Hopf Bifurcation Analysis and Implementation of Single Tunnel Diode Oscillator Circuit

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***Abstract*—In this paper, the simple LC tunnel diode is proved to oscillate at bias voltage in the range of the negative resistance region of tunnel diode. The Hopf bifurcation theorem is employed to prove the theoretical proof. The achieved analysis has been verified by circuit simulations and confirmed by experimental measurements. The results illustrates that for 1N3716 tunnel diode, oscillations occur at a bias voltage starts from 65 mV to 500 mV.**

***Keywords*—Hopf Bifurcation; Tunnel Diode; Oscillations.**

I. INTRODUCTION

One of the most powerful methods for studying periodic solutions in autonomous nonlinear systems is the Hopf bifurcation theorem. It shows that oscillations near an equilibrium point can be understood by looking at the eigenvalues of the linearized equations for perturbations from equilibrium and at certain crucial derivatives of the equation [1]. The problem discussed in this paper is applying Hopf analysis to an electronic circuit and verifying this using both simulation and hardware implementation and measurement.

Tunnel diodes are heavily doped p-n junctions only some 10 nm (100 Å) wide. The heavy doping results in a broken bandgap, where conduction band electron states on the n-side are more or less aligned with valence band hole states on the p-side. Under normal forward bias operation, as voltage begins to increase, electrons at first tunnel through the p-n junction barrier because electron states in the conduction band on the n-side become aligned with valence band hole states on the p-side of the pn junction. As voltage increases further, these states become more misaligned and the current drops — this is called negative resistance, because current decreases with increasing voltage. As voltage increases yet further, the diode begins to operate as a normal diode, where electrons travel by conduction across the pn junction, and no longer by tunneling through the pn junction barrier. Thus, the most important operating region for a tunnel diode is the negative resistance region.

This paper will be organized as follows: in Section-II Hopf bifurcation analysis is applied to the oscillator circuit [2].In Section-III, a SPICE model for the tunnel diode is built [3][4] and used in ORCAD software. The model is measured and compared with actual I-V characteristics measured with an I-V characterization device. The circuit is simulated using PSPICE. The circuit is implemented and the simulated results are compared with the measured ones.

II. CIRCUIT SCHEMATIC AND HOPF BIFURCATION ANALYSIS

The circuit consists of a single tunnel-diode, an inductor, and a capacitor as shown in Figure 1.

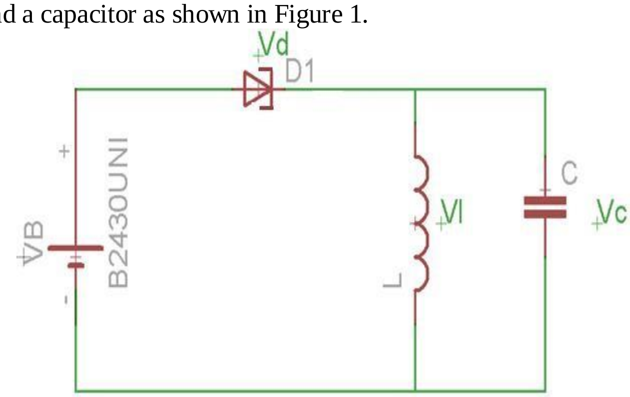

Figure 1. Circuit schematic of a single tunnel diode LC oscillator

The mathematical expressions for this circuit is [5]:

$$i_L' = \frac{1}{L} v_C \tag{1}$$

$$v_C' = \frac{1}{C}\left(g\left(v_B - v_C\right) - i_L\right) \tag{2}$$

Where g (V) describes the relation between current and voltage at tunnel diode terminals as follows:

$$I_D = I_{excess} + I_{tunnel} + I_{Diode} \tag{3}$$

$$I_D = \frac{v_D}{R_v}\, e^{(\frac{v_D - v_v}{v_{ex}})} + \frac{v_D}{R_o}\, e^{-(\frac{v_D}{v_o})} + I_s e^{(\frac{v_D}{V_T})} \tag{4}$$

Let: $$x_1 = i_L, x_2 = v_C \tag{5}$$

And VB is chosen to be μ, then (1) and (2) will be:

$$x_1^{'} = \frac{1}{L} x_2 \tag{6}$$

$$x_2^{'} = \frac{1}{C}\left(g\left(\mu - v_C\right) - x_1\right) \tag{7}$$

The equilibrium point is

$$x_2 = 0, x_1 = g\left(\mu\right) \tag{8}$$

$$y_2 = x_2, y_1 = x_1 - g\left(\mu\right) \tag{9}$$

The new ODE sys will be:

$$y_1^{'} = \frac{1}{L} y_2 \tag{10}$$

$$y_2^{'} = \frac{1}{C}\left[g\left(\mu - y_2\right) - y_1 - g\left(\mu\right)\right] \tag{11}$$

This system has a fixed point at the origin, and the linearized system will be:

$$A = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -f\left(\mu\right) \end{bmatrix} \tag{12}$$

Where f (μ) is the linear components of g (μ) and its eigenvalues are:

$$\lambda = \frac{-f(\mu) \pm \sqrt{f(\mu)^2 - \frac{4}{LC}}}{2} \tag{13}$$

$$\mathrm{f}\left(\mu_o\right) = 0, \lambda = \pm \mathrm{i}\frac{1}{\sqrt{\mathrm{LC}}} \tag{14}$$

The derivative of the eigenvalues is always positive for all values of μ. The index is calculated and proved to be negative. The bifurcation occurs at the negative resistance region of the tunnel diode.

III. CIRCUIT SIMULATION AND IMPLEMENTATION

In this section, circuit simulation is performed using OrCAD to verify the range of oscillation of the circuit. Then the circuit is implemented and measured to compare simulation results with experimental measurements.

A. *Tunnel Diode Spice Model*

The tunnel diode has no PSPICE model, so ABM is used to be able to simulate the tunnel diode as shown in Figure 2 using model equations in [3].

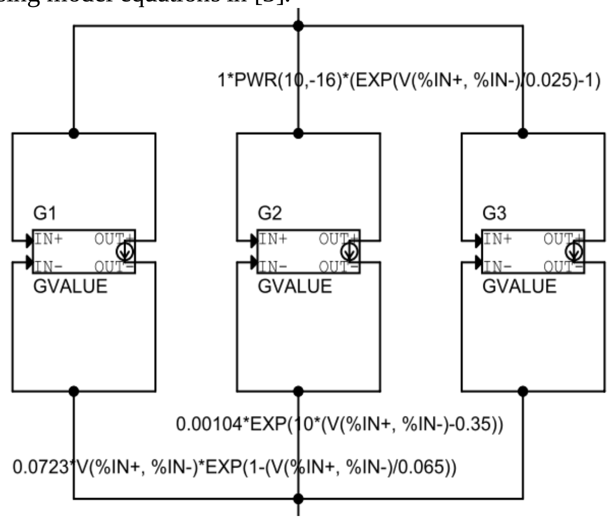

Figure 2. Analog behavioral model of a tunnel diode

The ABM model is a superposition of three currents:

$$I = I_{diode} + I_{tunnel} + I_{excess} \tag{15}$$

- The diode current (I_{diode}) is a regular p-n junction forward current due to injection of free electrons and holes from conduction band (CB) in n-type to CB in p-type, and from valence band (VB) in p-type to VB in type, respectively. This current is given by:

$$I_{diode} \approx I_s e^{\left[\left(\frac{V}{\eta V_{th}}\right) - 1\right]} \tag{16}$$

Where: Is denotes the saturation current, η represents the ideality factor, and Vth=KT/q.

- Tunneling current (I_{tunnel}) is the current due to tunneling of free electrons from CB in the n-type to free holes of the same energy in VB of the p-type. This current increases in the positive resistance region (ohmic region) due to the increase of the aligned energy states below Fermi-level in CB with that above Fermi-level in VB. At the same time, in the negative resistance region, the number of aligned energy states begin to decrease decreasing this current. This current is given by:

$$I_{tunnel} = \frac{V}{R_0} e^{\left[-\left(\frac{V}{V_0}\right)^m\right]} \tag{17}$$

Where: R_0 is the tunnel diode resistance in the ohmic region. m represents a factor with value ranges from 1 to 3. Also, V_0 ranges from 0.1V

- The excess current(I_{excess}) is an additional tunneling current related to parasitic tunneling via impurities and is given by:

$$I_{excess} = \frac{V}{R_V} e^{\left[\left(\frac{V-V_V}{V_{ex}}\right)\right]} \qquad (18)$$

Where: V_V denotes valley voltage (voltage at local minimum current). R_V and V_{ex} are empirical parameters in high quality diodes $R_V >> R_0$, V_{ex} ranges from 1 to 5V.

I-V characteristics were measured using Agilent I-V characterization system. Figure 3 shows the measured and simulated I-V characteristics.

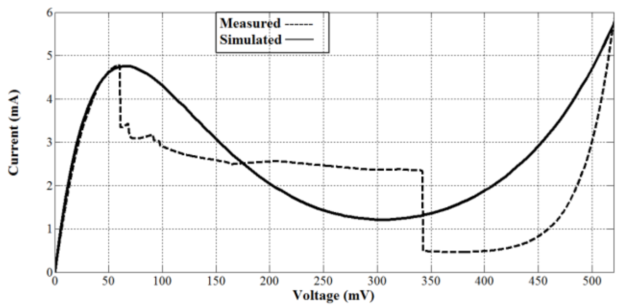

Figure 3. Simulated I-V characteristics of tunnel diode ABM on PSPICE vs measured characteristics using I-V characterization system

B. Simulation

The circuit was simulated using PSPICE a. A time domain analysis is done to the circuit. It was found that if the bias exceeds μ_0, the voltage across the inductor and capacitor oscillates with a frequency $\omega_o = \frac{1}{\sqrt{LC}}$ as mentioned before. This is shown in Figure 4. And when the bias exceeds V_V, the oscillation begins to die.

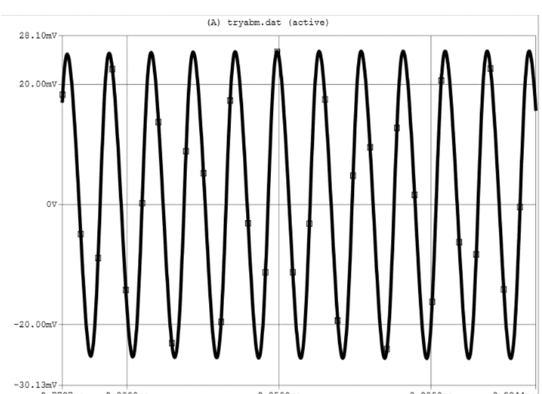

Figure 4. output voltage versus time in PSPICE shows that it performs oscillations at frequency ω_0

C. Circuit Implementation

The circuit is implemented using a simple breadboard, and measured using an oscilloscope.

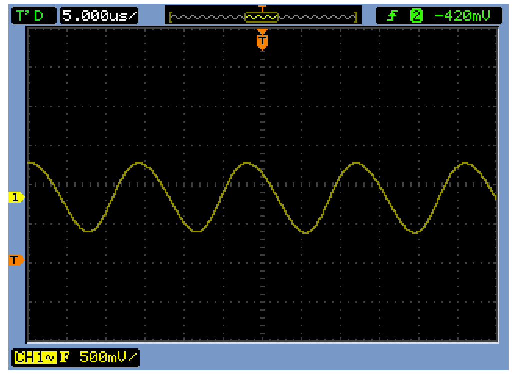

Figure 5. measured output voltage of the real circuit using oscilloscope and applying its internal digital filter.

It was found that the output voltage was very close to the simulated one except for some noise that appeared on the signal, so an internal digital filter built in the oscilloscope is used and the filtered output voltage is shown in Figure 5. We see that it is almost the same as the simulated circuit.

IV. CONCLUSION

Simple tunnel diode LC circuit oscillation was proved mathematically using the Hopf bifurcation theorem. The results showed that oscillation occurred at a bias voltage in the negative resistance region of the tunnel diode, an ABM of the tunnel diode was made to be able to simulate the circuit in PSPICE, and finally, the real circuit was implemented and its measured output was very close to the simulated one.

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