Multi-Antenna Energy Detector Under Unknown Primary User Traffic

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***Abstract*—In cognitive radio (CR) networks, the knowledge of primary user (PU) traffic plays a crucial role in designing the sensing slot duration and synchronization with PU traffic. However, the secondary user (SU) sensing unit usually does not have the knowledge of the exact time slot structure in the primary network. Moreover, it is also possible that the communication among PUs are not based on synchronous schemes at all. In this paper, the effect of unknown primary user (PU) traffic on the performance of multi-antenna spectrum sensing is evaluated under a flat fading channel. In contrast to the commonly used continuous time Markov model of the existing literature, a realistic and simple PU traffic model is proposed which is based only on the discrete time distribution of PU free and busy periods. Furthermore, in order to assess the effect of PU traffic on the detection performance, analytical expressions for the probability density functions of the decision statistic are derived considering Energy Detection (ED) test as spectrum sensing method. It is shown that the time varying PU traffic severely affects the spectrum sensing performance. Most importantly, our results show that the performance gain due to multiple antennas in the sensing unit is significantly reduced by the effect of PU traffic when the mean lengths of free and busy periods are of the same order of magnitude of the sensing slot.**

***Index Terms*—Energy Detection, Unknown Primary Traffic, Spectrum Sensing, Cognitive Radio**

I. Introduction

By accessing the unoccupied spectrum of licensed band, cognitive radio (CR) based dynamic spectrum sharing (DSS) is initially intended to alleviate the most challenging problems of future wireless communications, namely, spectrum scarcity. With real-time perception of surroundings and bandwidth availability and with the help of spectrum sensing functionality of CR, secondary users (unlicensed users) may dynamically use the vacant spectrum and perform opportunistic transmissions, by adapting the functionality intelligently to accommodate current wireless environments [1]. Thus, the domain of spectrum sensing techniques has long been investigated by many researchers: a detailed bibliography of the contributions in this area can be found in [2], [3]. Despite the significant volume of available literature on spectrum sensing under ideal scenarios, investigation under practical constraints and imperfections are still lacking [3]. Thus, recent research efforts are devoted to improve the accuracy and efficiency of sensing techniques under practical constraints and imperfections.

Currently, most of the existing research on cognitive radio spectrum sensing has been conducted based on the assumption that SUs are perfectly synchronized with PUs, thus providing a solid basis for guaranteeing that PU traffic transitions occur only at the beginning of the SU sensing frames. However, the SUs may not have the knowledge of the exact time slot structure in the primary network. Moreover, it is also possible that the communications among PUs are not based on synchronous schemes at all [5], [6]. Thus, under practical scenarios, the primary traffic transition may occur during the sensing period, especially when a long sensing period is used to achieve a good sensing performance, or when spectrum sensing is performed for a network with high traffic load.

Among a limited number of literature including [9]–[14] that deal with unknown primary traffic scenario, [9] was the first one to study the performance of well known semi-blind spectrum sensing algorithms including Energy Detection (ED) and Roy's Largest Root Test (RLRT) under bursty primary traffic, in which the burst interval is comparable to or smaller than the spectrum sensing interval. The traffic model used is limited to constant length bursts of the PU data, whose length is smaller than the SU sensing duration. However, the burst length of the PU may be varying with time following some stochastic models [7], [8]. A more general scenario, in which the PUs traffic transition is completely random, may affect the spectrum sensing performance. The analysis of the spectrum sensing performance has been presented in [11]–[14] by modeling the PU traffic as an independent and identically distributed two state Markov's model. Using this primary traffic model, authors in [11], [13], [14] analyzed the effect of PU traffic on the sensing performance and the sensing-throughput trade-off considering ED as a sensing technique under the half duplex scenario. Moreover, the effect of multiple PUs traffic on the sensing-throughput trade-off of the secondary system has been studied in [12]. Although all the aforementioned contributions recognized the fact that the PU traffic might affect the sensing performance including sensing-throughput trade-off, none of them considered the realistic scenario of multi-antenna spectrum sensing in a complex signal sample domain.

In this paper, the effect of PU traffic on the performance of multi-antenna spectrum sensing is evaluated under the complex domain of PU signal, noise and channel considering ED as a sensing technique. In contrast to the commonly used continuous time Markov model in the existing literature, a novel technique of modeling PU traffic is proposed which is only based on the discrete time distribution of PU free and busy periods. The proposed model is more realistic and simple compared to the continuous time Markov model proposed in the previous literature [11]–[14]. Moreover, an analytical performance evaluation of the decision statistic under the considered scenario is carried out. It is shown that the time varying PU traffic severely affects the performance of ED. More importantly, it is shown that the performance gain due to multiple antennas in the sensing unit is significantly suppressed by the effect of PU traffic when the mean lengths of free and busy periods are small.

The rest of the paper is organized as follows: A system model is presented in Section II. Simple characterization of the PU traffic model is described in Section III. The sensing performance is derived in Section IV. The simulation results are discussed in Section V and finally, the conclusion in Section VI.

II. SYSTEM MODEL

We consider a scenario where multiple antennas are employed by an SU. Suppose the SU has K antennas and each antenna receives N samples in each sensing slot. We focus on a single source scenario (single primary transmitter), which is of particular interest in many detection problems in CRNs. In a given sensing frame, the detector calculates its decision statistic T_D by collecting N samples from each one of the K antennas. Subsequently, the received samples are stored by the detector in the $K \times N$ matrix $\boldsymbol{Y}$.

As described in Section I, when the primary transmissions are not based on some synchronous schemes or the sensing unit at the SU does not have any information about the primary traffic structure, the received vector at the sensing unit may consist of partly the samples from one PU state and the remaining from alternate PU state as shown in Figure 1. To simplify the scenario, we begin with the following classification of the sensing slots based on the PU traffic status,

1) Steady State (SS) sensing slot: In such type of sensing slot, all the received samples in one sensing slot are obtained from the same PU state.
2) Transient State (TS) sensing slot: In such type of sensing slot, a part of the received samples within the sensing slot are obtained from one PU state and the remaining from the next PU state.

In general, the probabilities of receiving SS and TS sensing slots are dependent on the PUs traffic model. At the end of the sensing interval, based on the received samples, the detector must distinguish between null and alternate hypothesis.

$\mathcal{H}_0$: the channel is going to be free,
$\mathcal{H}_1$: the channel is going to be busy.

This hypothesis formulation implies that in a TS sensing slot, a transition from the PU busy state to the PU free state is considered $\mathcal{H}_0$, while a transition from the PU free state to the PU busy state is considered $\mathcal{H}_1$.

In the considered scenario, in an SS sensing interval, the generic received signal matrix under each hypothesis can be written as,

$$\boldsymbol{Y}_{SS} = \begin{cases} \boldsymbol{V} & (\mathcal{H}_0), \\ \boldsymbol{h}\boldsymbol{s} + \boldsymbol{V} & (\mathcal{H}_1), \end{cases} \tag{1}$$

where $\boldsymbol{V} \triangleq [\boldsymbol{v}(1) \cdots \boldsymbol{v}(n) \cdots \boldsymbol{v}(N)]$ is the $K \times N$ noise matrix , $\boldsymbol{h} = [h_1 \cdots h_K]^T$ is the channel vector and $\boldsymbol{s} \triangleq [s(1) \cdots s(n) \cdots s(N)]$ is a $1 \times N$ signal vector.

And in the TS sensing interval, the generic received signal matrix under each hypothesis can be written as,

$$\boldsymbol{Y}_{TS} = \begin{cases} \boldsymbol{h}\boldsymbol{s}_{N-D_0} + \boldsymbol{V} & (\mathcal{H}_0), \\ \boldsymbol{h}\boldsymbol{s}_{D_1} + \boldsymbol{V} & (\mathcal{H}_1), \end{cases} \tag{2}$$

where D_0 represents the number of pure noise samples in TS sensing slot under $\mathcal{H}_0$, D_1 represents the number of noise plus PU signal samples in TS sensing slot under $\mathcal{H}_1$, $\boldsymbol{s}_{D_0} \triangleq [\boldsymbol{s}_{1\times(N-D_0)} | \boldsymbol{0}_{1\times D_0}]$ with $\boldsymbol{s}_{1\times(N-D_0)}$ a $1 \times (N - D_0)$ signal vector and $\boldsymbol{0}_{1\times D_0}$ a $1 \times D_0$ zero vector. Similarly, $\boldsymbol{s}_{D_1} \triangleq [\boldsymbol{0}_{1\times(N-D_1)} | \boldsymbol{s}_{1\times(D_1)}]$ with $\boldsymbol{0}_{1\times(N-D_1)}$ a $1 \times (N - D_1)$ zero vector and $\boldsymbol{s}_{1\times D_1}$ a $1 \times D_1$ signal vector. In each of these, the unknown primary transmitted signal $s(n)$ at time instant n is modeled as independent and identically distributed (i.i.d.) complex Gaussian with zero mean and variance $\sigma_s^2 : s(n) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_s^2)$. The noise sample $v_k(n)$ at the k^{th} antenna of the SU at the time instant n is also modeled as complex Gaussian with mean zero and variance $\sigma_v^2 : v_k(n) \sim \mathcal{N}_{\mathbb{C}}(0, \sigma_v^2)$. The channel coefficient h_k of k^{th} antenna is assumed to be constant and memory-less during the sensing interval.

III. CHARACTERIZATION OF PRIMARY USER TRAFFIC

In this paper, the PU traffic is modeled as an i.i.d. on-off random process with geometrically distributed busy and free periods. To be in-line with the binary hypothesis testing of a spectrum sensing problem, a two state on-off modeling of the PU traffic is rather realistic especially when we are more concerned only about if the PU is transmitting or not. Furthermore, we are actually dealing with the discrete set of samples with a fixed sensing interval, thus the geometrically distributed busy and free periods are perfectly relevant in our considered scenario.

Let N_b be the geometrically distributed random variable denoting the number of consecutive busy samples with a parameter p_b. Similarly, let N_f be another identical and independent geometrically distributed random variable denoting the number of consecutive free samples with a parameter p_f. Then, the probability mass function (pmf) for each of them can be written as,

$$f_{N_b}(N_b = n_b) = (1 - p_b)^{n_b - 1} p_b, \quad \text{for } n_b = 1, 2, .., \infty \tag{3}$$

$$f_{N_f}(N_f = n_f) = (1 - p_f)^{n_f - 1} p_f, \quad \text{for } n_f = 1, 2, .., \infty \tag{4}$$

In the TS sensing slot, depending on the length of the free period or the busy period, the PU state can change anywhere within the sensing slot resulting the random variables (RVs) D_0 and D_1. For instance, suppose the PU is previously in

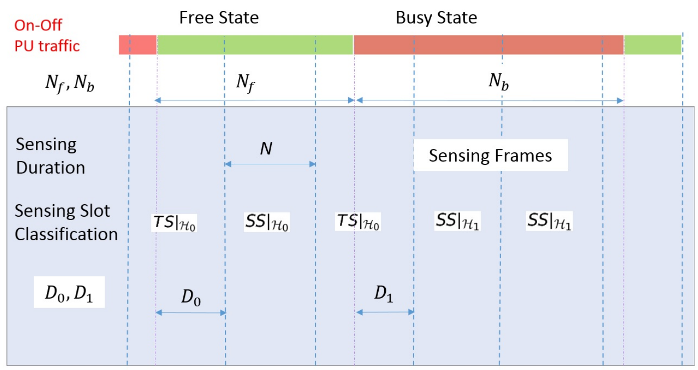


Fig. 1. Primary user traffic scenario and sensing slot classification

the busy state. The PU state transition from busy to the free state may occur anywhere, let's say after $(N - D_0)$ samples within the sensing slot. Thus, in each PU state transition from busy state to free state, the sensing unit has to decide based on D_0 pure noise samples and $(N - D_0)$ noise plus primary signal samples, which actually affects the overall sensing performance. The following Lemmas compute the pmfs of D_0 and D_1 respectively, based on the distribution of the busy period N_b, free period N_f and the sensing length N.

Lemma 1. Given the number of samples in a sensing duration N, the length of PU busy period N_b distributed as in (4), the probability of having D_0 noise only (PU signal free) samples in a TS sensing slot under $\mathcal{H}_0$ is given by,

$$P_{D_0}(D_0 = d_0)|_{\mathcal{H}_0} = \frac{p_b(1-p_b)^{N-d_0-1}}{1-(1-p_b)^N}. \tag{5}$$

Proof. As mentioned earlier during binary hypothesis formulation, the PU state transition from busy state to the free state corresponds to $\mathcal{H}_0$ sensing slot. We consider thus, without loss of generality, while dealing TS sensing slot under $\mathcal{H}_0$, PU state transition from busy to free state depends only on PU busy period N_b. Thus, for given N_b, the additional number of noise only samples D_0 which is required to complete a TS sensing slot under $\mathcal{H}_0$ is given by,

$$D_0 = \left\lceil \frac{N_b}{N} \right\rceil N - N_b. \tag{6}$$

Using the pmf of N_b, the probability of D_0 can be written as,

$$P_{D_0} = p_b(1-p_b)^{aN-D_0-1}, \tag{7}$$

where $a = \left\lceil \frac{N_f}{N} \right\rceil$. Now, from (6) and (7), it is clear that D_0 can be obtained from many different values of N_b. To be more precise, we obtain D_0 for all N_b such that $N_b = aN - D_0$. Thus, in order to evaluate the pmf of D_0, we need to sum the probability of occurrence of all the instances of N_b, i.e., $N_b = aN - D_0$, obtaining

$$P_{D_0}(D_0 = d_0) = \sum_{a=1}^{+\infty} p_b(1-p_b)^{aN-d_0-1}. \tag{8}$$

After some algebra and the truncation of infinite sum of geometric series, we obtain the pmf of D_0 as in (5). □

Lemma 2. Given the number of samples in a sensing duration N, the length of PU free period N_f distributed as in (3), the probability of having D_1 noise plus primary signal samples in a TS sensing slot under $\mathcal{H}_1$ is given by,

$$P_{D_1}(D_1 = d_1)|_{\mathcal{H}_1} = \frac{p_f(1-p_f)^{N-d_1-1}}{1-(1-p_f)^N} \tag{9}$$

Proof. Using the same line of reasoning as in the proof of Lemma 1, the proof of Lemma 2 is straightforward. □

As depicted from (1) and (2), to find the distribution of the decision statistic under different hypotheses, the prior deduction of the chances of occurrence of SS sensing interval, TS sensing interval, pmf of D_0 and pmf of D_1 is inevitable. The following proposition computes the probability of occurrence of SS sensing slot $p_{SS}|_{\mathcal{H}_0}$ under $\mathcal{H}_0$ and the probability of occurrence of TS sensing slot is normally the complementary of $p_{SS}|_{\mathcal{H}_0}$, i.e., $p_{TS}|_{\mathcal{H}_0} = 1 - p_{SS}|_{\mathcal{H}_0}$.

Proposition 1. Given the sample length of a sensing interval N, the length of PU free period N_f distributed as in (3), the probability of receiving SS sensing slot under $\mathcal{H}_0$ is given by,

$$p_{SS}|_{\mathcal{H}_0} = \frac{\sum_{s_0=0}^{+\infty} s_0 P(s_0)}{\sum_{s_0=0}^{+\infty} (s_0+1) P(s_0)}, \tag{10}$$

where $P(s_0) = \left[(1-p_f)^{s_0 N-1} - (1-p_f)^{N(s_0+1)-1}\right]$.

Proof. Under $\mathcal{H}_0$, the probability of receiving s_0 number of SS sensing slot is given by,

$$\begin{aligned} P(s_0) &= P(s_0 N \le N_f < (s_0+1)N) \\ &= F_{N_f}(N(s_0+1)-1) - F_{N_F}(Ns_0 - 1) \\ &= \left[(1-p_f)^{s_0 N-1} - (1-p_f)^{N(s_0+1)-1}\right], \end{aligned} \tag{11}$$

where $F_{N_f}(\cdot)$ denotes the Cumulative Distribution Function (CDF) of N_f.

For each free period N_f, there occurs one TS sensing slot unless the free period N_f is a perfect multiple of the sensing period N. Thus, the probability of receiving an SS sensing slot can be written as the ratio of the average number of SS sensing slot that can be received for a given distribution of N_f to the total number of sensing slots under consideration,

$$p_{SS}|_{\mathcal{H}_0} = \frac{\sum_{s_0=0}^{+\infty} s_0 P(s_0)}{\sum_{s_0=0}^{+\infty}(s_0+1)P(s_0) - \sum_{m=1}^{+\infty} p_{N_f}(N_F = mN)}. \tag{12}$$

Since the second summation in the denominator of (12) is negligibly small compared to the first summation, we can neglect this summation leading to (10). □

The following proposition computes the probability of occurrence of SS sensing slot $p_{SS}|_{\mathcal{H}_1}$ under $\mathcal{H}_1$ and the probability of occurrence of TS sensing slot is normally the complementary of $p_{SS}|_{\mathcal{H}_1}$, i.e., $p_{TS}|_{\mathcal{H}_1} = 1 - p_{SS}|_{\mathcal{H}_1}$.

Proposition 2. Given the sample length of a sensing interval N, the length of PU busy period N_b distributed as in (4), the probability of receiving SS sensing slot under $\mathcal{H}_1$ is given by,

$$p_{SS}|_{\mathcal{H}_1} = \frac{\sum_{s_1=0}^{+\infty} s_1 P(s_1)}{\sum_{s_1=0}^{+\infty}(s_1+1)P(s_1)}, \tag{13}$$

where $P(s_1) = \left[(1-p_b)^{s_1 N-1} - (1-p_b)^{N(s_1+1)-1}\right]$.

Proof. Using the same line of reasoning as in the proof of Proposition 1, the proof of Proposition 2 is straightforward. □

IV. Sensing Performance Analysis

Energy detection computes the average energy of the received signal matrix $\boldsymbol{Y}$ normalized by the noise variance σ_v^2 and compares it with a predefined threshold T_{ED} is given by,

$$T_{ED} = \frac{1}{\sigma_v^2}\sum_{k=1}^{K}\sum_{n=1}^{N}|y_k(n)|^2. \tag{14}$$

To analyze ED performance, it is necessary to express the probability density function (pdf) of the decision statistic in case of unknown primary traffic. The following theorem computes the pdf of the ED decision statistic under both the hypotheses using the PU traffic characterization presented in Section III.

Theorem 1. Given a multi-antenna sensing unit with K receiving antennas, N received samples in each slot and the random PU traffic with geometrically distributed free state duration, the pdf of the ED decision statistic under $\mathcal{H}_0$ and $\mathcal{H}_1$ is given by (15) and (16) (shown at the top of the next page), respectively, where $f_{\mathcal{G}}(x,\alpha,\beta)$ is a pdf of Gamma distribution with shape parameter α & rate parameter β and $f_{\mathcal{N}}(x,\mu,\sigma^2)$ is the pdf of Gaussian distribution with mean μ and variance σ^2.

Proof. As noted from Section II, the term within the summation in (14) is different for the SS sensing slot and TS sensing slot. Under the null hypothesis $\mathcal{H}_0$, the ED decision statistic in (14) can be decomposed as a probabilistic sum of $T_{ED}^{SS}|_{\mathcal{H}_0}$ and $T_{ED}^{TS}|_{\mathcal{H}_0}$.

$$T_{ED}|_{\mathcal{H}_0} = \frac{p_{SS}|_{\mathcal{H}_0}}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\left|\frac{v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \frac{p_{TS}|_{\mathcal{H}_0}}{2}\left[\sum_{k=1}^{K}\sum_{n=1}^{D_0}\left|\frac{v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \sum_{k=1}^{K}\sum_{n=N-D_0+1}^{N}\left|\frac{h_k s(n)+v_k(n)}{\sigma_v/\sqrt{2}}\right|^2\right]. \tag{17}$$

Next, the distribution of each sum in (17) can be derived as [15],

$$T_{ED}|_{\mathcal{H}_0} = \frac{p_{SS}|_{\mathcal{H}_0}}{2}\chi^2_{2KN} + \frac{p_{TS}|_{\mathcal{H}_0}}{2}\sum_{d_0=1}^{N-1}P_{D_0}(d_0)\left[\chi^2_{2Kd_0} + K\rho\chi^2_{2(N-d_0)} + \chi^2_{2K(N-d_0)} + \mathcal{N}(0,2\rho(N-d_0)K)\right], \tag{18}$$

where χ_A^2 represents a Chi-squared random variable with A degrees of freedom. and $\mathcal{N}(\mu,\sigma^2)$ represents the Normal random variable with mean μ and variance σ^2.

In fact, the product of a Chi-squared RV with a constant is a Gamma RV, thus, with this replacement we obtain,

$$T_{ED}|_{\mathcal{H}_0} = p_{SS}|_{\mathcal{H}_0}\mathcal{G}(KN,1) + p_{TS}|_{\mathcal{H}_0}\sum_{d_0=1}^{N-1}P_{D_0}(d_0)\left[\mathcal{G}(Kd_0,1) + \mathcal{G}(N-d_0,K\rho) + \mathcal{G}(K(N-d_0),1) + \mathcal{N}(0,2\rho(N-d_0)K)\right]. \tag{19}$$

In addition, $\mathcal{G}(\alpha,\beta)$ represents a Gamma random variable with a shape parameter α and a rate parameter β. Since the goal is to find the pdf of the sum in (14) under $\mathcal{H}_0$, we replace the random variables in (19) with their respective pdfs to obtain (15).

In the similar manner, under the alternate hypothesis $\mathcal{H}_1$, the ED decision statistic in (14) can be decomposed as a probabilistic sum of $T_{ED}^{SS}|_{\mathcal{H}_1}$ and $T_{ED}^{TS}|_{\mathcal{H}_1}$.

$$T_{ED}|_{\mathcal{H}_1} = \frac{p_{SS}|_{\mathcal{H}_1}}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\left|\frac{h_k s(n)+v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \frac{p_{TS}|_{\mathcal{H}_1}}{2} \cdot \left[\sum_{k=1}^{K}\sum_{n=1}^{D_1}\left|\frac{h_k s(n)+v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \sum_{k=1}^{K}\sum_{n=N-D_1+1}^{N}\left|\frac{v_k(n)}{\sigma_v/\sqrt{2}}\right|^2\right] \tag{20}$$

Using the fact that D_1 is a random variable,

$$T_{ED}|_{\mathcal{H}_1} = \frac{p_{SS}|_{\mathcal{H}_1}}{2}\sum_{k=1}^{K}\sum_{n=1}^{N}\left|\frac{h_k s(n)+v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \frac{p_{TS}|_{\mathcal{H}_1}}{2}\sum_{d_1=1}^{N-1}P_{D_1}(d_1) \cdot \left[\sum_{k=1}^{K}\sum_{n=1}^{d_1}\left|\frac{h_k s(n)+v_k(n)}{\sigma_v/\sqrt{2}}\right|^2 + \sum_{k=1}^{K}\sum_{n=N-d_1+1}^{N}\left|\frac{v_k(n)}{\sigma_v/\sqrt{2}}\right|^2\right]. \tag{21}$$

Deriving the distribution of each sum in (21) using [15],

$$T_{ED}|_{\mathcal{H}_1} = p_{SS}|_{\mathcal{H}_1}\left(\mathcal{G}(N,K\rho) + \mathcal{G}(KN,1) + \mathcal{N}(0,2\rho KN)\right) + p_{TS}|_{\mathcal{H}_1}\sum_{d_1=1}^{N-1}P_{D_1}(d_1)\left[\mathcal{G}(d_1,K\rho) + \mathcal{G}(Kd_1,1) + \mathcal{N}(0,2\rho Kd_1) + \mathcal{G}(K(N-d_1),1)\right]. \tag{22}$$

$$
\begin{aligned}
f_{T_{ED}|H_0}(x) &= p_{SS}|_{H_0} f_{\mathcal{G}}(x, KN, 1) + p_{TS}|_{H_0} \sum_{d_0=1}^{N-1} P_{D_0}(d_0) \left[f_{\mathcal{G}}(x, 2Kd_0, 1) + f_{\mathcal{G}}(x, N-d_0, K\rho) \right. \\
&\quad \left. + \; f_{\mathcal{G}}(x, K(N-d_0), 1) + f_{\mathcal{N}}(x, 0, 2\rho K(N-d_0)) \right], \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
f_{T_{ED}|H_1}(x) &= p_{SS}|_{\mathcal{H}_1} \left(f_{\mathcal{G}}(x, N, K\rho) + f_{\mathcal{G}}(x, KN, 1) + f_{\mathcal{N}}(x, 0, 2\rho KN) \right) \\
&\quad + p_{TS}|_{\mathcal{H}_1} \sum_{d_1=1}^{N-1} P_{D_1}(d_1) \left[f_{\mathcal{G}}(x, d_1, K\rho) + f_{\mathcal{G}}(x, Kd_1, 1) + f_{\mathcal{N}}(x, 0, 2\rho Kd_1) + f_{\mathcal{G}}(x, K(N-d_1), 1) \right]. \qquad (16)
\end{aligned}
$$

Finally, we replace the random variables in (22) with their respective pdfs to obtain (16). □

In essence, the pdfs in (15) and (16) consist of the sum of independent random variables. From a statistical point of view, the sum of two independent pdfs can be realized as a convolution of these pdfs [16]. Thus, the sum of pdfs can be computed using convolution or as an alternative, we can exploit the characteristic function approach by computing Fourier transform. In conclusion, (15) and (16) can be easily evaluated by using standard Fast Fourier Transform (FFT) techniques.

A. Probability of False Alarm: Given the pdf of the decision statistic in (15), we can compute the false-alarm probability. Under $\mathcal{H}_0$, the PU is in free state at the end of the sensing interval, but the decision statistic is erroneously above the threshold τ and the PU signal is declared present. For defining the probability of false-alarm P_F in our case, the following Corollary of Theorem 1 holds.

Corollary 1. The false-alarm probability of the ED test under unknown PU traffic and complex signal space scenario is:

$$
P_F = P(T_{ED}|_{\mathcal{H}_0} \geq \tau) \equiv \int_{\tau}^{+\infty} f_{T_{ED}|H_0}(x)dx. \qquad (23)
$$

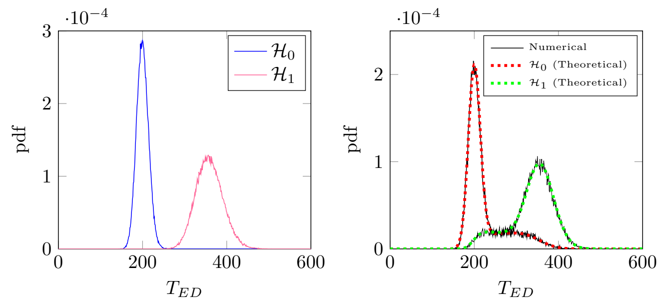
B. Probability of Detection: Given the pdf of the decision statistic in (16), we can compute the detection probability. Under $\mathcal{H}_1$, i.e., the PU is in busy state at the end of the sensing interval. Under this scenario, if the decision statistic is above the threshold, the PU signal is declared present. The following Corollary of Theorem 1 holds for defining the probability of detection P_D.

Corollary 2. The detection probability of the ED test under unknown PU traffic and the complex signal space scenario is:

$$
P_D = P(T_{ED}|_{\mathcal{H}_1} \geq \tau) \equiv \int_{\tau}^{+\infty} f_{T_{ED}|H_1}(x)dx. \qquad (24)
$$

V. Numerical Results and Discussion

In this section, the effect of PU traffic on the multi-antenna ED is analyzed based on the the traffic model developed in Section II. The length of the free and busy periods of the PU traffic are measured in terms of the discrete number of samples where each of them has Geometric distribution with probability of success parameters p_f and p_b, respectively. In this section, more often we use mean and busy period denoting $M_f = \frac{1}{p_f}$ and $M_b = \frac{1}{p_b}$, respectively. Under multiple antenna sensing scenario, the average SNR at the receiver is defined as, $\rho = \frac{\sigma_s^2 \|\boldsymbol{h}\|^2}{K\sigma_v^2}$, where $||.||$ denotes the Euclidean norm. The analytical expressions derived in Section III are validated via numerical simulation.



(a) Known PU Traffic (thus, synchronized PU-SU sensing slots)

(b) Unknown PU Traffic

Fig. 2. Pdfs of the ED decision statistic: Parameters: $N = 50$, $K = 4$, $M_f = 150$, $M_b = 150$ and SNR $= -6$ dB

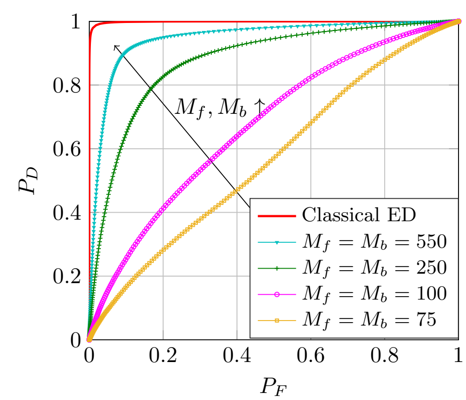


Fig. 3. ROC performance for the considered scenario, Parameters: $N = 100$, $K = 4$ and SNR $= -6$ dB

In Figure 2, the pdf of the decision statistic under ideal PU-SU sensing slot synchronization is compared with the pdf of the decision statistic under unknown PU traffic considering both hypotheses. In addition, the accuracy of derived analytical expressions of the pdfs is confirmed by the results presented in Fig. 1(b), where the theoretical formulas are compared against the numerical results obtained by Monte-Carlo simulation. The perfect match of the theoretical and the numerical pdfs validates the derived analytical expressions. Figure 3 illustrates

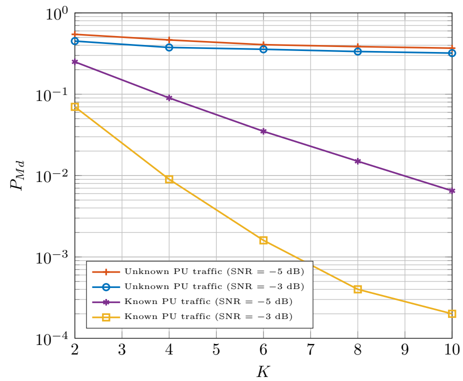


Fig. 4. Probability of missed detection vs. the number of Antennas, Parameters: $N = 25$, $M_f = 62$, $M_b = 62$ and $P_F = 0.1$

the Receiver Operating Characteristic (ROC) performance of the ED for different values of the mean free and busy period of the PU traffic. It shows that as the mean free and busy periods of the primary traffic increases, the detection performance of SU also increases. The conventional model with perfect synchronization of the PU-SU sensing slots performs better than the one with unknown PU traffic.

The variation of the sensing performance of the detector for different number of receiving antennas is plotted in Figure 4. It can be observed that unlike the rapid increase in sensing performance with the increasing number of receiving antennas under synchronized PU-SU sensing slot scenario (rapid decrease in missed-detection probability with the increasing number of receiving antennas), the sensing performance is almost constant even if we increase the number of antennas under unknown PU traffic. During a TS sensing slot, from each receiving antenna, the received signal samples are the mixture of pure noise samples and the samples with both noise and PU signal. Thus, even if we use multiple antennas, the nature of the received signal doesn't change much which is the reason the sensing performance improvement is suppressed by the unknown PU traffic (more specifically, the TS sensing performance) when the length of the free and busy periods of PU traffic are quite small (a few multiples of the length of the sensing window).

VI. Conclusion

In this paper, the effect of PU traffic on the performance of multi-antenna Energy Detector has been studied under a flat fading channel. A realistic and simple PU traffic model has been considered which is based only on the discrete time distribution of PU free and busy periods. Moreover, an analytical evaluation of the spectrum sensing performance under the considered scenario has been carried out. It has been shown that the performance gain due to multiple antenna in the sensing unit is significantly reduced by the effect of PU traffic when the mean lengths of free and busy periods are small (in the range of a few multiples of the sensing period).

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