: F_n(x) = F_0(x) \\ H_1 &: F_n(x) \neq F_0(x), \end{aligned} \tag{1}$$

for a random set of n independent and identically distributed observations and where $F_n(x)$ is the empirical CDF of the received sample and can be calculated by:

$$F_n(x) = |\{i : x_i \leq x, 1 \leq i \leq n\}/n|, \tag{2}$$

where $| \bullet |$ indicates cardinality, $x_1 \leq x_2 \leq .... \leq x_n$ are the samples under test and n represents the total number of samples.

Many goodness of fit tests are proposed in literature. The most important ones are the Kolmogorov- Smirnov test [14], the Cramer-von Mises test [15], the Shapiro-Wilk [21] test and the Anderson-Darling test [13]. In the following, we briefly recall these GoF tests.

A. Kolmogorov- Smirnov test (KS test): In this test, the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$D_n = max|F_n(x) - F_0(x)|, \tag{3}$$

where $F_n(x)$ is the empirical distribution which is defined in (2). If the samples under test are coming from $F_0(x)$, then, D_n converges to 0.

B. Cramer-Von Mises (CM test): In this test, the distance between $F_n(x)$ and $F_0(x)$ is defined as:

$$T_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, dF_0(x). \tag{4}$$

By breaking the integral in (4) into n parts, T_n^2 can be writen as:

$$T_n^2 = \sum_{i=1}^{n} [z_i - (2i-1)/2n]^2 + (1/12n), \tag{5}$$

with $z_i = F_0(x_i)$

C. Anderson-Darling test (AD test): This test can be considered as a weighted Cramer-Von Mises test where the distance between $F_n(x)$ and $F_0(x)$ is given by:

$$A_n^2 = \int_{-\infty}^{\infty} [F_n(x) - F_0(x)]^2 \, \frac{dF_0(x)}{F_0(x)(1 - F_0(x))}. \tag{6}$$

The expression of A_n^2 can also be simplified to:

$$A_n^2 = -n - \frac{\sum_{i=1}^{n} (2i-1)(\ln z_i + \ln(1 - z_{(n+1-i)}))}{n}, \tag{7}$$

with $z_i = F_0(x_i)$.

III. GoF Sensing Methods

We formulate the spectrum sensing problem as a binary hypothesis testing problem as follows:

$$\begin{aligned} H_0 : X_i &= W_i \\ H_1 : X_i &= S_i + W_i, \end{aligned} \tag{8}$$

where S_i are the received complex samples of the transmitted signal and W_i is the complex Gaussian noise. We now consider the random variable $Y_i = |X_i|^2$ which corresponds to the received energy. It is known that, if the real and the imaginary part of X_i are normally distributed, which is the case under H_0

hypothesis, the variable $Y_i = |X_i|^2$ is chi-squared distributed with 2 degrees of freedom.

As mentioned before, we will consider a normal noise, in order to be able to compare the different GoF sensing methods. This assumption is not limiting. The performance of the GoF sensing is independent of the noise distribution, as the distribution of GoF test statistic (A_n^2, T_n^2, D_n, ..) under H_0 is independent of the $F_0(y)$ [26] [27].

The spectrum sensing problem can now be reformulated as a hypothesis represented in (8) where we test whether the received energy $Y_i = |X_i|^2$ samples are drawn from a chi-square distribution with 2 degrees of freedom or not. The CDF of the chi-square distribution is given by:

$$F_0(y) = 1 - e^{-y/2\sigma_n^2} \sum_{k=0}^{m-1} \frac{1}{k!} (\frac{y}{2\sigma_n^2})^k, y > 0, \qquad (9)$$

where m is the degree of freedom (in our case $m = 1$) and σ_n^2 is the noise power.

In summary, GoF sensing methods follow these steps:

Step1 From the complex received samples X_i, calculate the energy samples $Y_i = |X_i|^2$

Step2 Sort the sequence $\{Y_i\}$ in increasing order such as $Y_1 \leq Y_2 \leq \cdots \leq Y_n$

Step3 Calculate the GoF test statistic T^* , with F_0 given in (9).
use (3) for KS GoF sensing
use (5) for CM GoF sensing
use (7) for AD GoF sensing

Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{T^* > \lambda | H_0\}. \qquad (10)$$

Step5 Accept the null hypothesis H_0 if $T^* \leq \lambda$, where T^* is the GoF test statistic (KS, CM or AD) . Otherwise, reject H_0 in favor of the presence of the signal.

The value of λ is determined for a specific value of P_{fa}. Tables listing values of λ corresponding to different false alarm probabilities P_{fa} are given according to the test considered [26]. Otherwise, these values can be computed by Monte Carlo approach [23] [25].

A. *Performance comparison of existing GoF sensing methods*

In this subsection, we will analyze and compare the performance of existing GoF sensing methods.

Thereafter, simulation results are presented to show the sensing performance of various GoF sensing methods compared to the conventional ED sensing. In Fig. 1, we show the ROC (Receiver Operating Characteristic) curves of GoF sensing methods (AD, CM and KS) and ED sensing for a fixed number of 80 samples and a given SNR equal to -6 dB. It is clear that ED sensing outperforms the considered GoF sensing methods. Likewise, AD sensing is the best among the considered GoF sensing methods. This is indeed confirmed in the simulation results as shown in Fig.2, where the detection probability versus SNR is plotted for a fixed number of 80 samples and at given false alarm probability $Pfa = 0.05$. ED sensing has better performance than the three GoF sensing methods. To achieve 90 % of detection probability, ED sensing outperforms AD sensing by about 1 dB, and AD sensing presents a slight difference in gain compared to CM sensing and KS sensing of about 0.2 dB and 0.5 dB respectively.

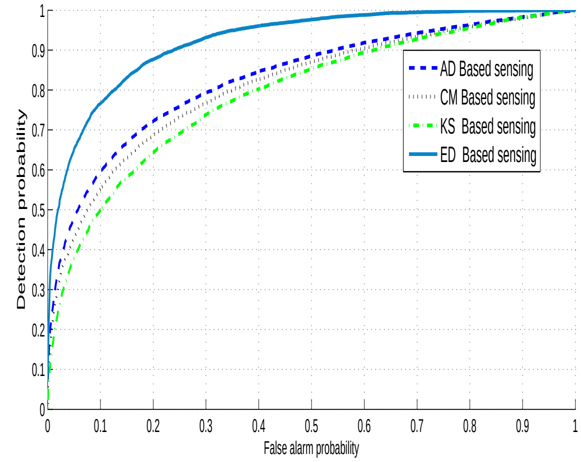

Figure 1. Detection probability versus false alarm probability of various GOF test based sensing at $SNR = -6dB$ and $n = 80$ samples

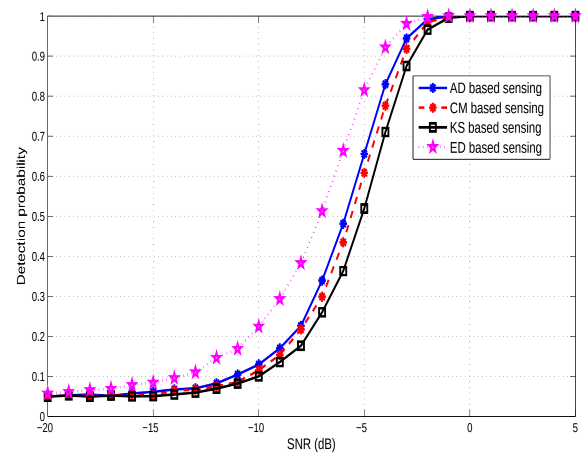

Figure 2. Detection probability versus SNR for different GOF tests based sensing with $Pfa = 0.05$ and $n = 80$ samples

IV. GoF Sensing Under Non Gaussian Noise, Noise Uncertainty and Rayleigh Channel

Although, its nice feature that it only needs a few samples to perform sensing, we have seen in the previous section that the conventional Energy Detection still outperforms the GoF based sensing (when considering a normal distribution of noise). However, the GoF sensing methods have the merit to be resistant to different impairments. This point is studied in this section.

A. *Impact of a non Gaussian noise (GM Model)*

It is worth to mention that the existing works on GoF for spectrum sensing [13] [15] [16] and [17] are focusing on detecting a signal in white Gaussian noise. In this paper, we will also focus on detecting signals in white non-Gaussian

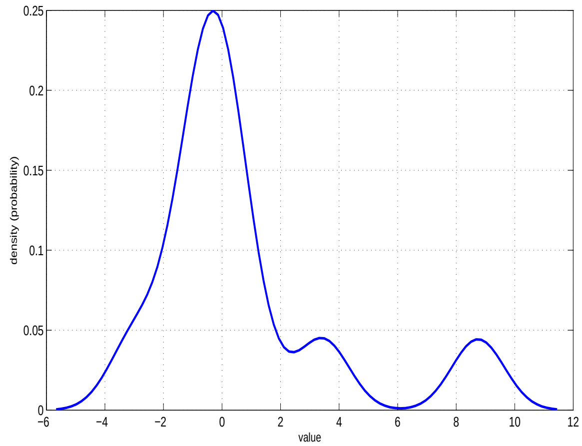

Figure 3. probability distribution function (pdf) of GM noise $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$

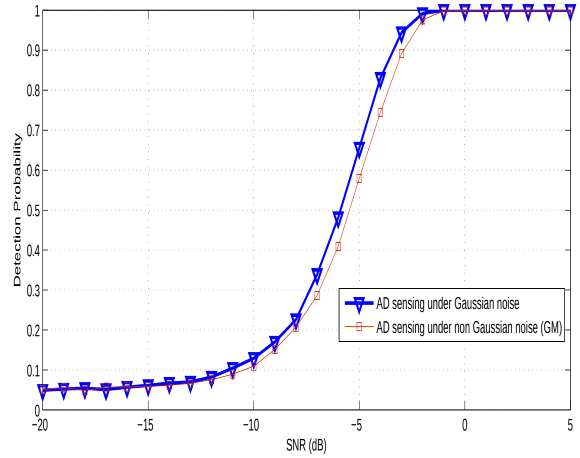

Figure 4. Detection probability versus SNR under Gaussian and non Gaussian noise for AD-GoF, with $Pfa = 0.05$ and $n = 80$ samples

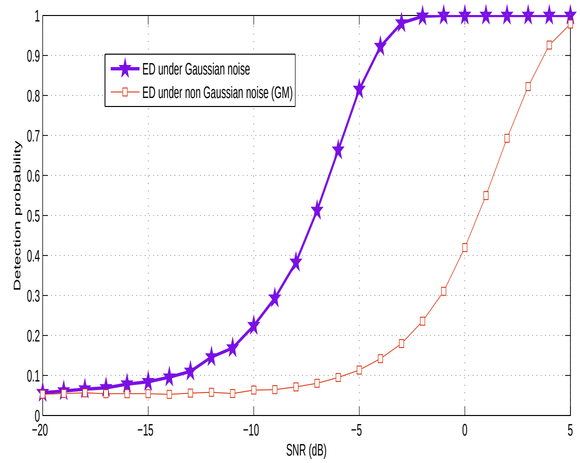

Figure 5. Detection probability versus SNR under Gaussian and non Gaussian noise for ED, with $Pfa = 0.05$ and $n = 80$ samples

noise. In literature, a lot of models are proposed to pattern a non Gaussian noise. The most used models are the Gaussian Mixture model (GM) and the Generalized Gaussian model (GG). For our spectrum sensing model, we will work with the GM model [28], as it has been used in practical applications in [30] and in radio signal detection applications in [31]. To apply the GoF test for spectrum sensing, we need to know the CDF of the non Gaussian noise (GM CDF). The Probability Density Function (PDF) of GM noise has three parameters α ,β , and σ and is defined as [31]:

$$f_w(w) = \frac{c}{\sigma\sqrt{2\Pi}}[\alpha exp(-\frac{c^2w^2}{2\sigma^2}) + \frac{1-\alpha}{\beta}exp(-\frac{c^2w^2}{2\sigma^2\beta^2})] \tag{11}$$

where $c = \sqrt{\alpha + (1-\alpha)\beta^2}$

In Fig. 3, we depict a PDF of a white non Gaussian noise (GM) with the following selected parameters $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$. The methodology explaining how the GM parameters may be estimated can be found in [29]. The CDF F_0 of the energy of the non-Gaussian noise samples under H_0 hypothesis can be derived from the GM's PDF. For that, we have: if $Y = X^2$ and X is GM noise with CDF $F_X(x)$

$$\begin{aligned} F_0(y) = &\quad P(Y \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) \\ = &\quad F_X(\sqrt{y}) - F_X(-\sqrt{y}) \end{aligned} \tag{12}$$

Once we get the CDF of the non Gaussian noise, we apply the proposed algorithm in section III. Note that the knowledge of F_0 is required to apply the GoF test, therefore, if the parameters of the GM model are unknown, they must be estimated first.

To evaluate the effect of a non Gaussian noise on the sensing performance, we have performed simulations with the selected GM noise. We set the parameters of the non Gaussian noise as: $\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$. Fig. 4 presents the results of the AD GoF sensing under Gaussian noise and non Gaussian noise. It is shown that the effect of considering a non Gaussian noise is to slightly decrease the performance of the AD GoF sensing. However, it can be seen in Fig. 5 that the performance of the ED is significantly influenced by the considered non Gaussian noise. It has to be noted that the considered non Gaussian noise ($\alpha = 0.9$, $\beta = 5$ and $\sigma = 1$) is very unfavorable for ED. In order to obtain a $P_{fa} = 0.05$, the threshold λ in the binary hypothesis test needs to be shifted to the right at a certain level. GoF sensing is less affected by the non Gaussian noise, as the test is performed on the mismatch between the measured CDF and the reference CDF F_0.

B. *Impact of a noise uncertainty*

One of the main issues with ED is the impact of noise uncertainty on the detection performance. It is shown in [33] and [32] that ED is very sensitive to noise uncertainty. The aim of this subsection it to study the effect of noise uncertainty on GoF sensing methods compared to ED.

Through simulation, we have compared the impact of noise uncertainty on both methods, ED based spectrum sensing and GoF sensing. The noise uncertainty is modeled by letting the actual noise variance be limited within a set given by a nominal noise variance and an uncertainty parameter ρ such

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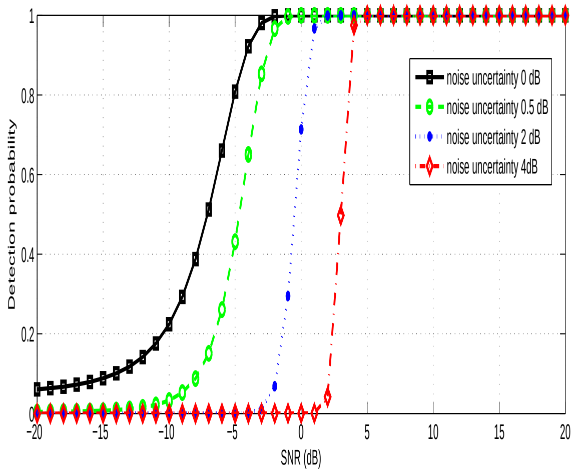

Figure 6. Impact of noise uncertainty on ED with $Pfa = 0.05$ and $n = 80$ samples

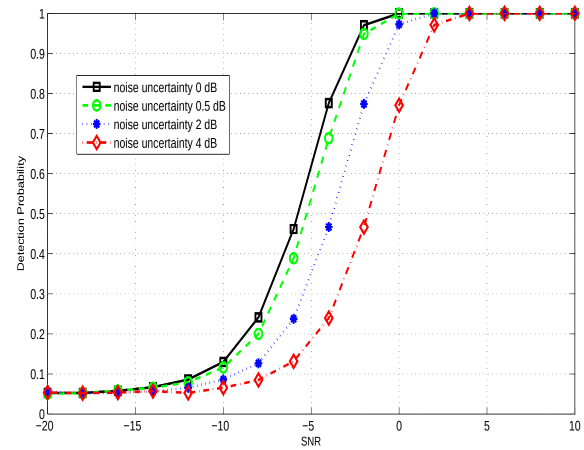

Figure 7. Impact of noise uncertainty on GoF test based sensing with $Pfa = 0.05$ and $n = 50$ samples

that $\sigma_n^2 \in [\frac{1}{\rho}\sigma^2, \rho\sigma^2]$.

There is a fundamental difference between ED and GoF sensing when it comes to noise uncertainty. The energy detector suffers under noise uncertainty because computing the threshold λ for the binary test requires knowledge of the underlying noise variance. In order to guarantee a given false alarm rate P_{fa}, the threshold λ will be calculated for the worst case, i.e., a noise variance of $\rho\sigma^2$, leading to higher values of λ and hence to a decrease in detection probability.

In GoF sensing, the distribution of the test statistic A_n^2 under the H_0 hypothesis is independent of the noise distribution. As a consequence, the value of the threshold λ for the GOF binary test will not be influenced by the noise uncertainty. However, the calculation of the test statistic (A_n^2) requires the exact knowledge of the underlying theoretical noise CDF F_0. In summary, for GoF sensing, noise uncertainty will, via F_0, indirectly affect the value of the test statistic, but not the detection threshold. For the simulation of the GoF sensing under noise uncertainty, we will also follow a worst case approach, by considering a reference noise CDF F_0 given in (9) based on the highest noise variance $\rho\sigma^2$, which will eventually lead to a reduction of the detection probability.

In Fig. 6, we have plotted the detection probability versus SNR for several values of noise uncertainty (0 dB, 0.5 dB, 2 dB, 4 dB) in the case of the ED spectrum sensing method. It is shown that the performance of the ED is significantly decreasing when the noise uncertainty level is increasing. At 80 % of detection probability, due to noise uncertainty of 0.5 dB, the SNR drops to about 2 dB.

In a similar way, in Fig. 7, we have plotted the detection probability as a function of SNR when considering a noise uncertainty for GoF based spectrum sensing. It can be seen that under uncertainty in the noise statistic of the CDF under hypothesis H_0 (F_0), the impact on the performance of the GoF based spectrum sensing is significantly less than the impact on energy detection. Intuitively, this can be explained by the fact that in ED, the values of P_{fa} and P_d are directly affected by the noise uncertainty. In case of GoF based sensing the statistic such as: A_n^2, is indirectly affected by the noise uncertainty via the CDF F_0 under hypothesis H_0.

Note also that, in Fig. 6, for high values of noise uncertainty, the P_d drops to 0. This effect is known as the SNR wall [33]. This effect is not observed in GoF based spectrum sensing for the given simulation parameters.

C. Impact of a Rayleigh fading channel

Under fading, the value of SNR may vary. In this case, the probability of detection must be given for the instantaneous SNR. This means that the resulting probability of detection may be derived by averaging over the fading statistics. Under Rayleigh fading, SNR has an exponential distribution [34].

In Fig. 8, we provide a plot of the ROC curve, under AWGN (Additive White Gaussian Noise) and Rayleigh fading scenarios. SNR_{avg} (the average over SNR values) and n are assumed to be -5 dB and 60 samples, respectively. It is shown that Rayleigh fading significantly degrades the performance of the energy detector.

To evaluate the impact of Rayleigh channel on GoF sensing methods, we have plotted in Fig. 9, the detection probability versus SNR_{avg} under AWGN and Rayleigh fading channel for AD GoF sensing with P_{fa} fixed to 0.05 and $n = 80$ samples. According to Fig. 9, it can be observed that the effect of considering a Rayleigh fading channel has a slight decrease in the performance of the AD GoF sensing.

V. New GoF spectrum sensing Methods

A. IQ GoF based spectrum sensing

We have proposed in [22] to calculate the energy samples $Y_i = |X_i|^2$, and then test the sequence Y_i against the chi-square distribution to determine if there exits a primary signal.

However, we could also form another sequence from the same observed complex samples by using its real and imaginary part, i.e.,$(Re(X_i), Im(X_i))$ and then test it against the Gaussian distribution to make a decision. The authors in [13] have considered a model (the received signal is real and $S_i = constant$) which does not reflect a realistic scenario for spectrum sensing in cognitive radio, as normally the received signal is complex and varies in time. Compared to the proposed

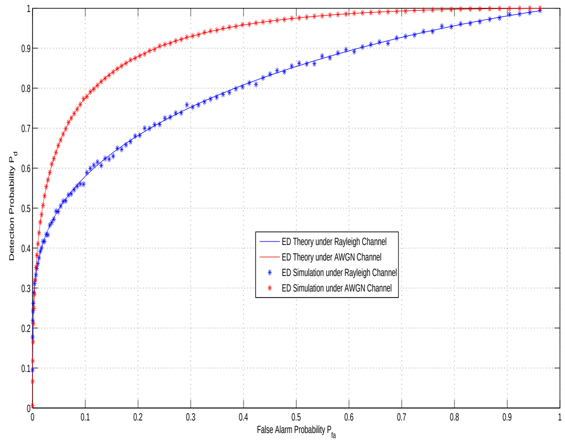

Figure 8. ROC curves for the energy detection under AWGN and Rayleigh fading channels

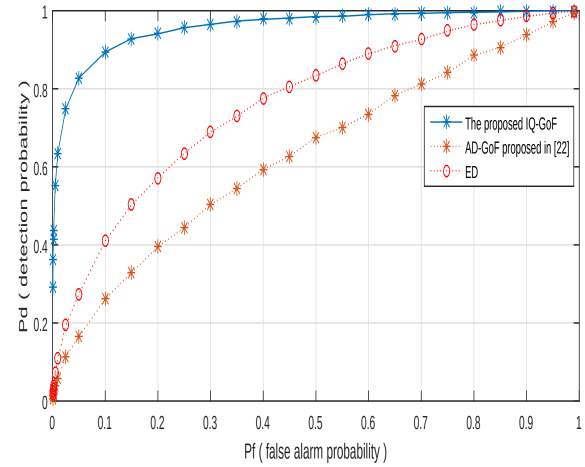

Figure 10. Detection probability versus false alarm probability with $SNR = -6 \quad dB$

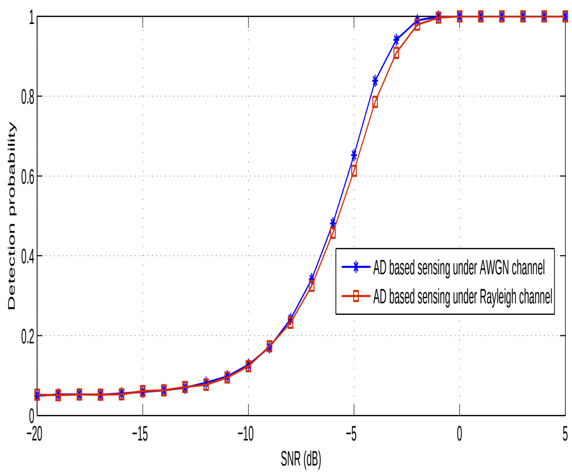

Figure 9. Detection probability versus SNR_{avg} under AWGN and Rayleigh fading channels for AD-GoF sensing, with $Pfa = 0.05$ and $n = 80$ samples

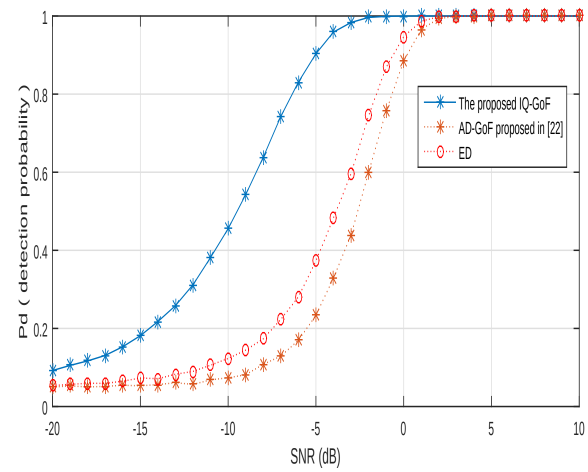

Figure 11. Detection probability versus SNR with $P_{fa} = 0.05$

method in [13], we proposed in this method to start from the more general model as in (8) and test the IQ samples against the Gaussian distribution to make a decision.

In summary, the proposed IQ GoF sensing methods follow these steps:

Step1 From the complex received samples X_i, separate the X_i to $(Re(X_i), Im(X_i))$.

Step2 Sort the sequence $\{Re(X_i)\}$ in increasing order such as $Re(X_1) \leq Re(X_2) \leq \cdots \leq Re(X_n)$. Perform the same thing for $Im(X_i)$.

Step3 Calculate the GoF test statistic using (7) for AD GoF sensing , with F_0 given in (9). We use the function 'Adtest' of Matlab, which combines the GoF from both real and imaginary parts, into a single GoF.

Step4 Find the threshold λ for a given probability of false alarm such that:

$$Pfa = P\{T^* > \lambda | H_0\}. \tag{13}$$

Step5 Accept the null hypothesis H_0 if $T^* \leq \lambda$, where T^* is the GoF test statistic (KS, CM or AD) . Otherwise, reject H_0 in favour of the presence of the signal.

The value of λ is determined for a specific value of P_{fa}. Tables listing values of λ corresponding to different false alarm probabilities P_{fa} are given according to the test considered [26]. Otherwise, these values can be computed by Monte Carlo approach.

The simulation results when $n = 20$ samples are displayed in Fig. 10 and Fig. 11. In both figures, 'IQ-GoF' denotes our proposed method and AD-GoF denotes the method proposed in [22]. The simulation results are obtained via 10000 Monte Carlo runs. Fig.10 shows the receiver operating characteristic (ROC) curves (detection probability against false alarm probability) with a SNR equal to -6 dB and the values of the detection probability against SNR are plotted in Fig. 11 with false alarm probability (Pf) set to 0.05. Both figures indicate that the proposed sensing method is more efficient compared to the conventional Energy Detection.

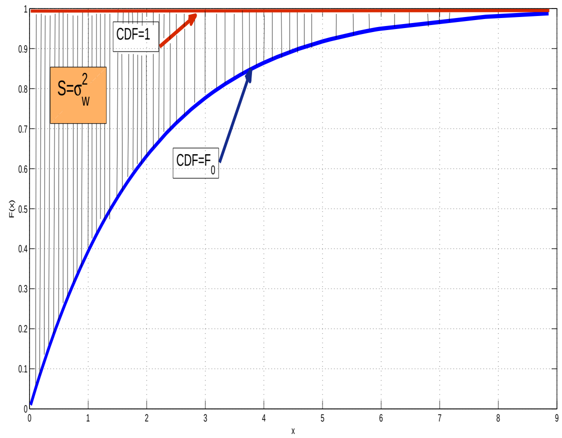

Figure 12. Noise power area

B. Spectrum sensing method based on the new GoF statistic test

The aforementioned GoF tests use the statistical hypothesis testing in (1) (which means testing the hypothesis H_0). However, in the H_1 hypothesis, it can be noted that the overall power of the received signal should always be larger than the noise power, as noise and signal are uncorrelated. This results in having a cumulative distribution function under hypothesis H_1 on the right of the cumulative distribution function of the noise, meaning that the area above the expected continuous CDF of the random variable (energy of samples in our case) will also increase. The above finding is based on the property of the expected value of a non-negative random variable.

$$E[X] = \int_0^{\infty} (1 - F_X(x))dx \tag{14}$$

In our sensing model as in [22], the received energy $Y_i = |X_i|^2$ is a non negative random variable and equation (14) is applicable. As the received signal $\{X_i\}$ has zero means, $E[Y] = E[|X_i|^2] = \sigma_X^2$. Hence, we find

$$\sigma_X^2 = \int_0^{\infty} (1 - F_Y(x))dx \tag{15}$$

In other words, the received signal power equals the area of the region lying above the CDF $F_Y(x)$ and below the line at height 1 to the right of the origin. Under H_0 hypothesis, this means that the area above F_0 equals the noise power σ_w^2 as depicted in Fig. 12. Under H_1 hypothesis, the total power in the received signal will increase to $\sigma_s^2 + \sigma_w^2$, meaning that the area above the expected continuous CDF of the random variable Y_i will also increase, shifting this CDF to the right. Therefore, the statistical hypothesis comes down to test one of the following inequalities:

$$\begin{array}{l} H_0 : F_n(y) \geq F_o(y) \\ H_1 : F_n(y) < F_o(y) \end{array} \tag{16}$$

The problem with the AD test (and also with the Von Mises test) is that the deviation of the empirical CDF $F_n(x)$ to the reference CDF $F_0(x)$ can be either to the left and to the right as the test is based on the square of the difference $[F_n(x) - F_0(x)]^2$. For spectrum sensing application, the sign of the difference is significant for the reason cited above. Therefore, the associated expression of the GoF test statistic can be given as:

$$S_n = n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y). \tag{17}$$

According to the choice of the weight function $\phi(t)$, we can derive the corresponding test statistic of the statistical hypothesis in (16). When $\phi(t) = 1$, the above equation (17) can be simplified as

$$\begin{aligned} S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]dF_0(y) \\ &= n \int_{-\infty}^{y_1} F_0(y)dF_0(y) \\ &+ \ldots \\ &+ n \int_{y(n)}^{+\infty} (F_0(y) - 1)dF_0(y) \\ &= -\frac{n}{2} + \sum_{i=1}^{n}((F_0(y)) \\ &= -\frac{n}{2} + \sum_{i=1}^{n}(z_i) \end{aligned} \tag{18}$$

When $\phi(t) = \dfrac{1}{t(1-t)}$, the above equation (17) can be simplified as

$$\begin{aligned} S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y) \\ &= n \int_{-\infty}^{y_1} \frac{F_0(y)}{F_0(y)(1 - F_0(y))}dF_0(y) \\ &+ \ldots \\ &+ n \int_{y(n)}^{+\infty} \frac{F_0(y) - 1}{F_0(y)(1 - F_0(y))}dF_0(y) \\ &= -\sum_{i=1}^{n}(ln(1 - F_0(y)) - ln(F_0(y))) \\ &= -\sum_{i=1}^{n}(ln(1 - z_i) - ln(z_i)) \end{aligned} \tag{19}$$

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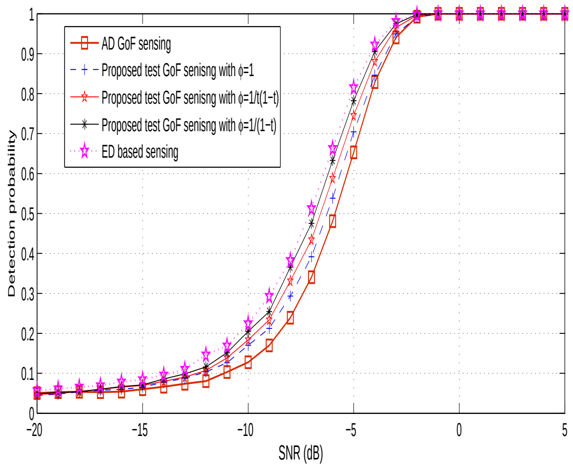

Figure 13. Detection probability versus SNR for the proposed GoF sensing under different weights, with $Pfa = 0.05$ and n=80 samples

When $\phi(t) = \dfrac{1}{(1-t)}$, the above equation (17) can be simplified as:

$$\begin{aligned} S_n &= n \int_{-\infty}^{+\infty} [F_0(y) - F_n(y)]\phi(F_0(y))dF_0(y) \\ &= n \int_{-\infty}^{y_1} \frac{F_0(y)}{(1-F_0(y))} dF_0(y) \\ &+ \ldots \\ &+ n \int_{y_(n)}^{+\infty} \frac{F_0(y)-1}{(1-F_0(y))} dF_0(y) \\ &= -n - \sum_{i=1}^{n} ln(1-F_0(y)) \\ &= -n - \sum_{i=1}^{n} ln(1-z_i) \end{aligned} \tag{20}$$

Once the test S_n is calculated, it will be compared with a decision threshold λ to decide whether to accept H_1 or reject it (accept H_0). The threshold λ can be determined according to the given value of the false alarm probability. The decision threshold λ is computed through Monte Carlo simulation.

In Fig. 13, the performance comparison between the new GoF sensing method, AD GoF sensing [22] and ED sensing is depicted. This figure shows detection performance in terms of detection probability as a function of SNR with $n = 80$ and $Pfa = 0.05$ for different weights. The new GoF sensing method outperforms the AD sensing method. The best performance is obtained with weight $\phi = \frac{1}{1-t}$ corresponding to (20) which has comparable detection performance with ED sensing. Table I gives a corresponding λ for some critical values of Pfa.

The simulations results show that the new GoF sensing method has the best performance and the lowest computational complexity.

TABLE I. THRESHOLD VALUES FOR SOME GIVEN Pfa AND $n = 80$ SAMPLES

$\phi = 1$	Pfa	0.1	0.05	0.01
	Threshold	3.536	4.480	6.295
$\phi = \frac{1}{t(1-t)}$	Pfa	0.1	0.05	0.01
	Threshold	21.875	28.165	39.484
$\phi = \frac{1}{1-t}$	Pfa	0.1	0.05	0.01
	Threshold	12.522	16.136	23.928

VI. Conclusion

In this paper, we present GoF sensing methods for CR. The paper has firstly provided a comparative study among existing GoF sensing methods. We have evaluated the performance of the GoF sensing methods through Monte-Carlo simulation. We have secondly studied some typical impairment for spectrum sensing, i.e., the effect of a non Gaussian noise, noise uncertainty and Rayleigh fading channel on the performance of GoF based sensing. As a model for the non Gaussian noise, we have used the Gaussian mixture (GM). It was observed that a non Gaussian noise can noticeably affect the performance of ED, but has only a limited influence on the performance of the GoF sensing methods. The same conclusion can be drawn for the impact of noise uncertainty and Rayleigh fading channel. This is mainly due to the fact that the test statistics in GoF testing is based on the difference of the measured CDF and the reference CDF and hence only indirectly influenced by noise parameters. Thirdly, we have proposed two new methods for GoF sensing. The first proposed method is the IQ GoF sensing method which consists in testing the real and the imaginary part of the received samples against the Gaussian distribution to make a decision. It was shown that this method exhibits better performance compared to ED. In the second method, we propose a new GoF test statistic by taking into account the physical characteristics of spectrum sensing. The derived GoF sensing method results in significant improvement in terms of sensing performance. Finally, this paper has shown the effectiveness of the GoF sensing methods in cognitive radio applications.

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