A Novel Unambiguous CBOC Correlation Function With an Improved Main-Peak

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***Abstract*—Conventionally, the design of a correlation function for unambiguous tracking of composite binary offset carrier (CBOC) signals has focused on only the elimination of the side-peaks causing the ambiguity in tracking without considering the loss in height and sharpness of the main-peak during the elimination process, thus resulting in a worse tracking performance compared with that corresponding to the CBOC-autocorrelation function. In this paper, we propose a novel correlation function with no side-peaks *and* a main-peak that is higher and sharper than those of the conventional correlation functions including the CBOC-autocorrelation function, thus enabling us to have not only unambiguity in tracking but also a better tracking performance over that of the CBOC-autocorrelation function. We first split the CBOC sub-carrier into multiple partial sub-carriers and correlate each of them with the received signal, yielding partial correlations. Then, we combine the partial correlations in a specialized way, where the side-peaks are canceled out and the main-peak becomes higher and sharper than that of the CBOC-autocorrelation function. Finally, the proposed correlation function is shown to have no side-peaks and to provide a better tracking performance than those of the conventional correlation functions including the CBOC-autocorrelation function.**

Keywords–Composite binary offset carrier; Tracking ambiguity; Galileo; global navigation satellite system;

I. INTRODUCTION

Recently, various global navigation satellite systems (GNSSs) have been developed due to increasing demands for location-based service [1]. Galileo is the GNSS developed by European Space Agency and is now operating with twelve satellites including six satellites launched in 2015 [2]. In Galileo, CBOC signals have been employed in E1 band to provide more precise location service than that of the conventional GNSSs: The CBOC signal provides an improved signal tracking performance compared with the phase shift keying (PSK) signals of the conventional global positioning system (GPS) due to its sharper correlation main-peak [3]. In addition, the CBOC signal enables Galileo to share the frequency band with GPS. The CBOC signal is generated by multiplying a pseudorandom noise (PRN) code and a CBOC sub-carrier obtained from a weighted sum of two sine-phased BOC sub-carriers, and is denoted by CBOC(x,y,α), where x and y are the ratios of the chip period $T_c = 1/(1.023 \times 10^6)$ of the PRN code to the sub-carrier periods of BOC(x,1) and BOC(y,1), respectively, and α represents that the power of the sub-carriers of BOC(x,1) and BOC(y,1) accounts for α and $1-\alpha$ of the power of the CBOC sub-carrier, respectively [3].

The main drawback of the CBOC signal is a problem of ambiguity in tracking caused by multiple side-peaks around the main-peak. The side-peaks could cause the tracking loop to be locked on one of the side-peaks, eventually incurring a biased tracking measurement. To tackle this problem, various unambiguous correlation functions have been proposed [4]-[12]. Several of them are for sine-phased or cosine-phased BOC signals only and are inapplicable to the CBOC signal [4]-[7]. Sousa proposed an unambiguous correlation function for the CBOC signal, removing the side-peaks completely [8]; however, the correlation function has a lower and blunter main-peak than that of the CBOC-autocorrelation function, and thus, leads to an inferior tracking performance compared with the CBOC-autocorrelation function. Although there are several correlation functions with a main-peak that is higher and sharper than that of Sousa, the improvement in height and sharpness of the main-peaks is not pronounced, and consequently, the tracking performances of the correlation functions do not exhibit a significant improvement over that of the CBOC-autocorrelation function [9]-[11]. In [12], a novel approach based on splitting the sub-carrier was presented for improvement of the main-peak and it was shown that the main-peak can be much higher and sharper compared with those of the correlation functions mentioned above through the approach. However, the splitting method is empirical and the number of the split sub-carriers is limited to four in the method.

Observing that a higher and sharper main-peak could be yielded by splitting the sub-carrier more, in this paper, we propose a *systematic* method for splitting the sub-carrier, by which the sub-carrier can be split into *any number* of partial sub-carriers, and consequently, a significantly improved main-peak can be obtained. We first split the CBOC sub-carrier into multiple partial sub-carriers, and subsequently, generate partial correlations by correlating each of the partial sub-carriers and the received signal. Then, we cancel out the side-peaks while making the correlation main-peak higher and sharper than that of the CBOC-autocorrelation function by combining the partial correlations in a specialized way.

The rest of this paper is organized as follows: In Section II, we describe the CBOC signal model. In Section III, we propose an unambiguous correlation function with an improved main-peak and no side-peaks. In Section IV, it is confirmed that the proposed correlation function provides a better tracking performance than those of the conventional correlation functions including the CBOC-autocorrelation function in terms of the tracking error standard deviation (TESD), and in Section V, conclusion is presented.

II. CBOC(6,1,1/11) SIGNAL MODEL

In this paper, the CBOC(6,1,1/11) signal, denoted by $B(t)$, is considered and it can be expressed as

$$B(t) = \sqrt{P} \sum_{i=-\infty}^{\infty} p_i r_{T_c}(t - iT_c) d(t) s_{sc}^i(t), \qquad (1)$$

where P is the signal power, $p_i \in \{-1, 1\}$ is the ith chip of a PRN code with a period T, $r_\alpha(t)$ denotes the unit rectangular

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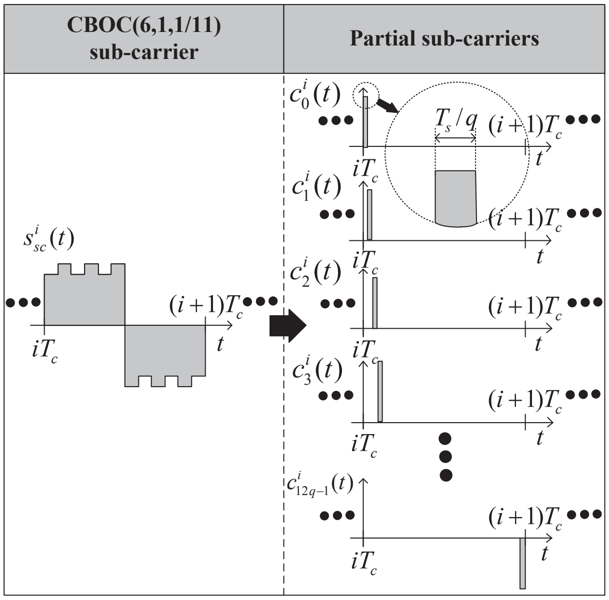

Figure 1. The sub-carrier and partial sub-carriers of the CBOC(6,1,1/11) signal.

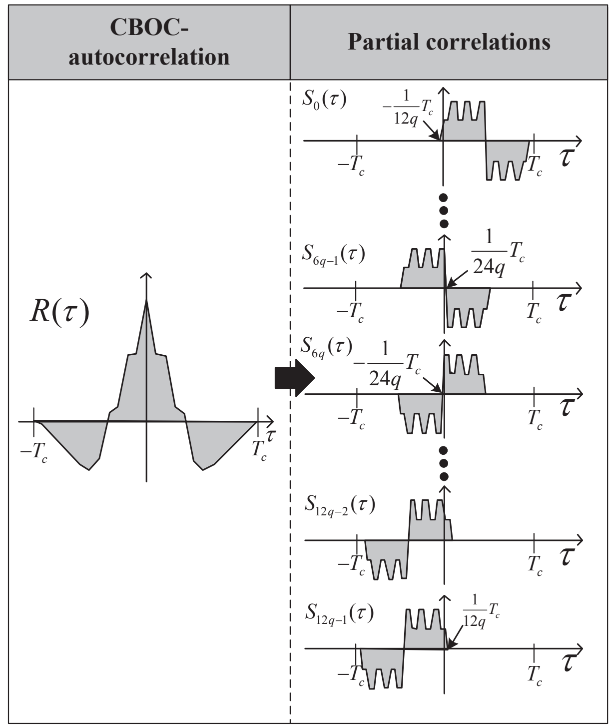

Figure 2. The autocorrelation and partial correlations for the CBOC(6,1,1/11) signal.

pulse over $[0, \alpha)$, T_c is the chip period of the PRN code, $d(t)$ denotes the navigation data, and $s_{sc}^i(t)$ is the CBOC sub-carrier for the ith PRN code chip. In this paper, we assume that every chip of the PRN code is an independent random variable taking on +1 and -1 with equal probability and the code period T is sufficiently large compared with the chip period T_c. It is also assumed that a pilot channel for signal tracking is provided so that no data modulation is present during the tracking process (i.e., $d(t) = 1$). The sub-carrier $s_{sc}^i(t)$ of the CBOC(6,1,1/11) signal can be expressed as a weighted sum of the BOC(1,1) sub-carrier and the BOC(6,1) sub-carrier with a power split ratio of 1/11. Thus, the CBOC(6,1,1/11) sub-carrier can be expressed as

$$s_{sc}^i(t) = \sqrt{\frac{10}{11}} s_{\text{BOC}(1,1)}^i(t) - \sqrt{\frac{1}{11}} s_{\text{BOC}(6,1)}^i(t), \tag{2}$$

where $s_{\text{BOC}(1,1)}^i(t)$ and $s_{\text{BOC}(6,1)}^i(t)$ are the BOC(1,1) and BOC(6,1) sub-carriers for the ith PRN code chip, respectively, and can be expressed as

$$s_{\text{BOC}(1,1)}^i(t) = \sum_{l=0}^{1} (-1)^l r_{6T_s}(t - iT_c - 6lT_s) \tag{3}$$

and

$$s_{\text{BOC}(6,1)}^i(t) = \sum_{l=0}^{11} (-1)^l r_{T_s}(t - iT_c - lT_s), \tag{4}$$

respectively, where $T_s = T_c/12$, i.e., the pulse period of the $s_{\text{BOC}(6,1)}^i(t)$. The CBOC(6,1,1/11) sub-carrier is depicted on the left-hand side of Figure 1.

III. Proposed unambiguous correlation function with an improved main-peak and no side-peaks

To obtain an unambiguous correlation function with an improved main-peak and no side-peaks, (i) we split the CBOC sub-carrier into multiple partial sub-carriers and (ii) combine the partial correlations in a specialized way.

A. Splitting the CBOC sub-carrier

First, we evenly split the CBOC(6,1,1/11) sub-carrier $s_{sc}^i(t)$ into $12q$ partial sub-carriers, where q is a natural number (i.e., $q = 1, 2, 3, \cdots$). Thus, the pulse duration of each partial sub-carrier is given by T_s/q, and the CBOC(6,1,1/11) sub-carrier can be expressed as the sum of the partial sub-carriers:

$$s_{sc}^i(t) = \sum_{m=0}^{12q-1} c_m^i(t), \tag{5}$$

where $c_m^i(t)$ is the mth partial sub-carrier for the ith PRN code chip as depicted on the right-hand side of Figure 1, and $\{c_m^i(t)\}_{m=0}^{12q-1}$ are used as locally-generated signals instead of the CBOC sub-carrier.

The normalized CBOC-autocorrelation function shown on the left-hand side of Figure 2 can be expressed as

$$\begin{aligned} R(\tau) &= \frac{1}{PT} \int_0^T B(t)B(t+\tau)dt \\ &= \frac{1}{\sqrt{P}T} \sum_{m=0}^{12q-1} \sum_{i=-\infty}^{\infty} \int_0^T B(t) c_m^i(t+\tau) p_i r_{T_c}(t+\tau-iT_c)dt \\ &= \sum_{m=0}^{12q-1} S_m(\tau), \end{aligned} \tag{6}$$

where $S_m(\tau)$ is the mth partial correlation shown on the right-hand side of Figure 2.

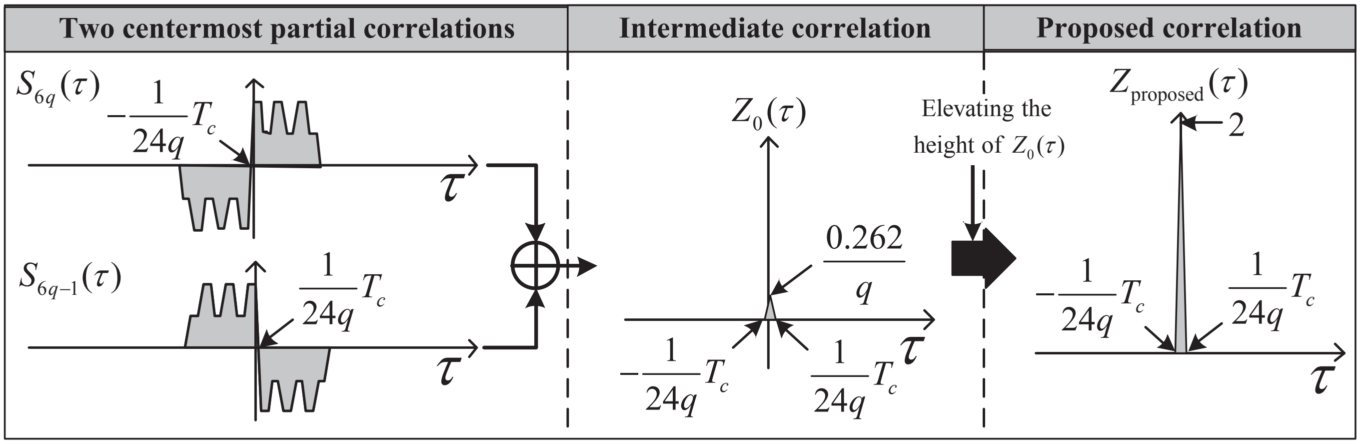

Figure 3. The cancelation of the side-peaks and improvement of the main-peak through combining of the partial correlations.

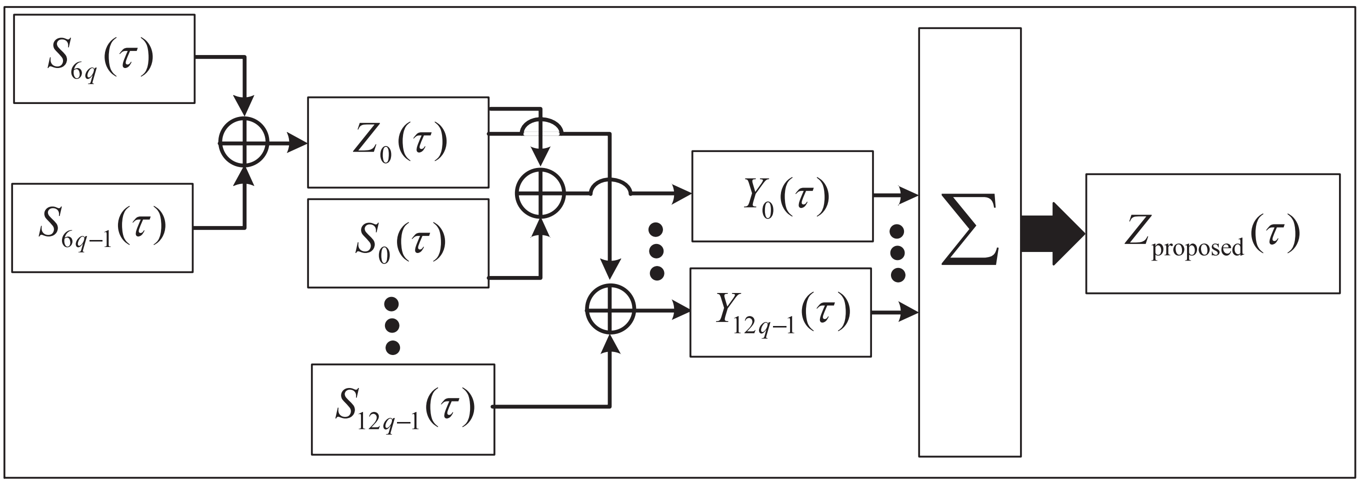

Figure 4. The generating process of the proposed unambiguous correlation function.

B. Combining the partial correlations

Now, we cancel out the side-peaks by combining two centermost partial correlations $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$. Specifically, we use the following arithmetic relation: $|x|+|y|-|x-y|=0$ for $xy \leq 0$ and $|x|+|y|-|x-y|>0$ otherwise. Since the product value of the partial correlations $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$ is positive and negative when $|\tau| < \frac{1}{24q}T_c$ and $|\tau| > \frac{1}{24q}T_c$, respectively, we can eliminate the side-peaks as follows:

$$Z_0(\tau) = S_{6q-1}(\tau) \oplus S_{6q}(\tau), \tag{7}$$

where $A(\tau) \oplus B(\tau) = |A(\tau)| + |B(\tau)| - |A(\tau) - B(\tau)|$, and $Z_0(\tau)$ is an intermediate correlation function obtained right after eliminating the side-peaks and is shown in Figure 3. In fact, we could employ other partial correlations besides $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$ in (7); yet, we found that the combination of $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$ yields the sharpest intermediate correlation function. For example, the half-width of the intermediate correlation function would be $\frac{1}{12q}$ if $S_0(\tau)$ and $S_{12q-1}(\tau)$ are used, which is twice the half-width ($\frac{1}{24q}$) of the intermediate correlation function obtained when $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$ are used, and so, the corresponding intermediate correlation function would be half as sharp as that obtained with $S_{6q-1}(\tau)$ and $S_{6q}(\tau)$.

Next, we increase the height of $Z_0(\tau)$, which is much lower than that of the CBOC-autocorrelation function; moreover, decreases as the value of q increases, and so, is not useful in obtaining a good tracking performance. We observe that similar correlation functions to $Z_0(\tau)$ are obtained by combining each of the partial correlations and $Z_0(\tau)$ as in (7), and propose the following correlation function

$$Z_{\text{proposed}}(\tau) = \sum_{m=0}^{12q-1} S_m(\tau) \oplus Z_0(\tau). \tag{8}$$

Figure 4 shows the whole process for generating $Z_{\text{proposed}}(\tau)$, where $Y_m(\tau) = S_m(\tau) \oplus Z_0(\tau)$. Figure 5 shows the normalized proposed and conventional correlation functions, where we can observe that the proposed correlation function is much sharper than the conventional correlation functions including the CBOC-autocorrelation function, and also, that the difference in sharpness becomes larger as the value of q increases, implying that we can further improve the tracking performance by using a larger value of q. However, it should be noted that the computational complexity is expected to increase as the value of q becomes larger, and thus, an appropriate value of q should be selected according to given system design requirements.

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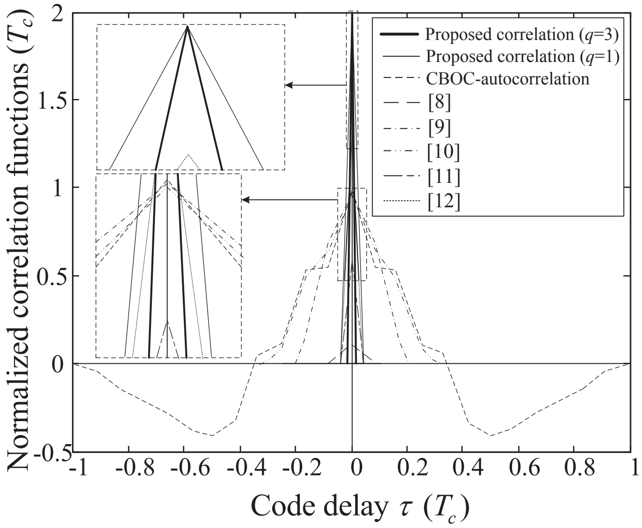

Figure 5. The normalized proposed and conventional correlation functions.

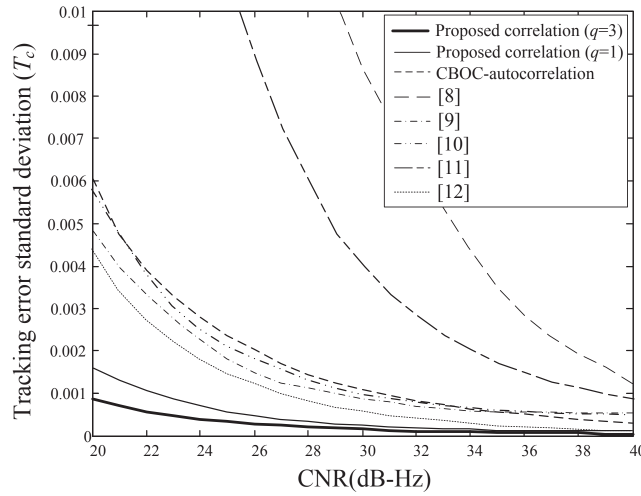

Figure 6. TESD performances of the proposed and conventional correlation functions as a function of the CNR when $\Delta = T_c/96$.

IV. Numerical results

In this section, we compare the tracking performances of the proposed and conventional correlation functions in terms of the TESD defined as

$$\frac{\sigma}{G}\sqrt{2B_L T_I}, \tag{9}$$

where σ is the standard deviation of the discriminator output $D(\tau)$ at $\tau = 0$, G is the discriminator gain at $\tau = 0$, i.e., $G = \frac{dD(\tau)}{d\tau}\big|_{\tau=0}$, B_L is the loop filter bandwidth, and T_I is the integration time [13]. The discriminator output $D(\tau)$ can be expressed as $D(\tau) = Z^2_{\text{proposed}}(\tau+\frac{\Delta}{2})-Z^2_{\text{proposed}}(\tau-\frac{\Delta}{2})$, where Δ is the early-late spacing for a delay lock loop (DLL). For simulations, we consider the following parameters: q=1 and 3, B_L = 1 Hz, $\Delta = \frac{T_c}{96}$, $T = T_I$, and 20,000 Monte Carlo runs are used for each carrier to noise ratio (CNR) defined as P/N_0 dB-Hz, where N_0 is the noise power spectral density. In addition, we consider T_c^{-1} = 1.023 MHz and T = 4 ms, which have been employed in the CBOC signal of Galileo E1 band [3]. For several conventional schemes which have additional system parameters [9][11], the optimized parameters of them are used for simulations, and thus, the best tracking performances of them are compared with those of other correlation functions including the proposed correlation function.

Figure 6 shows the TESD performances of the proposed and conventional correlation functions as a function of the CNR. From the figure, it is clearly confirmed that the proposed correlation function provides a significant improvement in TESD performance over the conventional correlation functions in the CNR range of $20 \sim 40$ dB-Hz of practical interest. Specifically, the proposed correlation function gives a performance improvement of more than 5 dB-Hz and 8 dB-Hz when q = 1 and 3, respectively, over all of the conventional correlation functions in the CNR range of practical interest. This stems from the fact that the proposed correlation function is not only unambiguous (i.e., the proposed correlation function has no side-peaks), but also is the highest and sharpest. In addition, as expected, the tracking performance becomes better, as the value of q increases. Specifically, the tracking performance of the proposed correlation function is improved by more than 3 dB-Hz when the value of q is changed from 1 to 3 as shown in Figure 6.

V. Conclusion

In this paper, we have proposed an unambiguous correlation function with an improved main-peak for tracking of the CBOC signal. Splitting the CBOC(6,1,1/11) sub-carrier into multiple partial sub-carriers and correlating each of the partial sub-carriers and the received CBOC(6,1,1/11) signal, we have obtained the partial correlations, and then, combining the partial correlations through a specialized way based on an arithmetic relation, we have canceled out the side-peaks completely, and also, have obtained a main-peak that is higher and sharper than those of the conventional correlation functions including the CBOC-autocorrelation function. Numerical results have confirmed that the CBOC tracking loop using the proposed correlation function offers a significant improvement over that using the conventional correlation functions in terms of the TESD in the CNR range of practical interest.

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