A Prospective Monotonic/Non-Monotonic Transition Zone Impediment for Concept Model-Centric Artificial Intelligence Systems

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***Abstract*—The increasing use of Artificial Intelligence (AI) has led to a myriad of Swarm Intelligence (SI) opportunities, wherein collective learning can occur, such as Machine Learning (ML) on ML, as well as collective Multi-Criteria Decision-Making (MCDM). Effective ML on ML tends to involve Knowledge Transfer (KT) via a Domain Knowledge Communication (DKC) channel, wherein successful interpretation of both the knowledge and the inferential processes involved is central. This is particularly important when temporal considerations matter. The conveyance of concepts, similar to the functioning of a Large Concept Model (LCM), exhibits promise, and various benchmarks — to ensure such a successful conveyance — have been scrutinized. However, while various efforts have been expended on the machine-centric side of the AI System (AIS) divide, a certain Achilles heel may reside on the human-centric side of the overarching Socio-Technical System (STS) in the form of non-concept model-centric Likert-derived information. This paper will progress through some machine-centric side experimental forays and then hone in on the Likert-centric repertoire on the other side of the AIS divide. A mitigation construct is proposed, and preliminary explorations exhibit some promise.**

Keywords-*artificial intelligence systems; machine learning; Lower Ambiguity Higher Uncertainty (LAHU); Higher Ambiguity Lower Uncertainty (HALU); isomorphic engine; domain knowledge communication; multi-criteria decision-making; decision quality; decision engineering*.

I. INTRODUCTION

The efficacy of certain Real World System (RWS) applications, such as Conversational Artificial Intelligence (AI), is often predicated upon consistency and reliability, and this particular facet can be referred to as Conversational AI (CAI) Robustness (CAIR). The responses/assertions provided by the involved CAI Agent (which should be designed to engage in "human-like conversations" by comprehending user intent, maintaining context, and putting forth pertinent responses) should adhere to the principle of CAIR; in other words, a core tenet of CAIR is that CAI Agent responses, once put forth, should maintain their validity (even amidst new user information provided). However, maintaining coherence and monotonicity is non-trivial, as the involved AIS might discern connections (particularly those that are non-monotonic) within the evolving dataset. In the context of CAIR, non-monotonic aspects can arise as incoming information can re-contextualize and/or contradict matters. Yet, enforcing a strict monotonic paradigm can segue to an unnatural rigidity and/or incorrect/irrelevant responses by the CAI. Accordingly, enhanced insight into the CAI behavior at Monotonic/Non-monotonic Transition Zones (MNTZ) can potentially be quite meaningful for elevating CAIR-related coherence and consistency (with the concomitant validity). Yet, this MNTZ element is often not part of CAI architectures, and the involved Repertoire of Likert-based Information (RLBI) training data and associated approach utilized do not necessarily have the benefit of various mitigation elements applied to them, such as in the form of Best-Worst Scaling (BWS) (which identifies "most preferred" and "least preferred" at the subset level), Q-methodology (which illuminates "opinion typologies"), and the like. Certain Subject Matter Experts (SMEs) in this arena attribute this to the hitherto success and novelty of CAI. However, Subsection A will highlight the potential downfall of reliance upon prior successes as architectural validation.

A. *Case study of a system-level Achilles heel*

The case study related to the Space Shuttle Columbia has been referred to often in various environs (e.g., academic), which focus upon a "learning culture." According to the National Aeronautics and Space Administration (NASA) commission that reviewed the Space Shuttle Columbia case, certain phenomena had become accepted over time; among these, was the "bipod ramp" (which connected the main external fuel tank to the spaceplane component of the Space Shuttle) thermal insulating foam (which prevented ice from forming when the external fuel tank was replete with liquid hydrogen and oxygen, as ice could damage the Space Shuttle, if shed during launch) that had been been observed falling off, in whole and/or in part, on several prior NASA missions (e.g., pertaining to the Challenger, Atlantis, and Columbia) prior to the Space Shuttle Columbia disintegration on 1 February 2003; ultimately, the cause of the disintegration could be attributed to a piece of thermal insulating foam breaking off from the external fuel tank after liftoff, striking the left wing, and causing a perforation that allowed "super-hot atmospheric gases" to enter the wing when the Space Shuttle Columbia later re-entered the atmosphere. As prior missions had been successful, NASA had grown accustomed to this "foam shedding" phenomena. After the Space Shuttle Columbia disintegration, the post-disaster investigation revealed that numerous NASA missions had indeed experienced thermal insulating foam loss, which had gone undetected. According to the NASA commission reviewing the Columbia case, the incident was at least partially attributed to the fact that "the Shuttle is now an aging system but still developmental in character," and "cultural traits and organizational practices detrimental to safety were allowed to develop, including reliance on past success as a substitute for sound engineering practices" [1].

In many ways, this lesson learned also seems to apply to those CAI architectures not treating the CAIR-related coherence, consistency, and validity issue. Indeed, it also seems to apply as a more generalized potential "foam shedding" aspect of contemporary Artificial Intelligence (AI) Systems (AIS); in particular, this may involve the Knowledge Transfer (KT) from an involved RLBI, which is not necessarily optimally conducive for a Large Concept Model (LCM) or concept model-centric Low Ambiguity High Uncertainty (LAHU)/Higher Ambiguity Lower Uncertainty (HALU) module that relies upon a quasi-isomorphic engine. In fact, preliminary experimentation shows that RLBI tends to aggravate matters in the MNTZ for AIS, as its non-concept model-centric paradigm seems to be problematic by introducing: heightened ambiguity, a less robust estimated parameter class (given the non-concept model-centric nature of RLBI), and a greater propensity for spawning towards the Non-deterministic Polynomial-time Hardness (NP-hard) non-continuous, non-polynomial, and non-monotonic side.

B. *Contemporary case of a prospective AIS Achilles heel*

The overarching rubric of AIS contains AI Control and Decision Systems (CDS) (AICDS), which in turn have Machine Learning (ML) constituent components. These components might involve, among others, LAHU/HALU components. The LAHU/HALU component has RWS application, as it accommodates the temporal element. By way of explanation, if the LAHU component deems that its repertoire [module] contains sufficient apriori experience (i.e., low ambiguity), then it might allow for an increased tolerance towards uncertainty, and likely, the need for more *Big Data* can be curtailed [2]; conversely, if the HALU component determines that there is insufficient apriori experience (i.e., high ambiguity), then it might necessitate more *Big Data*. In essence, the described paradigm equates to a quasi-isomorphic engine, which is better described as leveraging a quasi-LCM approach that segues to enhanced/nuanced semantic context, given the intrinsic ability to orchestrate multimodal inputs and extrapolate towards the desired notion/abstract concept class. The LAHU/HALU amalgam necessarily involves Multi-Criteria Decision-Making (MCDM). In turn, MCDM is typically comprised of Multi-Objective Decision-Making (MODM) and Multi-Attribute Decision-Making (MADM) modules. MODM usually involves multiple objectives, which are often conflicting, and MADM usually involves a singular objective. The counterpoising of the two is crucial. In turn, each of these can be comprised of Subjective Measures (SM), as well as Objective Measures (OM). Likewise, the counterpoising of these SM/OM is vital; otherwise, a variety of SM-related biases are likely to seep into the construct. The described AIS/AICDS is construed to reside within the realm of Decision Engineering (DE)/Decision-Making (DM), and the aforementioned is reflected in Figure 1. The enclosing purple boxes and font of that color pertain to some of the experimental forays presented herein. The blue boxes and font provide some pertinent ontological terms/concepts.

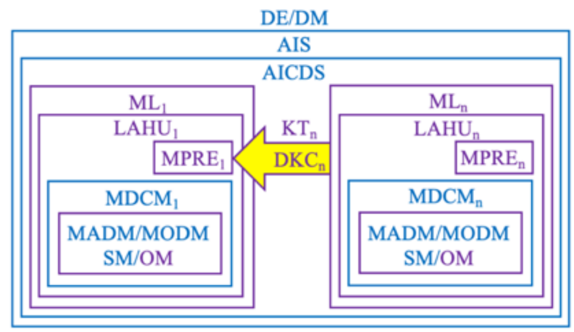

Figure 1. AIS/AICDS ML on ML KT (via DKC) with the LAHU/HALU MCDM (e.g., a MADM/MODM SM/OM counterpoising) supporting a Machine-Processed Repertoire of Experience (MPRE).

The LAHU/HALU's MPRE [module] is, ideally, enriched by KT, via the learnings of other precursor ML(s), such as shown in Figure 1 (e.g., ML_n). The channel by which the KT occurs is referred to as Domain Knowledge Communication (DKC) (e.g., DKC_n in Figure 1). Implicit in the reference to DKC is the prospective ability to convey the specialized knowledge of the involved domain(s); it is thought that a high efficacy approach centers upon the previously referenced notions/abstract concepts/estimated parameter class. Yet, these abstractions often also depend upon their interim notions/Inferred Latent Variables (ILVs), which in turn are, in some form or fashion, somewhat conveyed by various attributes. This paradigm is delineated via a prototypical Latent Variable Model (LVM) (Table I).

TABLE I. SAMPLE LVM CATEGORIES OF ESTIMATED PARAMETER CLASS, INTERIM NOTIONS, AND ATTRIBUTES CATEGORIES

Abstract Concept/ Notion/ Predicted Class/ Unknown Parameter/ Estimated Parameter Class	***Unobserved Variables/ Interim Notions/ Hidden Underlying Factors/ Latent Traits/ Inferred Latent Variables (ILVs)***	***Observed/Observation Variables/ Measured Variables/ Indicators Items/ Measures/ Attributes***

However, when the measured variables and ILVs are not 1-to-1, such as in the case wherein certain attributes are linked to many ILV (1-to-many) and/or many attributes are used to convey the essence of an ILV (many-to-1), then the causal

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relationships are no longer linear; they are non-linear. In cases such as this, Interpretability and Explainability (I&E) becomes paramount so as to better contextualize the causal pathways. This is non-trivial and the "quantification of joint contributions" is an active research area. Harris, by way of example, suggests joint Shapley values as a measure of joint feature importance within the involved feature sets [3]. Dhamdhere extends this by suggesting the utilization of "Shapley-Owen values" for this type of quantification [4]. Also, while the content related to the KT can indeed be significant, knowledge of the involved inferential process can, in a number of cases, be even more vital, as there may be certain bulwarks established, wherein KT across the DKC does not occur if the I&E threshold is not met.

It can then be ascertained that I&E and DKC for high efficacy ML on ML is a key thematic of this paper. Effective ML upon ML necessitates a certain degree of I&E for operationalizing DKC. I&E is construed to be part of the System Transparency, Explainability, and Accountability (STEA) rubric. In turn, STEA endeavors to mitigate against bias, and while machine-side AIS has been heavily scrutinized, oftentimes, the human-side elements (e.g., individual, institutional bias) "of the larger Socio-Technical System [STS]" have not been treated as robustly [5]. Exemplar biases impacting I&E/DKC include, but are not limited to:

1) Central Tendency Bias

Wang and Liu remind us that RWS data "often exhibit a long-tailed" distribution [6][7]. Within the STS paradigm, human input contributions (towards the repertoire of apriori experience) via modalities, such as Likert-derived information, often tend toward central tendency bias (i.e., a predilection towards the median and away from the min/max), and this is affirmed by Akbari and Sabolic [8][9]. This central tendency bias is likely to obscure/obfuscate the long-tail realities of the involved RWS.

2) Acquiescence Bias

Continuing along the vein of RLBI, Friborg reminds us that these forms "introduce acquiescence bias" [10]. To mitigate against this, it is customary to engage in negation transformations, but the "transformations may introduce errors, as negatives of positive constructs may appear contra-intuitive" (i.e., counter-intuitive) [10].

3) Anchoring Bias

As Yasseri reminds us, Kahneman and Tversky cautioned against the use of certain heuristics, wherein "certain information will be simplified, some ignored, and estimations will be made, thus increasing the likelihood of systematic errors in decisions" [11][12]. This predilection for gravitating towards "immediate examples" in one's mind is often referred to as a "mental shortcut" or "cognitive bias" [11]; an example includes anchoring bias, which involves gravitating towards "the first piece of information encountered" [11]. LAHU/HALU mitigates against this by examining MPRE and making the DE/DM determination on whether more *Big Data* is needed or not to lower the uncertainty.

4) Selection Bias

Of note, Berger reminds us that "the quality of randomization is an under-appreciated facet" and that "improper randomization" can segue to "selection bias" [13]. In essence, the choice of datasets, methods, design, programming, etc. as well as the individuals, groups, etc. selected for analysis (if subject to selection bias) can lead to a failure of "proper randomization" [14]. The significance is that the sample set obtained will no longer be representative of the population set to be scrutinized [15]. Hence, the results will likely be skewed. The LAHU/HALU MCDM can help mitigate against this, as the MCDM well considers the SM/OM counterpoisings for MADM/MODM.

The aforementioned referenced biases, among others, can challenge I&E/DKC and wreak havoc on an AIS/AICDS. To better illuminate this challenge, Figure 2 depicts the Human-Computer Interface (HCI) or Human-Machine Interface (HMI) zone, and the DKC channel is depicted as well.

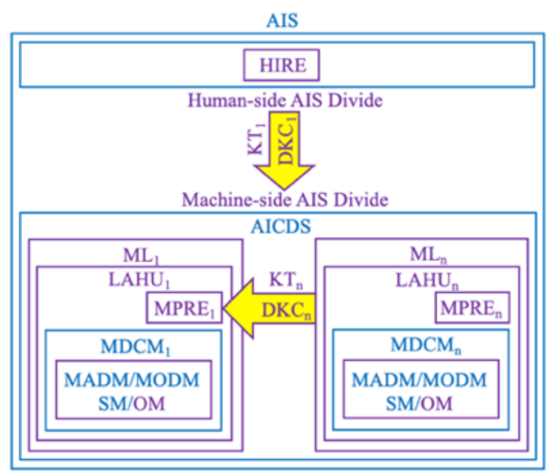

Figure 2. Human-side HIRE and Machine-side MPRE of the AIS Divide with KT/DKCs shown.

In the context of the referenced LAHU/HALU MPRE, the STS Human-Informed Repertoire of Experience (HIRE) side has high potential to skew/bias the STS MPRE side, thereby affecting the entire AIS/AICDS; hence, careful consideration of this issue is needed. *STS elements that can hinder I&E and degrade DKC are deemed to be disruptive for the AIS/AICS, and illuminating some of these STS elements is another key thematic of this paper.* Figure 2 shows that this can come via HIRE, across the KT_1/DKC_1, and affect the involved MPREs, such as $MPRE_1$ (it can also come from $MPRE_n$, across KT_n/DKC_n, and affect $MPRE_1$ in some cases).

Accordingly, this paper delineates the STS HIRE side of an AIS and its potential to adversely impact the STS MPRE side of an AIS. Section I provided an overview, which underscored the import of I&E and DKC for high efficacy

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ML on ML, as well as the fact that STS elements, such as HIRE, can potentially hinder I&E and degrade DKC so as to be disruptive for the AIS/AICS.

For the reader's convenience, a listing of acronyms utilized thus far and for the sections that follow is being provided in Table II below.

TABLE II. LISTING OF ACRONYMS UTILIZED

Acronym	***Expanded Form***
AI	Artificial Intelligence
AICDS	Artificial Intelligence Control and Decision Systems
AIS	Artificial Intelligence System
BLUF	Bottom Line Up Front
BWS	Best-Worst Scaling
C	Consistency
CAC	Current Architectural Construct
CAI	Conversational Artificial Intelligence
CAIR	Conversational Artificial Intelligence Robustness
COPRAS	Complex Proportional Assessment
CRITIC	CRiteria Importance through Intercriteria Correlation
D	Hoeffding's D Correlation Coefficient
dCor	Distance Correlation Coefficient
DE	Decision Engineering
DEA	Data Envelopment Analysis
DKC	Domain Knowledge Communication
DM	Decision-Making
ELECTRE	ELimination Et Choix Traduisant la Realité
F	Flexibility
F-VIKOR	Fuzzy VIseKriterijumska Optimizacija I Kompromisno Resenje
GP	Goal Programming
HALU	Higher Ambiguity Lower Uncertainty
HCI	Human-Computer Interface
HIRE	Human-Informed Repertoire of Experience
HMI	Human-Machine Interface
HV	Hypervolume
I	Interpretability
I&E	Interpretability and Explainability
ICC	Information Coefficient of Correlation
IGD	Inverted Generational Distance
ILVs	Inferred Latent Variables
KT	Knowledge Transfer
LAHU	Low Ambiguity High Uncertainty
LCM	Large Concept Model
LVM	Latent Variable Model
MADM	Multi-Attribute Decision-Making
MC	Maximal Correlation
MCDM	Multi-Criteria Decision-Making
MI	Mutual Information
MIC	Maximum Information Coefficient
MINLP	Mixed Integer Non-Linear Programming
ML	Machine Learning
MM	MULTIMOORA
MNTZ	Monotonic/Non-monotonic Transition Zones
MODM	Multi-Objective Decision-Making
MPRE	Machine-Processed Repertoire of Experience
NASA	National Aeronautics and Space Administration
NP-hard	Non-deterministic Polynomial-time Hardness
OM	Objective Measures
P	Performance
PAC	Previous Architectural Construct
PBCC	Percentage Bend Correlation Coefficient
PPMCC	Pearson's [Product]-Moment Correlation Coefficient
PROMETHEE	Preference Ranking Organization Method for Enrichment Evaluation
rho	Spearman's Rho Correlation Coefficient
RLBI	Repertoire of Likert-based Information
RNLBI	Repertoire of Non-Likert-based Information
ROM	Rough Order of Magnitude
ROYG	Red-Orange-Yellow-Green
RWS	Real World System
S	Sensitivity
S/R	Sorting/Ranking
SD	Semantic Differential
SDP	Semi-Definite Programming
SI	Swarm Intelligence
SM	Subjective Measures
SMEs	Subject Matter Experts
STEA	System Transparency, Explainability, and Accountability
STS	Socio-Technical System
tau	Kendall's Tau Correlation Coefficient
TOPSIS	Technique of Order Preference by Similarity to an Ideal Solution
U	Performance under Uncertainty
V	Validity
VC-dim	Vapnik-Chervonenkis dimension
VD	Verification/Discernment
WASPAS	Weighted Aggregated Sum Product Assessment

The remainder of this paper is organized as follows. Section II notes that RLBI potentially worsens conditions in the AIS MNTZ for AIS/AICDS, as its non-concept model-centric paradigm seems to be of some hindrance by introducing heightened ambiguity, a less robust estimated parameter class, and a greater propensity for spawning to the NP-hard, non-continuous, non-polynomial, and non-monotonic side. Section II also notes that Repertoire of Non-Likert-based Information (RNLBI) approaches, such as Semantic Differential (SD), necessitate a higher level of abstraction-level thinking, at the onset, that is more intrinsically akin to the ultimate notion/abstract concept to be expressed. Section III presents theoretical foundations as well as some precursor experimentation, an updated experimental setup (to account for some prospective quantitative speciousness in the literature), and an interim discussion regarding how RNLBI approaches (e.g., SD) — as they are more intrinsically akin to the concept model — will be more amenable for the MPRE via LAHU/HALU processing and may induce less spawning than RLBI. Section IV provides a discussion with some concluding remarks, and some proposed future work closes the paper.

II. BACKGROUND

Despite the criticality of I&E for operationalizing a high efficacy DKC, the treatment of I&E (and its overarching STEA) still remains in a fairly nascent state. By way of example, even the gauging of I&E still tends to be tied to the rudimentary metric of relating I&E to the complexity of the involved AIS/ML architecture. Along this vein, measures, such as "the Vapnik-Chervonenkis dimension (VC-dim)" are often utilized to gauge this complexity [16]. After all, the VC-dim can be emblematic, in a rough sense, of the Rough

Order of Magnitude (ROM) related to the involved number of weights, rules, etc.; indeed, an unwieldy number of, say, rules can readily segue to downstream brittleness issues. Brittleness, which had previously been explored in [17], necessitates a marked change in the paradigm, and I&E can fluctuate accordingly. Generally speaking, the ongoing tectonic shifts do not lend well toward enhancing I&E. Wood asserts that this type of "failure is due to brittle systems" [18]. Druce affirms, and of significance, Druce notes that "this lack of [I&E/]understandability in AIs precludes them from use in critical applications" [19]. Accordingly, this paper centers upon mission-critical RWS AIS/AICDS and revisits various potentially specious notions.

A. *Brittleness & Volatility in the MNTZ*

The described brittleness and volatility/unpredictability is especially prevalent within the MNTZ, wherein the shift of the involved variables from a monotonic to a non-monotonic paradigm can be quite unexpected and occur more frequently than anticipated/desired. In a sense, this seems to be aggravated when the involved repertoire is not intrinsically concept model-centric, such as in the case of RLBI. By way of background, Table III provides a simple depiction of: (1) "Monotonic," which denotes when an increase or decrease at one variable can segue to a corresponding change at the same rate (i.e., linear monotonic) or a different change of rate (i.e., non-linear monotonic) at the other variable, and (2) "Non-monotonic," which denotes when the ML model can alter direction at various points, such as when the first derivative switches signs (i.e., "a sign-changing first derivative") [20]. These are mapped against "Linear," wherein "the output is proportional to the input" and "Non-Linear," wherein the "relationship is more complex" (e.g., the relationship between/among the features is complex, the boundary areas are ambiguous, etc.) [21].

TABLE III. EXEMPLAR RESULTANTS AND MNTZ I&E

	Monotonic		***Non-monotonic***
Linear			
Non-linear			

The Monotonic/Non-monotonic and Linear/Non-linear resultants have varying degrees of I&E for the various complexities. The color coding for Table II utilizes the Red-Orange-Yellow-Green (ROYG) color coding schema, wherein the various shades of colors denote lowest to highest I&E. By way of example, red denotes low, orange denotes low/medium, yellow denotes medium, and green denotes high I&E. The shown boundary areas reflect the approximate encountered I&E, and as depicted in Table II, monotonic can be linear or non-linear. The crossing of "Linear Non-monotonic" is hatched, as technically, it cannot be linear (yet, over the long-term, a near-steady-state oscillation may appear, depending upon the magnification, so as to be quasi-linear); as Nicolaou points out, "in network systems, however, even at the level of linear dynamics, fundamental questions remain open concerning such transient growth — or, more generally, non-monotonic dynamics" [22].

B. *The Spawning of NP-Hard Non-Monotonic, Non-Polynomial, and Non-Continuous Functions within and abutting the MNTZ*

For the case of the RWS AIS/AICDS-related ML discussed herein, the spawning of "non-monotonic, non-polynomial, and even non-continuous functions" is not infrequent [23]. In other words, within the MNTZ, it is even more challenging to discern/ascertain what the I&E situation will be, for there is an even greater propensity for spawning to the NP-Hard side. This is not dissimilar to the paradigm, wherein the transformation of "non-convex Mixed Integer Non-Linear Programming (MINLP) to convex problems, often spawn further non-convex MINLP problems" that necessitate further handling, as is shown in Figure 3 [2]. In the context of Monotonic/Non-Monotonic and Linear/Non-linear, this is recast, as shown in Figure 4, wherein Non-Monotonic can be Continuous or Discontinuous, and Non-Linear can be Polynomial (e.g., which involves certain operations, such as addition, subtraction, and multiplication as well as non-negative integers as powers) or Non-Polynomial (e.g., wherein other operations are possible, and powers can be negative, fractional, or trigonometric, etc.).

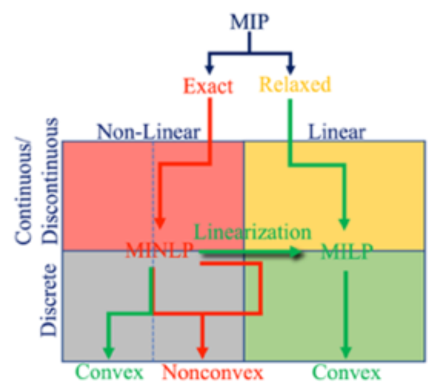

Figure 3. Non-convex to convex transformation pathways (e.g., non-convex discontinuous non-linear MINLPs to convex form)

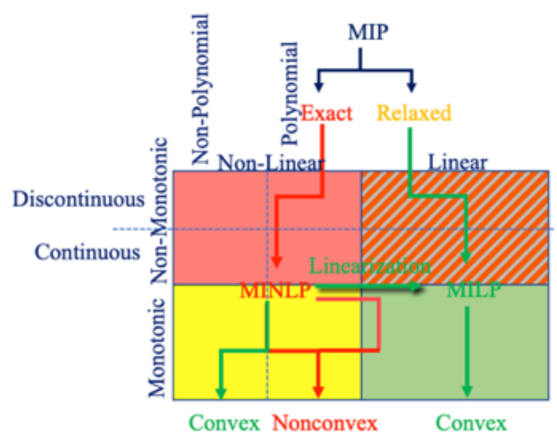

Figure 4. Non-convex to convex transformation pathways (e.g., non-convex non-monotonic, non-polynomial, non-continuous MINLPs to convex form)

Figure 3 and Figure 4 depict the pathways to convex form (e.g., linearization) in green font; once in a convex form, a myriad of Semi-Definite Programming (SDP) solvers can be brought to bear so as to resolve the involved optimization problems in polynomial time (presuming further spawning does not occur). In both cases, NP-hard-related spawn can potentially congest matters with an indefinite impasse.

C. Potentially, RLBI Aggravates & RNLBI Alleviates Matters in the MNTZ

RLBI potentially aggravates matters within the AIS MNTZ since its non-concept model-centric paradigm can potentially run counter to the overall construct by inducing increased ambiguity, increasing uncertainty regarding the estimated parameter class, and increasing the likelihood for spawning to the NP-hard, non-continuous, non-polynomial, and non-monotonic side. RNLBI, such as SD, involves a higher level of abstraction-level thinking that is more intrinsically akin to the ultimate notion/abstract concept to be expressed/articulated.

To summarize this section, the MNTZ must be treated for mission-critical RWS AIS/AICDS, as the transition of involved variables from a monotonic to a non-monotonic state can be quite unpredictable and occur at a higher frequency than anticipated/desired. In addition, without apropos mitigation bulwarks, the computational challenge may be inadvertently increased, as the spawning of "non-monotonic, non-polynomial, and even non-continuous functions" can occur at a higher than anticipated/desired rate. Moreover, a number of RWS, such as CAI, may not robustly distinguish between RLBI and RNLBI; this may be of detriment, as RLBI has been observed to potentially negatively impact matters within the AIS/AICDS MNTZ.

III. Experimentation

As a Bottom Line Up Front (BLUF) of the main outcome of this section, the experimental findings allude to a paradigm of decreased spawning as relates to RNLBI over RLBI. This should be of no great surprise, as RNLBI tends to be comprised of less subjective facets while RLBI tend to be comprised of inherently subjective elements (e.g., as respondents may construe the various scale points in a number of ways, thereby segueing to a set of data that is more difficult to compare and contrast). The logical progression that leads to these findings is presented as subsections A through B below.

A. Theoretical Foundations & Precursor Experimentation

As noted previously in Section IB, the issue of the quantification of joint contributions is non-trivial. Even taking the more simplistic case of RLBI, there may be a non-monotonic relationship between variables; for example, as one of the measured variables increases/decreases, the other variable may exhibit a complex curve, which may be challenging for I&E. Even for the seemingly simplistic case of the null hypothesis, wherein there exists no relationship between the two variables, it may be challenging to discern/affirm this paradigm. Of course, a monotonic linear relationship is more suitable for I&E. The task of I&E becomes increasingly challenged with a monotonic non-linear relationship, wherein the two variables increase/decrease, but at different rates of change. As discussed in Section IIA, while non-monotonic, technically, cannot be linear, linear dynamics can indeed transiently segue to non-monotonic dynamics, as Nicolaou points out [22]. This transient nature can be better understood via phase transitions, and to better ascertain when the segueing to non-monotonic non-linear paradigms occurs, various measures for monotonic/non-monotonic and linear/non-linear paradigms are utilized, such as presented and described in Table IV below. These Table IV measures are then sorted and presented by exemplar usage in Table V below and on the following page.

TABLE IV. VARIOUS MEASURES FOR MONOTONIC/NON-MONOTONIC AND LINEAR/NON-LINEAR PARADIGMS

Measure	***Descriptor***
Distance Correlation Coefficient (dCor) [23]	dCor is "better at revealing complex… relationships… compared with other correlation metrics" by "integrating both linear and non-linear dependence" [27].
Hoeffding's D Correlation Coefficient (D) [23][24]	D can reflect a certain degree of concordance and discordance.
Information Coefficient of Correlation (ICC) [25]	ICC can provide a gauge of alignment between the posited and actual value.
Kendall's Tau Correlation Coefficient (tau) [23][26]	Tau can illuminate correlations of import when the distributions of the sample set and population are not necessarily known.
Maximal Correlation (MC) [25]	MC pertains to transformations of the data, which are considered to maximize the correlation.
Maximum Information Coefficient (MIC) [25]	MIC encompasses both linear and nonlinear correlations between the "variable pairs."
Mutual Information (MI) [25]	MI is a paradigm, wherein one of the variables conveys a quantifiable amount of information about the other.
Pearson's [Product]-Moment Correlation Coefficient (PPMCC) [23]	PPMCC measures the relationship strength and direction between the "variable pairs."
Percentage Bend Correlation Coefficient (PBCC) [23]	PBCC refers to a paradigm, wherein a specified percentage of marginal observations deviating from the median are weighted downward [28].
Spearman's Rho Correlation Coefficient (rho) [23][26]	Rho scrutinizes the dependence between two random variables [29].

TABLE V. EXEMPLAR USAGE OF VARIOUS MEASURES

	Monotonic	***Non-monotonic***
Linear	D [23] rho [23] tau [23][26] PPMCC [23][25][26] PBCC [23] dCor [23]	N/A[2]
Non-linear	PPMCC[1][23][25][26] rho [23] tau [23][26] PBCC [23] dCor [23] D [23]	MC[3] [25] dCor[4] [23] D [23] PPMCC[5] [23][25] rho[5] [23][25]

	Curvilinear	rho [23] PBCC [23] dCor [23] PPMCC[1][23] tau [23]	dCor [23] D [23] PPMCC[5] [25] rho[5] [23][25]

[1] Heuvel notes the efficacy of PPMCC with "families of bivariate distribution functions with non-linear monotonic associations" [26].
[2] as noted previously in Section IIA, technically, non-monotonic cannot be linear; however, as noted by Nicolaou, linear dynamics may experience transient segueing "toward non-monotonic dynamics" [22].
[3] requires "greater than 100 observations" [25].
[4] requires "less than 50 observations," as "it is not susceptible to the exact number of observations" [25].
[5] of note, it does not "find non-monotonic dependence," given symmetry [25].

Generally speaking, the reflected ROYG results align with the findings of Mirtagioglu. For example, the following seem to hold: (1) "in cases where there is no relationship between the variables" (e.g., non-functional relationship, wherein "there is no function of one variable that interacts with the other and vice versa"), dCor, D, tau, PPMCC, PBCC, and rho "have given very satisfactory results," as well as MC, (2) "very low values (close to 0)" of rho, tau, PPMCC, and PBCC is emblematic of a "random relationship between the variables," and (3) "very low values (close to 0)" of tau, PPMCC, and PBCC, and rho when conjoined with "very high values (close to 1)" of dCoR is emblematic of a non-monotonic relationship between/among variables, such as shown in Table VI below [23][26].

TABLE VI. EXEMPLAR FINDINGS FROM MEASURES & POSITS

Close to 0	***Close to 1***	***Close to -1***	***Relationship Posits***
dCor, D, tau, PPMCC, PBCC, rho	N/A	N/A	None
rho, tau, PPMCC, PBCC	N/A	N/A	Random
N/A	rho, PPMCC	N/A	Strong Positive Monotonic
N/A	N/A	rho, PPMCC	Strong Negative Monotonic
tau, PPMCC, PBCC, rho,	dCor	N/A	Non-monotonic

The results also somewhat align with the findings of Fujita, Rainio, and Heuvel. However, the rankings and sortings, such as offered by Mirtagioglu (M), Rainio (R), and Heuvel (H) somewhat differ, as shown in Table VII below.

TABLE VII. POSITED RANKING/SORTINGS BY M, R, AND H

	M [23]	***R*** [25]	***H*** [26]
Linear Monotonic	rho PBCC PPMCC dCor tau	PPMCC[1] rho[2] tau[2]	PPMCC MIC
Non-linear Monotonic	rho PBCC dCor tau D	rho tau PPMCC	PPMCC MIC
Non-linear (e.g., curvilinear)	dCor D	N/A	PPMCC[3] rho[3]
Non-monotonic			MIC

[1] more oriented for "linear association" [26].
[2] more oriented for "monotonic association [26].
[3] however, this is N/A when the non-monotonic dependence is symmetric [25].

Of course, it would be ideal to first, ascertain the involved relationships (initial foray), second, apply the pertinent measures (verification/discernment), and then, perhaps, third, repeat this process recursively; however, this may not always be possible, as there are a number of subtleties/challenges amidst varying temporal conditions/constraints. In any case, the notional construct utilized is shown in Figure 5 on the following page. Furthermore, the exemplar organization and sequencing of the measures (a.k.a., Verification/Discernment or VD measures) of Figure 5 for monotonic transformation and MNTZ insights was treated with the various methods reflected in Table VIII, which applied specific methods (a.k.a., Sorting/Ranking or SR methods) to the VD measures of Figure 5.

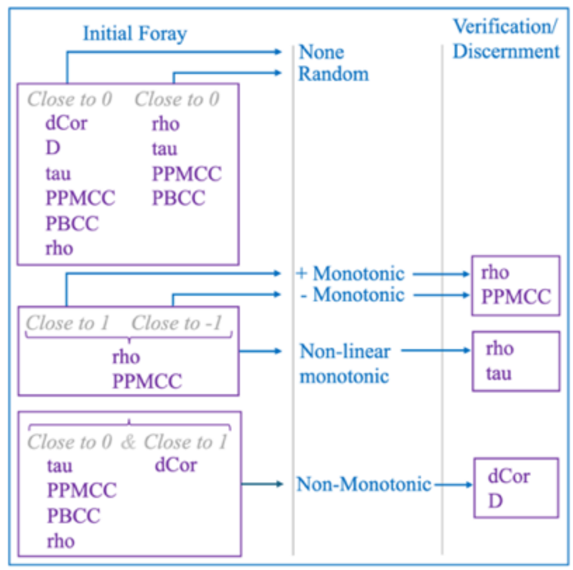

Figure 5. Exemplar sequencing regarding verifications/discernments for monotonic transformation/MNTZ insights

Specifically, the VD measures are needed for more robust insights into the monotonic transformations and ensuing monotonic/non-monotonic dynamics within/around the MNTZ. It should be noted that the SR methods were utilized in a MADM OM sense, since some of the methods are able to handle both SM and OM. As part of the experimentation, a bespoke experimental architectural construct was explored with the OM#1-7 of Table VIII as well as Figures 6 and 7. MULTIMOORA (MM), Goal Programming (GP), and Weighted Aggregated Sum Product Assessment (WASPAS) were presets utilized for MODM SM, MODM OM, and MADM SM, respectively, as shown in Figures 6 and 7 on the following page.

TABLE VIII. METHODS APPLIED TO THE VD MEASURES OF FIGURE 5.

#	*Methods*	*MADM*	*OM*
1	Complex Proportional Assessment (COPRAS)	[30]	[36]
2	CRiteria Importance through Intercriteria Correlation (CRITIC)	[31]	[37]
3	Data Envelopment Analysis (DEA)	[32]	[38]
4	ELimination Et Choix Traduisant la Realité (ELECTRE)	[33]	SM/OM [39]
5	Fuzzy VIseKriterijumska Optimizacija I Kompromisno Resenje (F-VIKOR)	[34]	SM/OM [40]
6	Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE) (e.g., I and II)	[35]	SM/OM [41]
7	Technique of Order Preference by Similarity to an Ideal Solution (TOPSIS)	[34]	[42]

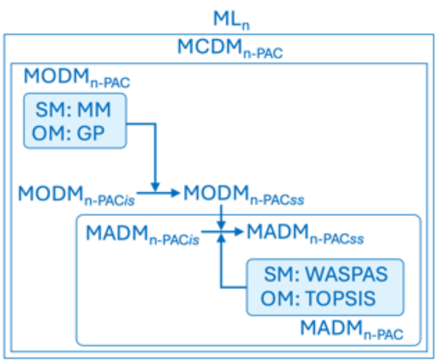

Figure 6. PAC with TOPSIS usage for the MADM OM

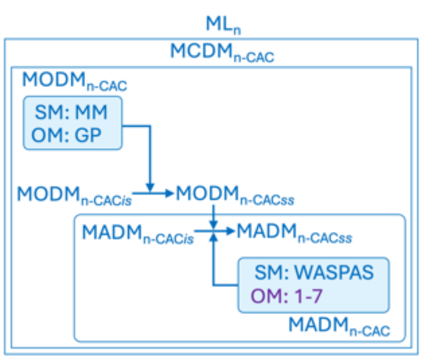

Figure 7. CAC with explicit OM#1-7 usage for MADM

For Figure 6, "PAC" refers to "Previous Architectural Construct," "*is*" equates to "input set," and "*ss*" equates to "solution set." Please note: the "MODM 'solution set' (MODMn-PAC$_{ss}$) facilitates the MADM 'input set' (MADMn-PAC$_{is}$) to MADM 'output solution set' (MADMn-PAC$_{ss}$) progression [2]. For Figure 7, "CAC" refers to "Current Architectural Construct," "*is*" equates to "input set," and "*ss*" equates to "solution set." Please note: the "MODM 'solution set' (MODMn-CAC$_{ss}$) facilitates the MADM 'input set' (MADMn-CAC$_{is}$) to MADM 'output solution set' (MADMn-CAC$_{ss}$) progression [2].

Taking the 7 metrics of **P**erformance (***P***) (i.e., execution time), **C**onsistency (***C***) ("which is a useful indicator" for stability as well as "the underlying convergence paradigm"), **F**lexibility (***F***) ("for adaptation, hybridization, etc."), **S**ensitivity (***S***), *P* under **U**ncertainty (***U***), **V**alidity (***V***), and **I**nterpretability (***I***), various comparative evaluations of OM#1-7 were conducted, and the interim findings are delineated in Figure 8 [43]. For ease of comparison, the relative values were normalized. In terms of benchmarking indicators, Inverted Generational Distance (IGD) and Hypervolume (HV) were utilized, where in the context of the multi-objective domain (e.g., MCDM, MODM, etc.), IGD is a metric that assesses the solution set quality by way of measures, such as convergence (distance of the solutions in the solution set to the Pareto front) and diversity (coverage by the solution set relating to the Pareto front), among others, and HV pertains to the volume of a hv-dimensional space populated by the solution set, where a higher HV alludes to a more robust solution set. This is consistent with Sun and Chugh opining that IGD "has been widely considered as a reliable performance indicator," and likewise, that HV "is one of the most used set-quality indicators" [44][45][46].

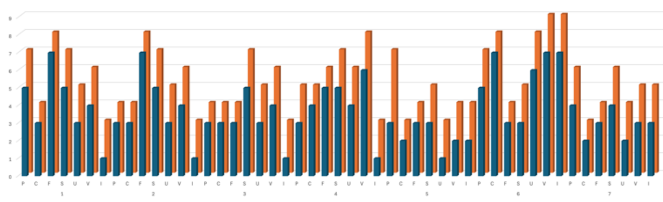

Figure 8. Preliminary Results from OM Benchmarking against IGD/HV

A literature review was conducted to ensure that the results were reasonable, and some example affirmations are shown in Table IX below.

TABLE IX. SAMPLE AFFIRMATIONS OF REASONABLENESS OF RESULTS

Metric	*Exemplar Affirmation*
P	• Varatharajulu favors the COPRAS/TOPSIS amalgam [47]. • Hezer favors COPRAS, TOPSIS, and VIKOR (in that order) [48].
C	• Salabun favors TOPSIS and PROMETHEE over VIKOR [49]. • Ezhilarasan favors ELECTRE over TOPSIS [50][51].
F	• Akram favors extensions of ELECTRE and TOPSIS [52].
S	• Kokaraki opines that TOPSIS may likely be the "most sensitive" (i.e., a high *S*) [53].
U	• Jordehi and Lofti favor ELECTRE [54][55]. • Ziemba favors PROMETHEE [56]. • Taherdoost, Oubahman, and Moreira favor PROMETHEE (e.g., PROMETHEE I for "partial ranking," and PROMETHEE II for "complete ranking"), as it can "accommodate complex qualitative and quantitative evaluations" [57][58][59]
V	• Ozmen favors PROMETHEE to ELECTRE [60].
I	• Leyva-Lopez and Yedjour favor ELECTRE and PROMETHEE [61][62].

Variables and parameters were established as suggested in [45][63].

B. *Updated Experimental Setup*

Initial experimentation had been predicated upon exemplar sample size recommendations, such as shown in Table X, rooted in Gunawan's work, and predicated upon Thompson's *Exploratory and Confirmatory Factor Analysis* [64][65][66].

TABLE X. EXEMPLAR SAMPLE SIZE RECOMMENDATIONS

Recommended Sample Sizes	***Investigators***
200-300	Guadagnoli & Velicer [70]
>=200	Hair et al. [71]
>=200-1000	Nevitt & Hancock [72]
300	Comrey & Lee [67]; Clark & Watson [73]; VanVoorhis & Morgan [74]; White [75]
>=300 is deemed "good enough" (e.g., sample sizes < 300 "tend to diverge") [64][66] [68][69]	Kyriazos & DeVellis [68][69]
>=400	Aleamoni [76]
500 is "very good" [64][67]	Comrey & Lee [67]
>=1,000 is "excellent"	Gunawan [64]; Comrey & Lee [67]

Initially, experimentation sample sizes, such as >=300, were deemed to be sufficient. However, according to Columbia University's Professor Gelman, it is opined that *"you need 16 times the sample size to estimate an interaction than to estimate a main effect*" [77]. As the MNTZ are likely rife with these described *interactions*, Gelman's point is taken. Also, Gelman's argument seems to dovetail with Rainio's thoughts on *power* (e.g., the efficacy to ascertain whether there is "some association between the variables or not"), *equitability* (e.g., the ability to ascertain "similar values for…relationships that are based on different functions but have the same level of noise"), and *generality* (e.g., the capability, at the involved sample quantity, to not only "detect linear, monotonic, or functional dependence," but also "recognize more complicated relationships between the variables") [25]. Interestingly, with an updated experimental setup predicated upon Gelman's and Rainio's thoughts, the findings allude to a paradigm of decreased spawning with RNLBI over RLBI. The pathways discussed within are summarized in Figure 9.

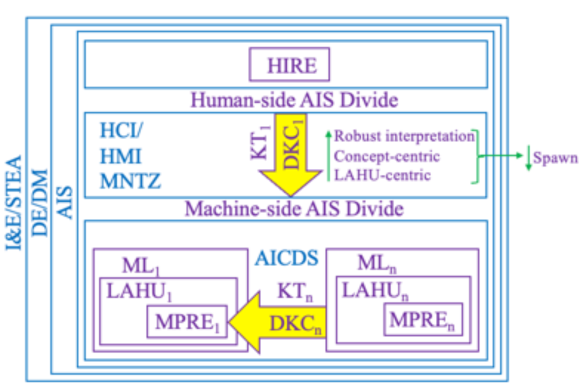

Figure 9. Overall MNTZ and spawn reduction observations/posits

Beyond the pathways, it might be fitting to also address certain aspects of the HIRE human-side and MPRE machine-side of the AIS Divide, particularly as pertains to prospective human-centric and machine-centric (i.e., AI-centric) biases. For example, human respondents might tend to agree with the crowd in the form of acquiescence bias; likewise human respondents might also avoid the extremes and tend towards the "safety" of the "middle of the scale" in the form of central tendency bias. Conversely, the machine might accentuate a particular historical bias and perpetuate that aspect (predicated upon the potentially specious notion that the temporal span of the historical data, albeit possibly biased, carries weight), and interestingly, this might be a long-tail perspective that is exacerbated in the form of selection bias; likewise omitted variable bias can occur, if the machine applies enough weighting to the long-tail perspective (as overfitting can also be a source of bias). For these reasons and others, the mitigation measures alluded to in Section I highlight the prospective robustness of RNLBI over RLBI. In addition, the handling/counterpoising of monotonicity/non-monotonicity can be central for RWS, such as CAIR for CAI; after all, monotonoic reasoning presumes that prior assertions should always hold true. In contrast, non-monotonic reasoning allows for revisions predicated upon new information. In this regard, the LAHU/HALU MCDM component is also critical for treating the temporal component, as RWS (such as CAI) applications tend to occur in real-time. By considering the presented machinations and mitigations, a more robust MNTZ discerning/understanding conjoined with the discussed LAHU/HALU MCDM counterpoisings can potentially segue to more graceful management of seeming contradictions, thereby better harmonizing/counterpoising monotonic/non-monotonic paradigms.

IV. DISCUSSION & CONCLUDING REMARKS

Auret put it well more than a decade ago: a "better understanding of process phenomena is dependent on the interpretation of models capturing the relationships between the process variables" [78]. As these relationships are central, Gelman's recommendations were taken into consideration. With this particular perspective, the explorations of this paper indicate that RLBI can likely be a prospective impediment, particularly within or around the MNTZ for concept model-centric AIS. Indeed, it seems that the spawn rate (e.g., the spawning of "non-monotonic, non-polynomial, and even non-continuous functions") for RLBI may be higher than that for RNLBI [23][79]; however, more quantitative and qualitative forays are needed in this regard (e.g., future works). For the mission-critical RWS AIS/AICDS focus of this paper, it was gleaned that an effective ML on ML paradigm necessitates robust STEA/I&E, which can facilitate the assurance of the intended interpretation, such as via the DKC channel, for KT. Nicolaou reminds us that "transient growth offers interesting alternative explanations for behavior usually attributed to nonlinearity, such as ignition dynamics" [22]; given the myriad of varied connotational interpretations (i.e., alternative explanations), enhanced interpretation, via the DKC interpretand, is particularly vital within/around the MNTZ. The DKC, in the case of this paper, is akin to the LCM in that it is more akin to being concept-based. Accordingly, for meaningful KT to occur, the I&E for the utilized hierarchical/non-hierarchical LVM needs to be of sufficient robustness.

HIRE, which is often replete with RLBI, can skew matters for the AIS/AICDS, as it is not intrinsically concept-based. However, RNLBI, such as SD, can intrinsically be more concept-based (e.g., via its bipolar dichotomy and the in-between continuum). With regards to the DKC recognition element, such as via the quasi-isomorphic engine, the various morphisms (e.g., automorphisms, homeomorphisms, diffeomorphisms, symplectomorphisms, etc.) as well as the various subgraph isomorphism relaxations need to be well treated. However, the utilized approach (e.g., robust convex relaxations) can also further spawn further non-convex MINLP problems, so a reduction in spawning is key. Overall, RNLBI seems to lend towards a reduction of this spawning, and given this prospective mitigation approach, the described machinations at/or abutting the DKC, such as within the MNTZ, can mitigate against the various inferences/predictions/posits/insights of the involved AIS. Of note, the intrinsic wherewithal to accommodate both discrete and continuous paradigms is critical. Along this vein, the LAHU/HALU MCDM, which is at the heart of DE/DM, encompasses MODM for "undetermined continuous alternatives" as well as MADM for "discrete alternatives." Axiomatically, these require continuous as well as discrete evaluations, respectively. It then follows that since RLBI do not contain "0," discrete testing is not possible; restated, only continuous distribution testing is possible. On the contrary, RNLBI (e.g., SD) do indeed contain "0" and are able to accommodate both discrete and continuous distribution testing. The preliminary experimental findings seem to affirm that RNLBI lend toward a higher P, C, F, U, V, I and a lower S than RLBI, particularly in and/or around the MNTZ (with less spawn observed).

To conclude, RNLBI are potentially more amenable to higher nuance/insight and seem to warrant further investigation. After all, this particular facet of AIS/AICDS addresses the important AI challenge of how biases and transition zones may potentially affect DE/DM. In particular, the frameworks for LAHU/HALU, ML on ML/DKC/KT, and MNTZ are central for addressing the challenge by facilitating the exploration of AIS behavior within transition zones (e.g., MNTZ) and ML on ML/DKC/KT frameworks. Future work will entail more quantitative investigation (with careful consideration given toward quantitative fallacy, as alluded to by Gelman), and a more extensive AIS/AICDS comparative literature survey with accompanying empirical evaluation will be conducted.

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