Blocking Probabilities in Multicast WDM Optical Networks With First-Fit Wavelength Assignment

Anwar Alyatama
Computer Engineering Department
Kuwait University
P. O. Box 5969, Safat 13060, Kuwait
Email: a.yatama@ku.edu.kw

***Abstract*—We present an approximate analytical method to evaluate the blocking probabilities in multicast Wavelength Division Multiplexing (WDM) networks without wavelength converters. Our approach is based on iteratively solving the multicast call blocking probabilities for fixed routing with First-Fit wavelength assignment algorithm. We divide the WDM network into layers (colors) and we use the moment matching method to characterize the overflow traffic from one layer to another. Analyzing blocking probabilities in each layer of the network is derived from an exact approach. Results are presented which indicate the accuracy of our method.**

***Index Terms*—Blocking probability, Mutlicast Routing, WDM.**

I. Introduction

Wavelength-division multiplexing (WDM) has the potential of delivering huge bandwidth by providing many lightpaths simultaneously on one fiber. Each lightpath is independent and located at a different wavelength. A lightpath may span multiple fiber links to provide a circuit-switched interconnection between two nodes. When the network does not have conversion capabilities, the same wavelength must be available on all links. Many applications such as distribution of video require a multicast connection. A multicast connection contains a source node and a group of destination nodes. The subnetwork spanning the source node and the group of destination nodes is called a multicast tree. Using such trees, signals are transmitted to the leaf (destination) nodes in the multicast tree. Signals pass through a non-leaf destination node are dropped locally, but a copy of it is also transmitted downstream to the next node. However, finding an optimal multicast tree is not easy, and many algorithms are introduced to solve the multicast tree problem.

A challenging issue is the multicast call blocking probability in which, given a multicast call (request) traffic rate that need to be established on the network, and given a constraint on the number of wavelengths, calculate the probability that no common wavelength is available on the predetermined multicast tree. The problem of evaluating the multicast call blocking probabilities has been studied in several studies [1] [2]. They differ in their underlying assumptions and have varying computational complexities and level of accuracy. The purpose of this paper is to derive an iterative model to calculate the call blocking probabilities for fixed routing in Multicast WDM networks. Our approach uses the wavelength independence assumption. We analyze a given wavelength-routing network by dividing it into layers (colors). The analysis of each layer is derived from an exact approach. The overflow traffic from one layer to another is characterized by the moment matching method (Section III). An equivalent path method is used to calculate the overflow moments. These moments are used to calculate the equivalent Poisson overflow load used in the calculation of the path blocking probabilities. The analytical model presented in this paper is based on the work in [3] for the unicast WDM. The model is applicable to arbitrary network topologies with static routing and First-Fit wavelength assignment.

The rest of the paper is organized as follows: next section presents the network model. In section III, we introduce our proposed solution. Section IV presents some numerical results. Lastly, we present our conclusion.

II. Network Model

A call is considered the basic unit of WDM traffic. A multicast call originating from a source node s to the set of destination nodes $D_t = \{d_1, d_2, \ldots, d_i\}$ is denoted as (s, D_t). A unicast call has a single destination $|D_t| = 1$. A predetermined light-tree (i.e. an all-optical multicast tree) $T(s, D_t)$ exists for each multicast request (s, D_t) at the design stage [4]. The call arrival process entering the network is assumed to be Poisson with rate λ_{s,D_t} calls/unit time. The call termination process is exponentially distributed with a mean $\mu = 1$. The arrival and termination rates are assumed to be equal. The call requires one wavelength (channel) to be available from each link along the predetermined fixed tree $T(s, D_t)$ from the source s to each destination $d \in D_t$. Since no conversion capability is assumed the same wavelength must be used in all links belonging to the tree; otherwise the call request is blocked. The nodes in the network are classified into two categories: split incapable or split capable. Multicast split incapable nodes are nodes which cannot split the incoming lightpath to more than one output link. However, the implementation of a split capable node may be expensive due to the large amount of amplification and fabrication [5]. All nodes of the network have Drop-and-Continue capability [6].

Let the order of wavelengths be numbered $w = 1, 2, 3, \ldots, W$. Upon the arrival of a call, the source s will

 ISBN: 978-1-61208-199-1

offer the call to the first wavelength (layer) $w = 1$ on the predetermined fixed tree. The call is accepted if the wavelength w is available on all links belonging to the predetermined fixed path. Otherwise, the call is offered to the next wavelength. Thus, the traffic which cannot be carried by a wavelength w is offered to the next wavelength $w+1$ and so on until the call is either accepted or blocked. Multicast call (s, D_t) blocking probability for a given wavelength w is denoted by P^w_{s,D_t}. The link i, j capacity is denoted as $C_{i,j}$ and the path $r(s, d)$ capacity is denoted as $C_{s,d}$, where $C_{s,d} = \min_{(i,j)\in r(s,d)} C_{i,j}$. Similarly, C_{s,D_t} denotes the capacity of tree $T(s, D_t)$.

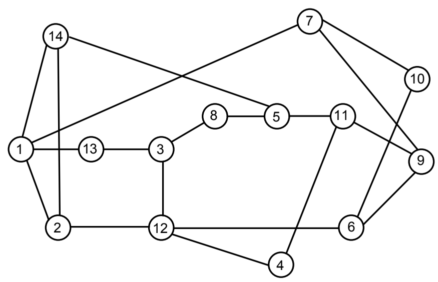

Fig. 1. The 14-Nodes NSFNET network topology

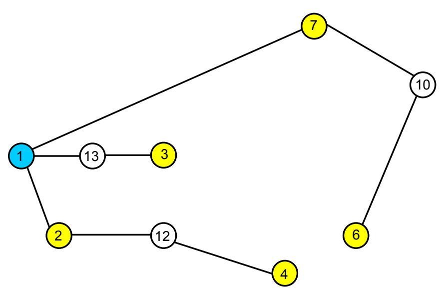

Fig. 2. A multicast tree $T(1, D_1)$, $D_1 = \{2, 3, 4, 6, 7\}$. Transient nodes are Drop-and-Continue capable but not splitting capable. The tree $T(1, D_1)$ contains the route $r(1, 3), r(1, 4), r(1, 6)$.

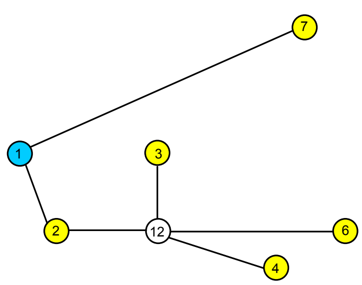

Fig. 3. A multicast tree $T(1, D)$, $D = \{2, 3, 4, 6, 7\}$. The tree $T(1, D)$ contains the route $r(1, 7), r(1, 12)$ and subtree $T(12, \{3, 4, 6\})$. Node 12 is a transient and split capable node.

Fig. 1 shows the NSFNET network topology. Fig. 2 and Fig. 3 show a multicast call $T(s, D)$, $D = \{2, 3, 4, 6, 7\}$ with two trees. All nodes in Fig. 2 are split incapable nodes, whereas node 12 is a split capable node in Fig. 3. The first tree can be descried by the set of paths $T(s, D) = r(s, d_i)$ and $r(s, d_i) \nsubseteq r(s, d_j)$. The second tree is described by the set of paths $r(s, d_i)$ and $r(d_s, d_j)$, where node d_s is a split capable node.

III. Proposed Solution

In this section, we present a basic description of the wavelength decomposition method for unicast calls introduced in [3]. Wavelength decomposition method analyzes the network by splitting it into layers and uses a moment matching method to calculate an equivalent Poisson overflow traffic to each layer. The analysis of blocking probabilities in each layer is derived from an exact approach. In this work, we extend the single layer unicast blocking probability calculations to solve the multicast call blocking probability.

A. The Single Layer Blocking Probability

First, we review the basic concept of calculating the single layer blocking probability for unicast calls in [3]. Consider the $k-1$ hop route shown in Fig. 4, denoted as $r(1, k)$. Let the state of path $r(1, k)$ in a single wavelength (layer) w at time t be described by the $k(k-1)/2$ dimensional process

$$X^w_{r(1,k)}(t) = (n^w_{1,2}(t), n^w_{1,3}(t), \dots, n^w_{k-1,k}(t)) \tag{1}$$

The state of the $k-1$ hop path $r(1, k)$ for wavelength w is thus denoted by the number of calls $n^w_{i,j} \in \{0, 1\}$ in progress for each segment $r(i, j)$, $1 \le i < k$, $1 < j \le k$, $i < j$, where

$$\begin{aligned} n^w_{i,j} + n^w_{l,m} \le 1 \quad \forall\, r(i, j) \cap r(l, m) \ne \emptyset \quad \text{and} \\ 1 \le l < k, 1 < m \le k, l < m \end{aligned} \tag{2}$$

Process $X^w_{r(1,k)}(t)$ is a time-reversible Markov process and the stationary vector π is given by [7]

$$\begin{aligned} \pi(n^w_{1,2}, n^w_{1,3}, \dots, n^w_{k-1,k}) = \frac{1}{G^w_{r(1,k)}} \Bigg[\frac{(a^w_{1,2})^{n^w_{1,2}}}{n^w_{1,2}!} \cdot \\ \frac{(a^w_{1,3})^{n^w_{1,3}}}{n^w_{1,3}!} \cdots \frac{(a^w_{k-1,k})^{n^w_{k-1,k}}}{n^w_{k-1,k}!} \Bigg] \end{aligned} \tag{3}$$

where $a^w_{i,j}$ is the background traffic in each segment i, j for a given wavelength w. $G^w_{r(1,k)}$ is the normalization constant for wavelength w on the path $r(1, k)$ and is given by

$$G^w_{r(1,k)} = \sum_{\substack{n^w_{i,j}+n^w_{l,m}\le 1 \\ \forall\ r(i,j)\cap r(l,m)\ne\emptyset \\ r(i,j)\subseteq r(s,d),\ r(l,m)\subseteq r(1,k)}} \prod_{r(i,j)\subseteq r(1,k)} \frac{(a^w_{i,j})^{n^w_{i,j}}}{n^w_{i,j}!} \tag{4}$$

by using the reduced blocking path model [8], the background traffic $a^w_{i,j}$ for segment $r(i, j) \subseteq T(s, D_t)$ is calculated as [3],

$$a^w_{i,j} = \sum_{\substack{r(i,j)\subseteq T(s,D_t) \\ r(i,j)\leftarrow A^w_{s,D}\ \text{then}\ r(m,l)\not\leftarrow A^w_{s,D_t} \\ \forall\ r(l,m)\subseteq r(1,k)}} \frac{A^w_{s,D_t}\cdot(1-P^w_{s,D_t})}{1-P^w_{i,j}} \tag{5}$$

where A^w_{s,D_t} is the source s to group D_t offered load at wavelength w, $A^1_{s,D_t} = \lambda_{s,D_t}$. The normalization constant $G^w_{r(1,k)}$ can be calculated recursively [3] as

$$G^w_{r(1,k)} = G^w_{r(1,k-1)} + \sum_{i=1}^{k-1} G^w_{r(1,k-i)} a^w_{k-i,k} \tag{6}$$

where $G^w_{r(1,1)} = 1$. Thus, path $r(1,k)$ blocking probability $P^w_{r(1,k)}$ *(or $P^w_{1,k}$ for short)* in a single layer $w \le C_{1,k}$ is calculated as

$$P^w_{r(1,k)} = 1 - \pi(0,0,\ldots,0) = 1 - \frac{1}{G^w_{r(1,k)}} \tag{7}$$

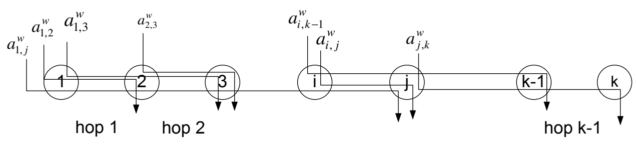

Fig. 4. A $k-1$ hop path $r(1,k)$. The state of the path in wavelength (layer) w at time t is described by the $k(k-1)/2$ dimensional process $X^w_{r(1,k)}(t) = (n^w_{1,2}(t), n^w_{1,3}(t), \ldots, n^w_{k-1,k}(t))$. Where $n^w_{i,j}(t)$ is the number of calls using the segment $r(i,j)$, that are currently active in wavelength w at time t i.e., $n^w_{i,j}(t) \in \{0,1\}$. The background offered traffic in each segment i,j for a given wavelength w is denoted as $a^w_{i,j}$.

Now, to extend the unicast wavelength decomposition method to multicast WDM network let us consider two disjoint routes *(paths)* $r(s_1,d_1)$ and $r(s_2,d_2)$ as shown in figure 5. Common traffic passing through both routes are denoted as $a^w_{(i,j),(x,y)}$ where $i,j \in r(s_1,d_1)$ and $x,y \in r(s_2,d_2)$. We can calculate the single layer joint normalization constant for the two routes $G^w_{r(s_1,d_1)} \cap G^w_{r(s_2,d_2)}$ as follow.

$$\begin{aligned} G^w_{r(s_1,d_1)} \cap G^w_{r(s_2,d_2)} &= g^w_{r(s_1,d_1)} \cdot g^w_{r(s_2,d_2)} \\ &+ \sum_{\substack{\forall\, i,j \in r(s_1,d_1) \\ \forall\, x,y \in r(s_2,d_2)}} g^w_{r(s_1,i)} \cdot g^w_{r(j,d_1)} \cdot a^w_{(i,j),(x,y)} \cdot g^w_{r(s_2,x)} \cdot g^w_{r(y,d_2)} \end{aligned} \tag{8}$$

where, $g^w_{r(i,j)}$ represents the normalization constant for segment $r(i,j)$ excluding all common traffic between route $r(s_1,d_1)$ and route $r(s_2,d_2)$.

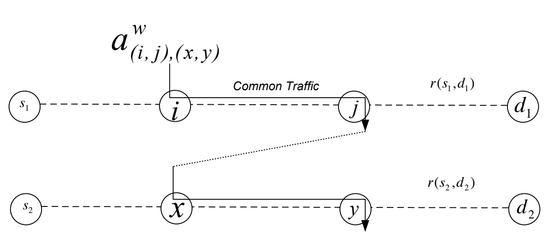

Fig. 5. Two disjoint routes *(paths)* $r(s_1,d_1)$ and $r(s_2,d_2)$

Consider a tree $T(s,D)$ (e.g., Fig. 2) for a given wavelength w. The event that a wavelength w is available to the multicast call $T(s,D)$ is conditioned on the availability of wavelength w on all links belonging to the tree. Therefore, a multicast call (s,D_t) blocking probability for a given wavelength w is given by

$$P^w_{s,D_t} = 1 - \frac{1}{\mathbf{G}^w_{s,D_t}} \tag{9}$$

where, $\mathbf{G}^w_{s,D_t}$ is the multicast call $T(s,D_t)$ normalization constant. If all nodes are split incapable, then the tree is a group of disjoint paths $r(s,d_i)$. Hence,

$$\mathbf{G}^w_{s,D_t} = \bigcap_{\forall i} G^w_{r(s,d_i)} + A^w_{s,D_t} \tag{10}$$

For example, the normalization constant $\mathbf{G}^w_{1,D_1}$ for the tree in Fig. 2 is

$$\mathbf{G}^w_{1,\{2,3,4,6,7\}} = G^w_{r(1,3)} \cap G^w_{r(1,4)} \cap G^w_{r(1,6)} + A^w_{1,\{2,3,4,6,7\}}$$

Since $r(1,2) \subseteq r(1,4)$ and $r(1,7) \subseteq r(1,6)$.

For networks with split capable nodes, trees will be a mix of disjoint paths with no split capable node and subtrees. The split capable nodes d_s will be the root of these subtrees and the leaves will be a subset $D_{t'} \subseteq D_t$. Therefore, the normalization constant will be the product of disjoint paths $G^w_{r(s,d_s)}$ and subtrees $\mathbf{G}^w_{d_s,D_{t'}}$ calculated from Eq. 12. Now, consider a branch with a split capable node d_s to a subset $D_{t'}$ as shown in Fig. 6. The normalization constant, $\mathbf{G}^w_{d_{s-1},D_{t'}}$ can be calculated form $\mathbf{G}^w_{d_s,D_{t'}}$ as

$$\begin{aligned} \mathbf{G}^w_{d_{s-1},D_{t'}} &= G^w_{r(d_{s-1},d_s)} \cdot \mathbf{G}^w_{d_s,D_{t'}} + \sum_{\forall k \in D_{t'}} \\ & \sum_{\forall j \in r(d_{s+1},k)} a^w_{d_{s-1},j}\, \mathbf{G}^w_{d_s,D_{t'}-\{k\}} + A^w_{d_{s-1},D_{t'}} \end{aligned}$$

Generally, the normalization constant, $\mathbf{G}^w_{s,D_{t'}}$ is

$$\begin{aligned} \mathbf{G}^w_{s,D_{t'}} &= G^w_{r(d_i,d_{i+1})} \cdot \mathbf{G}^w_{d_{i+1},D_{t'}} \\ &+ \sum_{\forall\, i \in r(s,d_s-1)} \sum_{\forall k \in D_{t'}} \sum_{\forall j \in r(d_{s+1},k)} G^w_{r(s,i)} \\ &\cdot\; a^w_{i,j}\; \mathbf{G}^w_{d_s,D_{t'}-\{k\}} + A^w_{s,D_{t'}} \end{aligned} \tag{11}$$

Finally, $\mathbf{G}^w_{s,D_t}$

$$\mathbf{G}^w_{s,D_t} = \prod_{\forall D_{t'}} \mathbf{G}^w_{s,D_{t'}} \cdot \prod_{\substack{\forall d_i \in D_t, d_i \notin D_{t'} \\ r(s,d_i) \nsubseteq r(s,d_j)\, \forall\, d_j \in D_t}} G^w_{r(s,d_i)} + A^w_{s,D_t} \tag{12}$$

For example, the normalization constant $\mathbf{G}^w_{1,D}$ for the tree

 ISBN: 978-1-61208-199-1

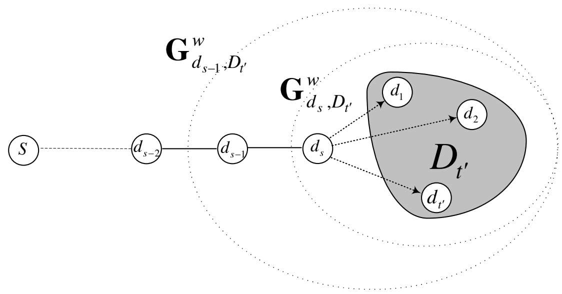

Fig. 6. Calculating the normalization constant recursively, for a multicast tree with a split capable node d_s.

in Fig. 3 is

$$\mathbf{G}^w_{1,\{2,3,4,6,7\}} = G^w_{r(1,7)} \cap \mathbf{G}^w_{1,\{3,4,6\}} + A^w_{1,D}$$

$$\begin{aligned}\mathbf{G}^w_{1,\{3,4,6\}} &= G^w_{r(1,12)} \cdot \mathbf{G}^w_{12,\{3,4,6\}} \\ &+ G^w_{r(1,1)} \cdot a^w_{1,3} \cdot \mathbf{G}^w_{12,\{4,6\}} + G^w_{r(1,1)} \cdot a^w_{1,4} \cdot \mathbf{G}^w_{12,\{3,6\}} \\ &+ G^w_{r(1,1)} \cdot a^w_{1,6} \cdot \mathbf{G}^w_{12,\{3,4\}} + G^w_{r(1,2)} \cdot a^w_{2,3} \cdot \mathbf{G}^w_{12,\{4,6\}} \\ &+ G^w_{r(1,2)} \cdot a^w_{2,4} \cdot \mathbf{G}^w_{12,\{3,6\}} + G^w_{r(1,2)} \cdot a^w_{2,6} \cdot \mathbf{G}^w_{12,\{3,4\}}\end{aligned}$$

To simplify notations we will drop the subscript t from D_t.

B. Calculating the Moments of the Overflow Traffic

Since the overflow load from wavelength $w+1$ is non Poisson, we need to calculate both the first and the second overflow traffic moments (*mean* $\bar{A}^{w+1}$ *and variance* $\bar{V}^{w+1}$) to the next layer $w+1$. For this, we construct an equivalent single-link system such that the blocking of the Poisson traffic in this system will approximate the blocking on the tree $T(s,D)$. We know that the total offered load to the tree is $\lambda_{s,D}$ and the overflow mean, up to the current wavelength w is $A^w_{s,D}P^w_{s,D}$. Hence,

$$\lambda_{s,D} \cdot E_r(\lambda_{s,D}, N^w_{s,D}) = A^w_{s,D} \cdot P^w_{s,D} \tag{13}$$

where, E_r is the generalized (not integral) Erlang-B formula [9]. The overflow mean to wavelength $w+1$ is

$$\bar{A}^{w+1}_{s,D} = \lambda_{s,D} \cdot E_r(\lambda_{s,D}, N^w_{s,D}) \tag{14}$$

The variance $\bar{V}^{w+1}_{s,D}$ is calculated using Riordan's formula [8],

$$\bar{V}^{w+1}_{s,D} = \bar{A}^{w+1}_{s,D}\left(1 - \bar{A}^{w+1}_{s,D} + \frac{\lambda_{s,D}}{N^w_{s,D} + 1 + \bar{A}^{w+1}_{s,D} - \lambda_{s,D}}\right) \tag{15}$$

The peakedness is defined as, $\bar{Z}^{w+1}_{s,D} = \frac{\bar{V}^{w+1}_{s,D}}{\bar{A}^{w+1}_{s,D}}$.

C. Calculating the Equivalent Poisson Traffic

The path blocking probabilities calculated in Eq. 5 through Eq. 12 assume Poisson traffic with $\bar{Z}^w_{s,D} = 1$. However, the overflow traffic to layer $w+1$ calculated from equations 14 and 15 is in general non Poisson $\bar{Z}^{w+1}_{s,D} \neq 1$. We again use an equivalent single-link system with $\bar{N}^{w+1}_{s,D} \leq 1$ wavelengths to find an equivalent Poisson traffic with mean $A^{w+1}_{s,D}$ and $Z^{w+1}_{s,D} = 1$ that matches the overflow traffic with mean $\bar{A}^{w+1}_{s,D}$ and variance $\bar{V}^{w+1}_{s,D}$.

Fredricks and Hayward's equivalence method was used with the original wavelength decomposition method described in [3]. It attempts to describe a non-Poisson traffic $Z \neq 1$ by an equivalent Poisson traffic $Z = 1$ [9]. Mainly, the blocking probability of the actual system with $\bar{N}^w_{s,d}$ channels offered non-Poisson traffic with rate $\bar{A}^w_{s,d}$ and peakedness $\bar{Z}^w_{s,d} \neq 1$ has the same blocking probability with $\bar{N}^w_{s,d}/\bar{Z}^w_{s,d}$ channels, offered $\bar{A}^w_{s,d}/\bar{Z}^w_{s,d}$ traffic, and peakedness $Z^w_{s,d} = 1$ (Poisson).

In this work, we combine Fredricks and Hayward's equivalence method with Berkeley's Equivalent Random Traffic (ERT) approximation. Combining moment matching functions seems to be more suitable to calculate the equivalent Poisson traffic [10]. The idea of the Equivalent Random Traffic (ERT) method is to think that the traffic with mean $\bar{A}^w_{s,d}$ and variance $\bar{V}^w_{s,d} \neq \bar{A}^w_{s,d}$ is obtained as overflow traffic from a fictitious system with $\hat{\mathbf{N}}^w_{s,d}$ channels offered a Poisson traffic with mean $\hat{\mathbf{A}}^w_{s,d}$. Hence, the blocking probability of the non-Poison secondary system is the same as the blocking probability of the equivalent system with $\hat{\mathbf{N}}^w_{s,d}$ channels, mean $\hat{\mathbf{A}}^w_{s,d}$ and peakedness $Z^w_{s,d} = 1$. Berkeley's ERT approximation is considered as a single parameter ERT method where, we fix $\hat{\mathbf{A}}^w_{s,d} = \lambda_{s,d}$ [8] [11]. Hence, $\hat{\mathbf{N}}^w_{s,d}$ is as follows

$$\hat{\mathbf{N}}^w_{s,d} = \frac{\lambda_{s,d}(\bar{A}^w_{s,d} + \bar{Z}^w_{s,d})}{\bar{A}^w_{s,d} + \bar{Z}^w_{s,d} - 1} - \bar{A}^w_{s,d} - 1 \tag{16}$$

D. Calculating the Overall Path Blocking Probability

The overall path blocking probability is calculated as

$$P_{s,D} = \frac{A^{C_{s,D}}_{s,D} \cdot P^{C_{s,D}}_{s,D}}{\lambda_{s,D}} \tag{17}$$

IV. NUMERICAL RESULTS

First, we present a five node network shown in Fig. 7 as an illustrative example for First-Fit WA. There are two multicast calls, $T(1,D_1)$, $D_1 = \{3,5\}$ and $T(2,D_2)$, $D_2 = \{3,4\}$ with arrival rate of $\lambda_{1,D_1} = \lambda_{1,D_2} = 0.5$ (calls/unit time). The unicast call arrival rates are $\lambda_{1,2} = \lambda_{1,3} = \lambda_{1,4} = \lambda_{1,5} = \lambda_{2,3} = \lambda_{2,4} = 0.5$ (calls/unit time). Link capacities are 4 channels.

The normalization constant for the multicast tree $T(1,D_1)$ is

$$\begin{aligned}\mathbf{G}^w_{1,D_1} &= G^w_{r(1,3)} * G^w_{r(1,5)} + A^w_{1,D_1} \\ &= (1 + a^w_{1,2} + a^w_{2,3} + a^w_{1,2}a^w_{2,3} + A^w_{1,3}) * \\ &\quad (1 + A^w_{1,5}) + A^w_{1,D_1}\end{aligned}$$

where

$$a^w_{1,2} = \frac{A^w_{1,2}(1-P^w_{1,2}) + A^w_{1,4}(1-P^w_{1,4})}{1-P^w_{1,4}}$$

$$a^w_{2,3} = \frac{A^w_{2,3}(1-P^w_{2,3}) + A^w_{2,D_2}(1-P^w_{2,D_2})}{1-P^w_{2,3}}$$

Copyright (c) IARIA, 2012. ISBN: 978-1-61208-199-1

The normalization constant for the multicast tree $T(2, D_2)$ is

$$\mathbf{G}^w_{2,D_2} = 1 + a^w_{2,3} + a^w_{2,4} + a^w_{2,3}a^w_{2,4} + A^w_{2,D_2}$$

where

$$a^w_{2,3} = \frac{A^w_{2,3}(1-P^w_{2,3})+A^w_{1,3}(1-P^w_{1,3})+A^w_{1,D_1}(1-P^w_{1,D_1})}{1-P^w_{2,3}}$$
$$a^w_{2,4} = \frac{A^w_{2,4}(1-P^w_{2,4})+A^w_{1,4}(1-P^w_{1,4})}{1-P^w_{2,4}}$$

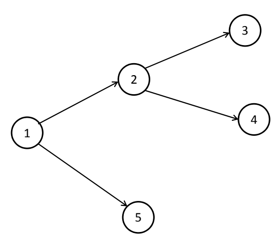

Fig. 7. A network example with five nodes

The path blocking probabilities in the first layer and the overall blocking probabilities are shown in table I.

TABLE I
THE PATH BLOCKING PROBABILITIES IN THE FIRST LAYER AND THE OVERALL BLOCKING PROBABILITIES FOR AN EXAMPLE OF A TREE WITH SPLIT INCAPABLE NODES.

layer	First		Overall	
Path	Simulation	Calculation	Simulation	Calculation
$P^1_{1,2}$	0.5519	0.5559	0.0664	0.0633
$P^1_{1,3}$	0.7557	0.7512	0.1744	0.1672
$P^1_{1,4}$	0.7557	0.7469	0.1613	0.1531
$P^1_{1,5}$	0.3887	0.3886	0.00814	0.0060
$P^1_{2,3}$	0.5517	0.5559	0.0663	0.0633
$P^1_{2,4}$	0.4977	0.5021	0.0302	0.0306
P^1_{1,D_1}	0.8376	0.8342	0.2032	0.2159
P^1_{2,D_2}	0.7557	0.7469	0.1615	0.1531

Now, let us assume node 2 is split capable node and $D_1 = \{3, 4, 5\}$ for the multicast call $T(1, D_1)$. The normalization constant for the multicast tree $T(1, \{3, 4, 5\})$ is

$$\begin{aligned} \mathbf{G}^w_{1,\{3,4,5\}} &= G^w_{r(1,5)} \cdot \mathbf{G}^w_{1,\{3,4\}} + A^w_{1,D_1} \\ \mathbf{G}^w_{1,\{3,4\}} &= G^w_{r(1,2)} \cdot \mathbf{G}^w_{2,\{3,4\}} \\ &+ G^w_{r(1,1)} \cdot a^w_{1,3} \cdot \mathbf{G}^w_{2,\{4\}} \\ &+ G^w_{r(1,1)} \cdot a^w_{1,4} \cdot \mathbf{G}^w_{2,\{3\}} \end{aligned}$$

The normalization constant for the multicast tree $T(2, D_2)$ is

$$\begin{aligned} \mathbf{G}^w_{2,D_2} &= 1 + a^w_{2,3} + a^w_{2,4} + a^w_{2,3}a^w_{2,4} \\ &+ A^w_{1,D_1} * (1 - P^w_{1,D_1})/(1 - P^w_{2,D_2}) + A^w_{2,D_2} \end{aligned}$$

where $a^w_{2,3}$ is

$$a^w_{2,3} = \frac{A^w_{2,3}(1-P^w_{2,3})+A^w_{1,3}(1-P^w_{1,3})}{1-P^w_{2,3}}$$

TABLE II
THE PATH BLOCKING PROBABILITIES IN THE FIRST LAYER AND THE OVERALL BLOCKING PROBABILITIES FOR AN EXAMPLE OF A TREE WITH SPLIT CAPABLE NODES.

layer	First		Overall	
Path	Simulation	Calculation	Simulation	Calculation
$P^1_{1,2}$	0.5362	0.5421	0.0639	0.0581
$P^1_{1,3}$	0.7469	0.7428	0.1686	0.1583
$P^1_{1,4}$	0.7459	0.7428	0.1685	0.1583
$P^1_{1,5}$	0.3684	0.3706	0.0083	0.0046
$P^1_{2,3}$	0.5361	0.5421	0.0638	0.0581
$P^1_{2,4}$	0.5370	0.5421	0.0641	0.0581
P^1_{1,D_1}	0.8862	0.8881	0.2478	0.2660
P^1_{2,D_2}	0.7467	0.7428	0.1688	0.1583

The path blocking probabilities in the first layer and the overall blocking probabilities are shown in table II.

Our next test vehicle is the 14 nodes NSFNET as shown in Fig. 1. The network traffic for unicast calls is similar to a realistic network with realistic traffic, and the network has been dimensioned using shortest path [12]. The multicast traffic for split incapable nodes is given in Table III. For split capable network, we arbitrary chose nodes 12 as split capable node. Fig. 3 shows the new tree for the multicast call $D_1 = \{2, 3, 4, 6, 7\}$.

Simulation results are run 10 times and each run starts with a different random seed where, each seed simulation runs for 10,000 holding times. The overall average result is obtained with 95% confidence. For the analytical techniques, the iterative algorithm terminates when all blocking probability values have converged within 10^{-5}.

TABLE III
MULTICAST TRAFFIC WITH LOAD FACTOR=12 FOR SPLIT INCAPABLE NODES.

Source	D	$\lambda_{s,D}$	Routes	Links
1	2,3,4,6,7	0.1	$r(1,3), r(1,4), r(1,6)$	8
1	7,10,12	4.0	$r(1,10), r(1,12)$	4
2	3,5,14	2.5	$r(2,3), r(2,5)$	4
4	1,9,11	5.0	$r(4,1), r(4,9)$	5
5	2,6,9,10	1.5	$r(5,2), r(5,10)$	6
6	2,5,10,11	7.0	$r(6,12), r(6,5), r(6,10)$	6
7	1,4,9,14	0.5	$r(7,4), r(7,14)$	5
8	1,5,13,14	2.0	$r(8,13), r(8,14)$	4
9	8,6	2.0	$r(9,6), r(9,8)$	4
10	1,4,12	4.0	$r(10,1), r(10,4)$	5
12	1,2,11	5.0	$r(12,1), r(12,11)$	4
12	3,8,10	4.0	$r(12,8), r(12,10)$	4
13	3,4,7,10,11	0.1	$r(13,10), r(13,11)$	7
14	2,3,5,8,12	0.25	$r(14,3), r(14,12)$	5

 ISBN: 978-1-61208-199-1

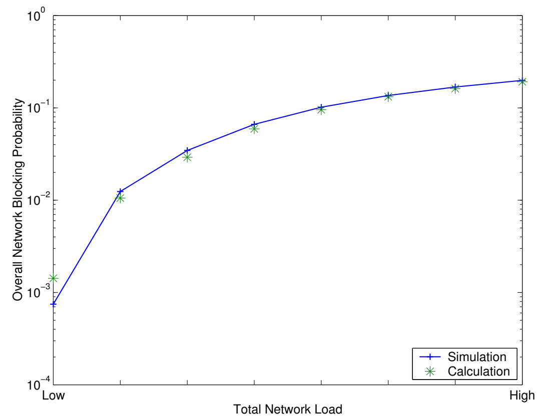

Fig. 8. Overall network end-to-end blocking probability for the NSFNET mesh network with no split capability.

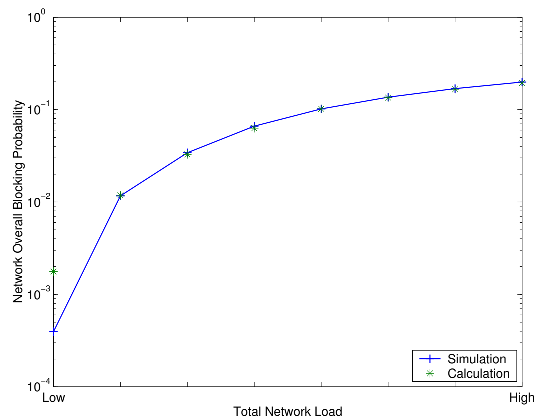

Fig. 9. Overall network end-to-end blocking probability for the NSFNET mesh network with a split capable node 12.

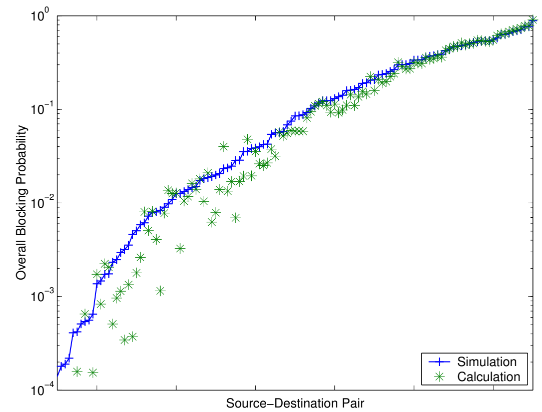

Fig. 10. The end-to-end blocking probabilities for various unicast calls in the NSFNET mesh network with no split capability.

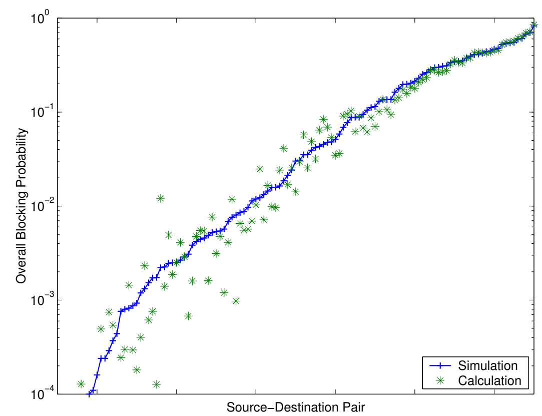

Fig. 11. The end-to-end blocking probabilities for various unicast calls in the NSFNET mesh network with a split capable node 12.

In Figs. 8 and 9, we plot the overall network blocking probability against the total network load for the split incapable and the split capable network respectively. In both figures, the first curve is obtained from simulation, and the second curve is plotted from our new approach. Figs. 10 and 11 show the end-to-end blocking probabilities for various unicast calls for both cases. The source/destination pairs are numbered in ascending order of their blocking probability values obtained from simulations. There are 170 source/destination pairs that have none zero load. Source/destination pair that yields an end-to-end blocking probability of at least 10^{-4} in the simulation is shown in the figure. Similarly, Fig. 12 and Fig. 13 show the end-to-end blocking probabilities for various multicast calls in the network. Again, trees are numbered in ascending order of their blocking probability values obtained from simulations. We can notice that the outputs of our calculations are very close to simulation results.

V. The Conclusion

We have presented a new analytical approach to evaluate more accurately the call blocking probabilities of a multicast Wavelength Division Multiplexing (WDM) network. Our approach assumes fixed routing, First-Fit wavelength assignment with split incapable or capable nodes. The new approach views the WDM network as a set of different layers (colors) where, blocked traffic in one layer is overflowed to another layer. Simulation results show the accuracy of our approach. Applying our new method to random wavelength assignment

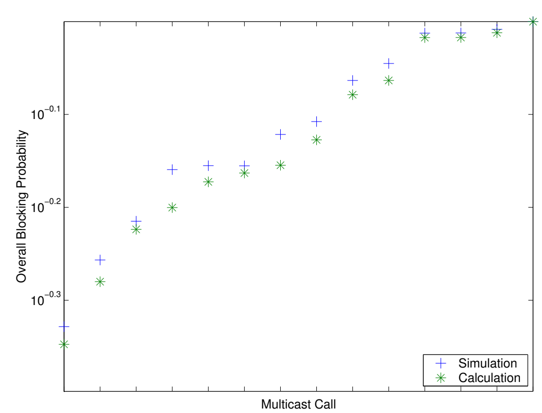

Fig. 12. The end-to-end blocking probabilities for various multicast calls in the NSFNET mesh network.

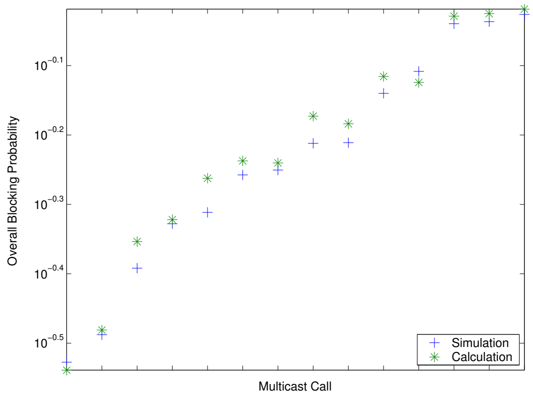

Fig. 13. The end-to-end blocking probabilities for various multicast calls in the NSFNET mesh network with a split capable node 12.

in multicast networks will be considered in future research.

References

[1] A. Kamal and A. Alyatama, “Blocking probabilities in circuit-switched wavelength division multiplexing networks under multicast service,” *Performance Evaluation*, vol. 47, no. 1, pp. 43-71, 2002.

[2] S. Ramesh, G. Rouskas and H. Perros, “Computing blocking probabilities in multiclass wavelength routing network,” *IEEE Journal on Selected Areas in Communications*, vol. 20, Jan., 2002.

[3] A. Alyatama, “Wavelength decomposition approach for computing blocking probabilities in WDM optical networks without wavelength conversions,” *Computer Networks*, vol. 49, pp. 727-742, Dec., 2005.

[4] J. He, S. Chan, and D. Tsang, “Multicasting in WDM Networks,” *EEE Communications Surveys & Tutorials*, vol. 4, no. 1, 2002.

[5] J. Siregar, Y. Zhang and H. Takagi, “Optimal multicast routing using genetic algorithm for WDM optical networks,” *IEEE Transactions on Communications*, vol. E88B, no. 1, pp. 219-226, Jan., 2005.

[6] R. Ramaswami and K. Sivarajan, *Optical Nerworks: A practical Prospective*, Second Edition, Morgan Kaufmann.

[7] C. Lea and A. Alyatama, “Bandwidth quantization and states reduction in the broadband ISDN,” *IEEE/ACM Transactions on Networking*, vol. 3, no. 3, June, 1995.

[8] A. Girard, *Routing and Dimensioning in Circuit-Switched Networks*, Addison-Wesley, 1990.

[9] ITU-D Study Group 2, *Teletraffic Engineering Handbook*, Geneva, Dec., 2002.

[10] A. Alyatama, “Computing the equivalent Poisson traffic for the Wavelength Decomposition Method”, *Journal of High Speed Networks*, vol. 15, no. 4, pp. 399-414, 2006.

[11] E. Karasan and E. Ayanoglu, “Effects of wavelength routing and selection algorithms on wavelength conversion gain in WDM optical networks,” *IEEE/ACM Transactions on Networking*, vol. 6, no. 2, April, 1998.

[12] R. Hulsermann, M. Jager, S. Krumke, D. Poensgen, J. Ramboui and A. Turchscherer, “Dynamic routing algorithms in transparent optical networks,” *Proceedings of ONDM 2003*, pp. 293-312, Feb., 2003, Budapest, Hungary.