# On the Capacity of a Cognitive User with Subcarrier Collisions over Rayleigh Fading Channels

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***Abstract*—The paper evaluates the capacity of a cognitive user in an orthogonal frequency-division multiplexing-based spectrum sharing communication system that assumes random subcarrier allocation and absence of spectrum sensing information at the secondary (cognitive) user. In the absence of the primary user's channel occupation information, i.e., no spectrum sensing mechanism is used, the secondary user randomly accesses the subcarriers of the primary network and collides with the primary users' subcarriers. The capacity of a secondary user under such a random access that assumes subcarrier collisions is evaluated herein paper and used as a performance benchmark to investigate the proposed communication scheme over the Rayleigh fading channel.**

***Keywords-cognitive radio; OFDM; spectrum sharing; capacity; random access;***

## I. INTRODUCTION

Recent measurements have highlighted that the radio frequency (RF) spectrum is being under-utilized. Therefore, the cognitive radio (CR) technology has recently received huge interest because it has the potential to yield more efficient RF spectrum utilization [1]. This paper focuses on evaluating the capacity of a cognitive user in an orthogonal frequency-division multiplexing (OFDM)-based underlay spectrum sharing communication system, where primary users (PUs) are allocated a higher priority to access the RF spectrum than secondary users (SUs), and the coexistence of primary and secondary users is enabled under the PU's predefined interference constraint [2], also called interference temperature.

One of the critical issues faced in the adoption of CR networks is to know whether at a certain physical location and moment of time the RF spectrum is occupied by PU(s), i.e., if there is a sensing mechanism in place for the available spectrum [3], [4]. The challenge in deploying spectrum sensing mechanisms is due to the uncertainties ranging from channel randomness at device and network-level, hidden primary users, and issues pertaining to sensing duration and security [5].

Because of the difficulties faced in the acquisition of the spectrum sensing information, this paper focuses on assessing the capacity of a cognitive user in an OFDM-based CR spectrum sharing communication system that assumes random allocation and absence of the PU's channel occupation information, i.e., lack of spectrum sensing information. In other words, the SU's subcarriers are allowed to collide with PUs' subcarriers. As a major benefit of the proposed random access scheme, random subcarrier utilization helps to uniformly distribute the SU's interference among the PUs' subcarriers, and hence, to equalize uniformly the performance of all cognitive users across the whole network. So far, no studies have been reported to assess the effects of subcarrier collisions in OFDM-based cognitive spectrum sharing systems. Therefore, there is a critical need for a more comprehensive system analysis including the development of a stochastic model to capture the subcarrier collisions and protection of the operation of PUs in OFDM-based cognitive systems.

The outcomes of the analysis conducted herein paper will help in understanding the performance limits of random access OFDM-based cognitive communication systems and could be also utilized as performance benchmarks to assess the performance of other cognitive spectrum sharing systems that assume the availability of spectrum sensing information. The rest of this paper is organized as follows. Section II introduces the notations and assumptions that will be used throughout this paper, and describes the basic features of the adopted OFDM-based cognitive radio system. Section III is dedicated to the evaluation of the capacity of a secondary cognitive user in the presence of subcarrier collisions and Rayleigh flat fading channels. Computer simulations corroborating the proposed capacity evaluation study are presented in Section IV. Finally, Section V concludes the paper with some future possible extensions of the current results.

## II. SYSTEM MODEL

In the considered system model, the primary and cognitive (secondary) networks consist of $N$ PUs with a primary base station (PBS), and $M$ SUs with a secondary base station (SBS), respectively. To preserve the quality of service (QoS) requirements of PUs in the proposed random access OFDM spectrum sharing communication system, the interference

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power levels caused by the SU-transmitters at the primary receiver (PBS) are enforced to be smaller than a predefined interference temperature (IT) ($\Psi_i$) at the $i$-th subcarrier.

The channel power gains from the $m$th SU to SBS and PBS are represented in terms of variables $h_m$ and $h_{mp}$, respectively. Also, $g_n$ and $g_{ns}$ denote the channel power gains from the $n$th PU to PBS and SBS, respectively. All the channel gains are assumed to have unit mean and be independent and identically distributed (i.i.d.) flat Rayleigh fading random variables. It is also supposed that perfect information about the interference channel power gains, $h_{mp}$, is available at SUs. The SU can access the channel side information (CSI), through various means such as the channel reciprocity condition [6], [7], mediate band mechanism or cognitive radio network manager that coordinates the operation of PBS and SU [2]. The thermal additive white Gaussian noise (AWGN) at PUs and SUs is assumed circularly symmetric complex Gaussian stochastic process with zero mean and variance $\eta$, i.e., $\mathcal{CN}(0,\eta)$. The parameters $h_{m,i}$, $h_{mp,i}$, $g_{n,i}$ and $g_{ns,i}$ will represent the channel power gains associated with the $i$th subcarrier.

The total number of available subcarriers in the primary network is denoted by $F$. The subcarrier set of each PU is assigned by ensuring the orthogonality among the sets of subcarriers for all PUs, $F_n^P$ for $n = 1, \ldots, N$. SU randomly accesses the subcarriers from the available subcarriers set $F$ without knowing which channels (subcarriers) are occupied by PUs. Therefore, SU will collide with the subcarriers of the PUs with a certain probability. The probabilistic model adopted for the number of subcarrier collisions is very general and follows a multivariate hypergeometric distribution. During the evaluation study of the SU capacity (to be described in Section III), it will be assumed that there is only a single SU (which can be any SU in the system) in the cognitive network. The reason for focusing on such a simplified set-up is because such a framework enables to conduct a more simplified analysis without introducing a very cumbersome and hard-to-follow analysis, and to draw very general conclusions. In the same time, the proposed framework can be easily extended to multiple SUs with the assumption of no mutual interference among SUs.

## III. Cognitive User Capacity Analysis

### A. Secondary User Capacity with Subcarrier Collisions

The Rayleigh fading channel model is adopted to investigate the impact on performance of the system parameters and to evaluate the expressions for the probability density function (PDF) and cumulative distribution function (CDF) of SU capacity. We will focus first on the SU capacity expression with subcarrier collisions.

*1) Probability Mass Function of the Number of Subcarrier Collisions:* If any arbitrary (consider the $m$th) SU randomly accesses $F_m^S$ subcarriers from a set of $F$ available subcarriers without replacement while $\sum_{n=1}^N F_n^P$ subcarriers are being used by the $N$ PUs, then the joint probability mass function (PMF) of the number of subcarrier collisions, $\mathbf{k}_m$, follows the modified multivariate hypergeometric distribution:

$$p(\mathbf{k}_m) = \binom{F_f}{k_{fm}} \binom{F}{F_m^S}^{-1} \prod_{n=1}^{N} \binom{F_n^P}{k_{nm}}, \qquad (1)$$

where the notation $\binom{\cdot}{\cdot}$ stands for the binomial coefficient, and $\mathbf{k}_m = [k_{1m}, k_{2m}, \ldots, k_{Nm}, k_{fm}]^T \in \mathbb{Z}_{0+}^{N+1}$ represents the number of collisions of the $m$th SU with $N$ PUs and with the collision-free subcarriers, $k_{fm}$. Parameter $F_f = F - \sum_{n=1}^N F_n^P$ stands for the number of free subcarriers.

*2) Capacity with Collisions:* Let $k_{nm}$ denote the number of ($m$th) SU's subcarriers that collide with the $n$th PU's subcarriers, then the capacity of SU with subcarrier collisions can be expressed as:

$$\begin{aligned} C_m &= \sum_{i=1}^{k_{1m}} \log\left(1 + S_{m,i}^{I,1}\right) + \cdots + \sum_{i=1}^{k_{Nm}} \log\left(1 + S_{m,i}^{I,N}\right) \\ &+ \sum_{i=1}^{k_{fm}} \log\left(1 + S_{m,i}^{NI}\right) = \underbrace{\sum_{n=1}^{N} \overbrace{\sum_{i=1}^{k_{nm}} C_{m,i}^{I,n}}^{C_m^{I,n}}}_{C_m^I} + \underbrace{\sum_{i=1}^{k_{fm}} C_{m,i}^{NI}}_{C_m^{NI}} \end{aligned} \qquad (2)$$

where $S_{m,i}^{I,n}$ and $S_{m,i}^{NI}$ represent the signal-to-interference plus noise ratio (SINR) for the $i$th subcarrier of the $m$th SU with "interference" and "no-interference" from the $n$th PU, respectively. We make the remark that $S_{m,i}^{NI}$ is indeed the signal-to-noise ratio (SNR) for the $i$th subcarrier. However, throughout this paper to emphasize the subcarrier collision and collision-free cases, it will be referred to as the SINR with "no-interference" from PU. All logarithms herein paper are with respect to Euler's constant $e$.

### B. Cognitive Capacity over Rayleigh Fading Channels

The peak power interference constraint is considered herein paper, and an adaptive scheme is used to adjust the transmit power of SU to preserve the QoS of PUs. Hence, the transmit power of the $m$th SU corresponding to the $i$th subcarrier is given by $P_{m,i}^T = \min\{P_{m,i}, \Psi_i/h_{mp,i}\}$, for $i = 1, \ldots, F$.

Define the variable $\lambda_{m,i} := h_{m,i} P_{m,i}^T$, then the received SINR of the $m$th SU's $i$th subcarrier takes the form:

$$S_{m,i}^{I,n} = \frac{\lambda_{m,i}}{I_{n,i}^P + \eta}, \quad \text{for } n = 1, \ldots, N, \qquad (3)$$

where $I_{n,i}^P = P_{n,i} g_{ns,i}$ denotes the mutual interference caused by $n$th PU on the $i$th subcarrier. In (3), $S_{m,i}^{I,n}$ represents the SINR when subcarrier collision happens. Therefore, when there is no collision, i.e., the subcarrier is


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not being used by two users, there is no interference caused by PUs. Thus, $S_{m,i}^{NI} = \lambda_{m,i}/\eta$.

The PDF and CDF of $S_{m,i}^{NI}$ are given, respectively, by

$$f_{S_{m,i}^{NI}}(x) = \frac{\eta e^{-\frac{\eta x}{P_{m,i}}}}{P_{m,i}} \left[1 - e^{-\frac{\Psi_i}{P_{m,i}}} \left(\frac{(\eta x)^2 + \Psi_i \eta x - \Psi_i P_{m,i}}{(\Psi_i + \eta x)^2}\right)\right],$$

$$F_{S_{m,i}^{NI}}(x) = 1 - e^{-\frac{\eta x}{P_{m,i}}} + \frac{\eta x}{\Psi_i + \eta x} e^{-\frac{\eta x + \Psi_i}{P_{m,i}}}.$$

The derivations of the expressions for the PDF and CDF are omitted due to lack of space and are delegated to [8].

Similarly, in the presence of primary interference, the PDF and CDF of $S_{m,i}^{I,n}$ can be expressed as

$$\begin{aligned} f_{S_{m,i}^{I,n}}(x) &= \frac{x\eta P_{n,i} + P_{m,i}(\eta + P_{n,i})}{(xP_{n,i} + P_{m,i})^2} \left(e^{\frac{\Psi_i}{P_{m,i}}} - 1\right) e^{-\frac{x\eta + \Psi_i}{P_{m,i}}} + \frac{\Psi_i}{x^3 P_{n,i}^2} \\ &\times e^{\frac{x\eta + \Psi_i}{xP_{n,i}}} \left[(\Psi_i + xP_{n,i})\Gamma\left(0, \left(\eta + \frac{\Psi_i}{x}\right)\left(\frac{1}{P_{n,i}} + \frac{x}{P_{m,i}}\right)\right)\right. \\ &\left. + \frac{xP_{n,i}(x^2 \eta P_{n,i} - \Psi_i P_{m,i})}{(x\eta + \Psi_i)(xP_{n,i} + P_{m,i})} e^{-\left(\eta + \frac{\Psi_i}{x}\right)\left(\frac{1}{P_{n,i}} + \frac{x}{P_{m,i}}\right)}\right]. \end{aligned}$$

$$\begin{aligned} F_{S_{m,i}^{I,n}}(x) &= 1 - \frac{\left(1 - e^{-\frac{\Psi_i}{P_{m,i}}}\right) e^{-\frac{x\eta}{P_{m,i}}}}{1 + \frac{xP_{n,i}}{P_{m,i}}} - \frac{\Psi_i}{xP_{n,i}} e^{\frac{\Psi_i}{xP_{n,i}} + \frac{\eta}{P_{n,i}}} \\ &\times \Gamma\left(0, \left(\eta + \frac{\Psi_i}{x}\right)\left(\frac{1}{P_{n,i}} + \frac{x}{P_{m,i}}\right)\right), \end{aligned}$$

Finally, the desired expressions for the PDFs of $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ can be obtained through the transformation of appropriately defined RVs as follows:

$$f_{C_{m,i}^{I,n}}(x) = \left|\frac{dy}{dx}\right| f_{S_{m,i}^{I,n}}(y)\Big|_{y=e^x-1} = e^x f_{S_{m,i}^{I,n}}(e^x - 1),$$
$$f_{C_{m,i}^{NI}}(x) = e^x f_{S_{m,i}^{NI}}(e^x - 1).$$

Recalling the SU capacity expression in (2), in the presence of $N$ interfering PUs, there are two types of well known methods available to evaluate the distribution for sum of variates, namely, the characteristic function (CF) and the moment generating function (MGF) method. Unfortunately, employing these methods leads to intractable results and no explicit closed form expressions for the PDF and CDF of SU capacity in (2) can be achieved.

We will resort in this regard to an alternative approach. To sum up the rates for the cases of interference and no-interference, we will approximate the PDFs of $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ using the Gamma distribution. There are several desirable properties of the Gamma distribution that are fit for approximating the PDFs of the variables $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$. First, the sum of Gamma distributed RVs with the same scale parameters is another Gamma distributed RVs. Second, the skewness and tail of distribution are similar for the whole range of interest and are determined by mean and variance [9]. Last but not least, Gamma distribution is a Type-III Pearson distribution, which is widely used in fitting *positive* RVs [9], [10]. In addition, since Gamma distribution is uniquely determined by its mean and variance, we will make use of the moment matching method to match the first two moments of the RV, namely its mean and variance.

*Definition 1:* Random variable $X$ follows a Gamma distribution, $X \sim \mathcal{G}(\alpha, \beta)$ with scale and shape parameters $\beta > 0$ and $\alpha > 0$, respectively, if: $f_X(x) = \frac{x^{\alpha-1}\exp\left(-\frac{x}{\beta}\right)}{\beta^\alpha \Gamma(\alpha)} U(x)$, where $U(\cdot)$ represents the unit step function, and the Gamma function is defined as $\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} \mathrm{d}t$.

Since the mean and variance of Gamma distribution are $\alpha\beta$ and $\alpha\beta^2$, respectively, matching the first two moments with the PDFs of $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ leads to

$$\alpha_n^I = \frac{\left(\mathbb{E}\left[C_{m,i}^{I,n}\right]\right)^2}{\mathbf{var}\left[C_{m,i}^{I,n}\right]}, \quad \beta_n^I = \frac{\mathbf{var}\left[C_{m,i}^{I,n}\right]}{\mathbb{E}\left[C_{m,i}^{I,n}\right]},$$
$$\alpha^{NI} = \frac{\left(\mathbb{E}\left[C_{m,i}^{NI}\right]\right)^2}{\mathbf{var}\left[C_{m,i}^{NI}\right]}, \quad \beta^{NI} = \frac{\mathbf{var}\left[C_{m,i}^{NI}\right]}{\mathbb{E}\left[C_{m,i}^{NI}\right]},$$

for $n = 1, 2, \ldots, N$, and $\mathbf{var}(x)$ denotes the variance of $x$.

From [11], by employing the derived PDFs and CDFs of $S_{m,i}^{I,n}$ and $S_{m,i}^{NI}$, the average capacity of $C_{m,i}^{NI}$ and $C_{m,i}^{I,n}$ are expressed, respectively, as

$$\begin{aligned} \mathbb{E}\left[C_{m,i}^{NI}\right] &= \int_0^\infty x f_{C_{m,i}^{NI}}(x)\mathrm{d}x = \int_0^\infty \log(1+x) f_{S_{m,i}^{NI}}(x)\mathrm{d}x \\ &= \Gamma\left(0, \frac{\eta}{P_{m,i}}\right) e^{\frac{\eta}{P_{m,i}}} \left(1 + \frac{e^{-\frac{\Psi_i}{P_{m,i}}}\eta}{\Psi_i - \eta}\right) \\ &+ \frac{\Psi_i}{\eta - \Psi_i} \Gamma\left(0, \frac{\Psi_i}{P_{m,i}}\right). \end{aligned}$$

and

$$\begin{aligned} \mathbb{E}\left[C_{m,i}^{I,n}\right] &= \int_0^\infty \log(1+x) f_{S_{m,i}^{I,n}}(x)\mathrm{d}x = \frac{1 - e^{-\frac{\Psi_i}{P_{m,i}}}}{1 - \frac{P_{n,i}}{P_{m,i}}} \\ &\times \left(\Gamma\left(0, \frac{\eta}{P_{m,i}}\right) e^{\frac{\eta}{P_{m,i}}} - \Gamma\left(0, \frac{\eta}{P_{n,i}}\right) e^{\frac{\eta}{P_{n,i}}}\right) \\ &+ \frac{\Psi_i}{P_{n,i}} e^{\frac{\eta}{P_{n,i}}} \int_0^\infty \Gamma\left(0, \left(\eta + \frac{\Psi_i}{x}\right)\right. \\ &\times \left.\left(\frac{1}{P_{n,i}} + \frac{x}{P_{m,i}}\right)\right) \frac{e^{\frac{\Psi_i}{xP_{n,i}}}}{x(1+x)} \mathrm{d}x, \end{aligned}$$

The variance of $C_{m,i}^{I,n}$ is given by

$$\mathbf{var}\left[C_{m,i}^{I,n}\right] = \mathbb{E}\left[\left(C_{m,i}^{I,n}\right)^2\right] - \left(\mathbb{E}\left[C_{m,i}^{I,n}\right]\right)^2,$$

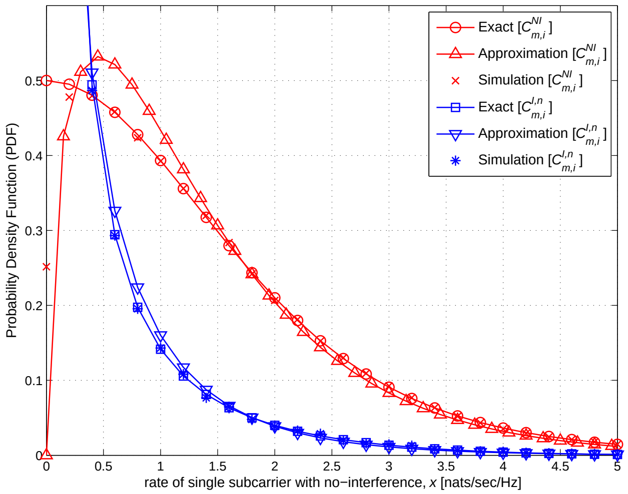


Figure 1. Comparison between the exact, approximation and simulation of $f_{C_{m,i}^{I,n}}(x)$ and $f_{C_{m,i}^{NI}}(x)$ using the PDF of Gamma distribution for $P_{m,i} = 20$ dB, $P_{n,i} = 10$ dB, $\Psi_i = 0$ dB and $\eta = 0.5$.

where the second moment of $C_{m,i}^{I,n}$ is expressed as

$$\begin{aligned}\mathbb{E}\left[\left(C_{m,i}^{I,n}\right)^2\right] &= \int_0^\infty [\log(1+x)]^2 f_{S_{m,i}^{I,n}}(x)\mathrm{d}x\\ &= \int_0^\infty \frac{2\log(1+x)}{1+x}\left[1-F_{S_{m,i}^{I,n}}(x)\right]\mathrm{d}x\\ &\simeq \sum_{j=1}^{N_p} w_j \frac{2\log(1+s_j)}{1+s_j}\left[1-F_{S_{m,i}^{I,n}}(s_j)\right],\end{aligned}$$

where the second equality is obtained by using integration by parts [11]. The resulting integral is estimated via Gauss-Chebyshev quadrature (GCQ), where the weights ($w_j$) and abscissas ($s_j$) are given by [12, Eqs. (22) and (23)], respectively. Similarly, the variance of $C_{m,i}^{NI}$ is expressed by adopting the same approach.

Based on the adopted Gamma approximation, the capacities are approximated as $C_{m,i}^{I,n} \sim \mathcal{G}\left(\alpha_n^I, \beta_n^I\right)$ and $C_{m,i}^{NI} \sim \mathcal{G}\left(\alpha^{NI}, \beta^{NI}\right)$.

In Figure 1, the exact and approximative expressions of $f_{C_{m,i}^{I,n}}(x)$ and $f_{C_{m,i}^{NI}}(x)$, including the simulations results, for different system parameters are depicted. It can be observed that the proposed approximation is very close to the exact results. Since both $C_{m,i}^{I,n}$ and $C_{m,i}^{NI}$ are i.i.d. for given $k_{nm}$, the conditional characteristic functions for the rate sums $\sum_{i=1}^{k_{nm}} C_{m,i}^I$ and $\sum_{i=1}^{k_{fm}} C_{m,i}^{NI}$ can be expressed as follows

$$\begin{aligned}\Phi_{C_m^{I,n}}(\omega|k_{nm}) &= \left(\Phi_{C_{m,i}^{I,n}}(\omega)\right)^{k_{nm}} = \left(1-j\omega\beta_n^I\right)^{-\alpha_n^I k_{nm}},\\ \Phi_{C_m^{NI}}(\omega|k_{nm}) &= \left(\Phi_{C_{m,i}^{NI}}(\omega)\right)^{k_{fm}} = \left(1-j\omega\beta^{NI}\right)^{-\alpha^{NI} k_{fm}},\end{aligned}$$

where $\Phi_{C_{m,i}^{I,n}}(\omega|k_{nm})$ and $\Phi_{C_{m,i}^{NI}}(\omega|k_{nm})$ are the characteristic functions of $f_{C_{m,i}^{I,n}}(x|k_{nm})$ and $f_{C_{m,i}^{NI}}(x|k_{nm})$, respectively. Using the property of the Gamma distribution that the sum of i.i.d. Gamma distributed RVs, with the same scale parameters ($\beta$) is another Gamma distributed RV, the conditional PDFs take the form:

$$\begin{aligned}f_{C_m^{I,n}|k_{nm}}(x|k_{nm}) &= \mathcal{G}\left(\alpha_n^I k_{nm}, \beta_n^I\right),\\ f_{C_m^{NI}|k_{nm}}(x|k_{nm}) &= \mathcal{G}\left(\alpha^{NI} k_{fm}, \beta^{NI}\right).\end{aligned} \tag{4}$$

In (2), even though the conditional PDFs of $C_m^{I,n}$ and $C_m^{NI}$ are calculated, to find the PDF expression for $C_m$, one first needs to evaluate the PDF of $C_m^I$, and then the PDF of its sum with $C_m^{NI}$. Notice that there are $N+1$ terms in (2), and each follows a Gamma distribution where the shape ($\alpha$) and scale ($\beta$) parameters can be arbitrary. Therefore, the aforementioned property of Gamma distribution for a sum of Gamma variates cannot be employed anymore.

The expression for the PDF of a sum of Gamma RVs was obtained by Moschopoulos in [13], where a mathematically tractable solution that does not restrict the scale and shape parameters to be integer-valued or all distinct is presented. Therefore, the following theorem will be used next.

*Theorem 1 (Moschopoulos, 1985):* Let $\{X_s\}_{s=1}^{\mathcal{S}}$ be independent but not necessarily identically distributed Gamma variates with parameters $\alpha_s$ and $\beta_s$, respectively, then the PDF of $Y = \sum_{s=1}^{\mathcal{S}} X_s$ can be expressed as

$$f_Y(y) = \prod_{s=1}^{\mathcal{S}}\left(\frac{\beta_1}{\beta_s}\right)^{\alpha_s}\sum_{k=0}^{\infty}\frac{\delta_k y^{\sum_{s=1}^{\mathcal{S}}\alpha_s+k-1}\exp\left(-\frac{y}{\beta_1}\right)}{\beta_1^{\sum_{s=1}^{\mathcal{S}}\alpha_s+k}\Gamma\left(\sum_{s=1}^{\mathcal{S}}\alpha_s+k\right)}U(y), \tag{5}$$

where $\beta_1 = \min_s\{\beta_s\}$, and the coefficients $\delta_k$ can be obtained recursively by the formula

$$\delta_k = \frac{1}{k+1}\sum_{i=1}^{k+1}\left[\sum_{j=1}^{\mathcal{S}}\alpha_j\left(1-\frac{\beta_1}{\beta_j}\right)^i\right]\delta_{k+1-i}$$

where $\delta_0 = 1$, and for $k = 0, 1, 2, \ldots$.

*Proof:* See [13]. ■

The Moschopoulos PDF provides a tractable representation for the sum of Gamma variates in terms of a single Gamma series via a recursive formula to evaluate iteratively the representation coefficients. This approach is applicable for any arbitrary shape parameters $\{\alpha_s\}_{s=1}^{\mathcal{S}}$ and scale parameters $\{\beta_s\}_{s=1}^{\mathcal{S}}$ including the possibility of having some of the parameters identical. Notice that in the considered communication system, with some probability the transmit power of PUs $P_{n,i}$ for $n = 1, \ldots, N$, can be the same, which means that the corresponding $\alpha_n^I$ and $\beta_n^I$ are the same. Such a set-up might arise when the PUs are at the same distance from their corresponding common PBS.

 

The CDF of $Y$ can be obtained from the PDF as $F_Y(y) = \int_{-\infty}^{y} f_Y(x)\mathrm{d}x$. Therefore,

$$F_Y(y) = \prod_{s=1}^{\mathcal{S}} \left(\frac{\beta_1}{\beta_s}\right)^{\alpha_s} \sum_{k=0}^{\infty} \frac{\delta_k}{\beta_1^{\sum_{s=1}^{\mathcal{S}} \alpha_s + k} \Gamma\left(\sum_{s=1}^{\mathcal{S}} \alpha_s + k\right)} \times \int_0^y x^{\sum_{s=1}^{\mathcal{S}} \alpha_s + k - 1} \exp\left(-\frac{x}{\beta_1}\right) \mathrm{d}x. \tag{6}$$

Notice that the interchange of summation and integration operators is justified due to the uniform convergence of (5). From [14], we can simplify (6) by using $\int_0^u x^{\nu-1}e^{-\mu x}\mathrm{d}x = \mu^{-\nu}\gamma(\nu, \mu u)$ for $\Re[\nu > 0]$ [15, pg. 346, Sec. 3.381, Eq. 1], where $\gamma(\cdot,\cdot)$ is the lower incomplete Gamma function and is defined as $\gamma(x, y) = \int_0^y t^{x-1}e^{-t}dt$. Hence,

$$\begin{aligned} F_Y(y) &= \prod_{s=1}^{\mathcal{S}} \left(\frac{\beta_1}{\beta_s}\right)^{\alpha_s} \sum_{k=0}^{\infty} \delta_k \frac{\gamma\left(\sum_{s=1}^{\mathcal{S}} \alpha_s + k, \frac{y}{\beta_1}\right)}{\Gamma\left(\sum_{s=1}^{\mathcal{S}} \alpha_s + k\right)} \\ &= \prod_{s=1}^{\mathcal{S}} \left(\frac{\beta_1}{\beta_s}\right)^{\alpha_s} \sum_{k=0}^{\infty} \delta_k \mathcal{P}\left(\sum_{s=1}^{\mathcal{S}} \alpha_s + k, \frac{y}{\beta_1}\right), \end{aligned} \tag{7}$$

where $\mathcal{P}(\cdot,\cdot)$ is referred to as the regularized (also termed normalized) incomplete Gamma function: $\mathcal{P}(a, z) = \frac{\gamma(a,z)}{\Gamma(a)} = 1 - \frac{\Gamma(a,z)}{\Gamma(a)}$. Based on the required accuracy, one may consider the first $h$, i.e., $k = h - 1$, terms in the sum series (5). The expression for truncation error is given in [13].

Recall that from (2) and (4), we have to determine the PDF of the sum $C_m^{I,1} + C_m^{I,2} + \cdots + C_m^{I,N} + C_m^{NI}$, for a given number of subcarrier collisions $\mathbf{k}_m = [k_{1m}, k_{2m}, \ldots, k_{Nm}, k_{fm}]$. Recall also that $C_m^I$ and $C_m^{NI}$ are Gamma distributed and independent but not necessarily identical. Therefore, the conditional PDF of their sum can be obtained by means of *Theorem* 1 as given in (8). Equation (8) displayed at the top of the next page describes the sought result. where $\beta_{\min} = \min\{\beta_1^I, \beta_2^I, \ldots, \beta_N^I, \beta^{NI}\}$, and the coefficients $\delta_k$ are obtained recursively:

$$\delta_k = \frac{1}{k+1} \sum_{i=1}^{k+1} \left[\sum_{j=1}^{N} \alpha_i^I k_{jm} \left(1 - \frac{\beta_{\min}}{\beta_j^I}\right)^i + \alpha^{NI} k_{fm} \left(1 - \frac{\beta_{\min}}{\beta^{NI}}\right)^i\right] \delta_{k+1-i} \quad \text{for } k = 0, 1, 2, \ldots$$

where $\delta_0 = 1$.

Now, the PDF of $C_m$ can be determined by averaging over the PMF of subcarrier collisions:

$$\begin{aligned} f_{C_m}(x) &= \sum_{\mathbf{k}_m} f_{C_m,\mathbf{K}_m}(x, \mathbf{k}_m) \\ &= \sum_{\mathbf{k}_m} f_{C_m|\mathbf{K}_m}(x|\mathbf{k}_m) p(\mathbf{k}_m). \end{aligned} \tag{10}$$

Plugging (1) and (8) into (10) yields the sought PDF in (9).

The outage probability is a often used performance metric in channels subject to fading conditions. Hence, herein paper we will determine the outage probability of SU capacity in terms of $P_{C_m}^{\mathrm{out}}(\varphi_{\mathrm{th}}) = Pr\left(C_m < \varphi_{\mathrm{th}}\right) = \int_0^{\varphi_{\mathrm{th}}} f_{C_m}(x)\mathrm{d}x$, which represents the CDF of the SU capacity over the outage threshold $\varphi_{\mathrm{th}}$ [dB].

Using (7) and (9), the CDF of $C_m$ takes the form:

$$F_{C_m}(x) = \sum_{k_{1m}} \sum_{k_{2m}} \cdots \sum_{k_{Nm}} \sum_{k_{fm}} \left\{ \binom{F_f}{k_{fm}} \binom{F}{F_m^S}^{-1} \times \prod_{n=1}^{N} \binom{F_n^P}{k_{nm}} \left(\frac{\beta_{\min}}{\beta^{NI}}\right)^{\alpha^{NI} k_{fm}} \prod_{n=1}^{N} \left(\frac{\beta_{\min}}{\beta_n^I}\right)^{\alpha_n^I k_{nm}} \times \sum_{k=0}^{\infty} \delta_k \mathcal{P}\left(\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI} k_{fm} + k, \frac{x}{\beta_{\min}}\right) \right\}.$$

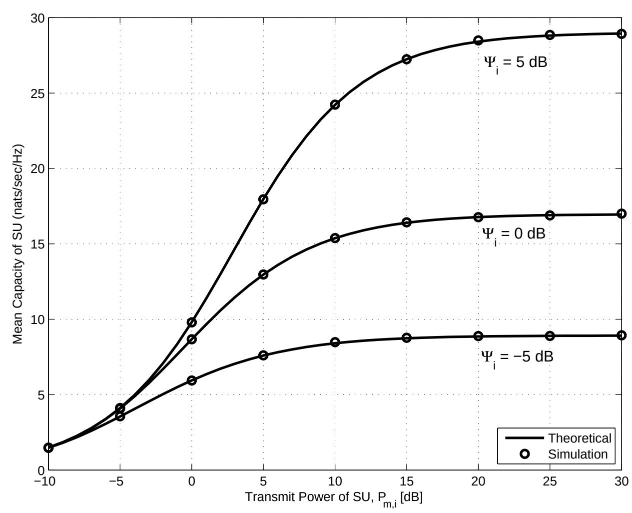


Figure 2. SU mean capacity versus the transmit power $P_{m,i}$ with different IT $\Psi_i$ values for $F_m^S = 20$, $F_n^P = 30$, $F = 128$ and $P_{n,i} = 10$ dB.

## IV. Computer Simulations

The influence of the secondary user peak transmit power $P_{m,i}$ (in dB) on its average capacity (in nats per second per hertz) is illustrated for different values of IT $\Psi_i$ in Fig. 2. It turns out that the cognitive user's average capacity gets saturated after a certain value of peak SU transmit power because of the IT constraint. Fig. 2 corroborates the fact that the analytical results agree well with the simulation results.

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$$f_{C_m|\mathbf{K}_m}(x|\mathbf{k}_m) = \left(\frac{\beta_{\min}}{\beta^{NI}}\right)^{\alpha^{NI}k_{fm}} \prod_{n=1}^{N} \left(\frac{\beta_{\min}}{\beta_n^I}\right)^{\alpha_n^I k_{nm}} \sum_{k=0}^{\infty} \frac{\delta_k x^{\sum_{n=1}^N \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k - 1} \exp\left(-\frac{x}{\beta_{\min}}\right) U(x)}{\beta_{\min}^{\sum_{n=1}^N \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k}\ \Gamma\left(\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k\right)}, \tag{8}$$

$$\begin{aligned} f_{C_m}(x) = &\sum_{k_{1m}}\sum_{k_{2m}}\cdots\sum_{k_{Nm}}\sum_{k_{fm}} \left\{ \binom{F_f}{k_{fm}} \binom{F}{F_m^S}^{-1} \prod_{n=1}^{N} \binom{F_n^P}{k_{nm}} \left(\frac{\beta_{\min}}{\beta^{NI}}\right)^{\alpha^{NI}k_{fm}} \prod_{n=1}^{N} \left(\frac{\beta_{\min}}{\beta_n^I}\right)^{\alpha_n^I k_{nm}} \right. \\ &\left. \times \sum_{k=0}^{\infty} \frac{\delta_k x^{\sum_{n=1}^N \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k - 1} \exp\left(-\frac{x}{\beta_{\min}}\right)}{\beta_{\min}^{\sum_{n=1}^N \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k}\ \Gamma\left(\sum_{n=1}^{N} \alpha_n^I k_{nm} + \alpha^{NI}k_{fm} + k\right)} U(x) \right\}. \end{aligned} \tag{9}$$

The plots in Figure 2 are in the presence of a single PU, i.e., $n \in [1, N]$, and unit variance AWGN ($\eta = 1$). The number of subcarriers in sets $F$, $F_m^S$ and $F_n^S$ is selected arbitrarily. Fig. 2 highlights also the fact that the saturation level of capacity increases as the IT constraint relaxes, and the capacity keeps growing until a saturation level as the transmit power of SU increases. However, the capacity gains due to the relaxation in the IT constraint disappears in the low SU transmit power regime.

## V. Conclusions

This paper assessed the capacity of a secondary (cognitive) user in a random access OFDM-based cognitive radio system with spectrum sharing features such as random subcarrier allocation and absence of spectrum sensing information. The adopted model for the number of subcarrier collisions in the presence of multiple interfering primary users is the general multivariate hypergeometric distribution. The PDF and CDF expressions of the secondary user capacity over a Rayleigh fading channel are derived. It turns out that the closed-form expression for the instantaneous secondary user capacity over Rayleigh channel fading is intractable. Therefore, a Gamma approximation of the secondary user capacity is obtained by employing the moment matching method and the concept of Moschopoulos PDF representation. The work conducted in this paper subscribes along the lines of our preliminary results [8], and we are hoping to extend these results to a more general OFDM-based cognitive radio network that assumes an arbitrary number of primary and secondary users, and general channel fading conditions.

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