

Unambiguous BOC Signal Acquisition Based on Recombination of Sub-Correlations

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Abstract—In this paper, we propose a novel unambiguous acquisition scheme for binary offset carrier (BOC) signals. Specifically, we first find out that the side-peaks arise due to the fact that the BOC autocorrelation is made up of the sum of the irregularly shaped sub-correlations, and then, propose an unambiguous acquisition scheme by recombining the sub-correlations. The proposed scheme is shown to remove the side-peaks completely for any type of BOC signal and to provide a better performance than the conventional scheme in terms of the incorrect acquisition probability and mean acquisition time.

Keywords—acquisition; ambiguous; binary offset carrier; correlation function; side-peak

I. INTRODUCTION

Recently, new global navigation satellite systems (GNSSs) such as Galileo and global positioning system (GPS) modernization are being developed to satisfy the increasing demand for GNSS-based services such as location-based service (LBS) and emergency rescue service (ERS) and complement the existing GNSSs such as GPS [1]-[2]. Currently, new GNSSs are designed to use the same frequency band of the existing GNSSs: for example, the E1 and E5 bands of Galileo are overlapped with the L1 and L5 bands of GPS, respectively [1], [3]. Thus, if a Galileo signal is modulated by a conventional scheme such as phase shift keying (PSK) used in GPS, it would suffer from co-channel interference. To overcome these problems, binary offset carrier (BOC) modulation has been proposed, where a high degree of spectral separation between the BOC-modulated signals and the others is achieved by shifting the signal energy from the band center [4]. The BOC signal is generated through the product of a spreading pseudo random noise (PRN) code and a sine-phased or cosine-phased square wave sub-carrier, and denoted by $\text{BOC}_{\sin}(kn, n)$ or $\text{BOC}_{\cos}(kn, n)$ depending on which of the sine-phased or cosine-phased sub-carriers are used, where k and n are the ratios of the PRN code chip period to the sub-carrier period and the PRN code chip rate to 1.023 MHz, respectively [3], [5]. For larger values of k , more separated spectrums are obtained, reducing the co-channel interference more effectively.

However, the BOC signal has multiple side-peaks on both sides of the main-peak of its autocorrelation function. Moreover, the number of side-peaks increases as the value of k becomes larger. Thus, the correlation-based synchronization schemes [6]-[11] originally proposed for PRN code synchronization would suffer from the ambiguous problem in the BOC signal synchronization due to the side-peaks in the BOC autocorrelation.

Several unambiguous acquisition schemes [12]-[16] have been proposed in order to tackle the problem. In [12]-[14], sideband filtering was used to deal with the ambiguous problem in the BOC signal acquisition; however, these schemes destroy the sharpness of the main-peak of the BOC autocorrelation function, degrading the BOC signal tracking performance severely. In [15], an interesting unambiguous acquisition scheme that maintains the sharp main-peak of the BOC autocorrelation function was proposed combining the correlation between the BOC and PRN signals with the BOC autocorrelation; however, this scheme is applicable to only $\text{BOC}_{\sin}(n, n)$ signals. In [16], a generalized unambiguous acquisition scheme including the scheme in [15] as a special case was proposed. This scheme is applicable to generic $\text{BOC}_{\sin}(kn, n)$ signals; however, its extension to generic $\text{BOC}_{\cos}(kn, n)$ signals is not straightforward. In addition, in [17], an unambiguous scheme applicable to both sine-phased and cosine-phased BOC signals was proposed by the authors preliminarily; however, the performance of the scheme becomes worse as the value of k increases.

In this paper, a novel unambiguous acquisition scheme applicable to both $\text{BOC}_{\sin}(kn, n)$ and $\text{BOC}_{\cos}(kn, n)$ signals is proposed based on a recombination of the sub-correlations making up the BOC autocorrelation. The scheme is found to remove the side-peaks of the BOC autocorrelation completely, while keeping the sharp shape of the main-peak. Moreover, it is demonstrated that the scheme offers a performance improvement over the scheme in [16] in terms of the incorrect acquisition probability and mean acquisition time (MAT).

The remainder of this paper is organized as follows. Section II analyzes the sub-correlations making up the BOC autocorrelation. In Section III, an unambiguous acquisition scheme with no side-peak is proposed by recombining the

sub-correlations. Section IV presents numerical results, and finally, Section V concludes this paper.

II. BOC SUB-CORRELATIONS

The BOC signal $b(t)$ can be expressed as

$$b(t) = \sqrt{P} \sum_{i=-\infty}^{\infty} c_i p_{T_c}(t - iT_c) d_{\lfloor iT_c/T \rfloor}(t) s(t), \quad (1)$$

where P is the signal power, $c_i \in \{-1, 1\}$ is the i th chip of a PRN code with a period of T , T_c is the PRN code chip period, $p_{T_c}(t)$ is the PRN code waveform defined as a unit rectangular pulse over $[0, T_c)$, $d_{\lfloor iT_c/T \rfloor}(t)$ is the navigation data, where $d_x(t)$ is the x th navigation data and $\lfloor x \rfloor$ is the largest integer not larger than x , and

$$s(t) = \begin{cases} \sum_{u=0}^{2k-1} (-1)^u p_{T_s}(t - iT_c - uT_s), & \text{for BOC}_{\sin}(kn, n), \\ \sum_{u=0}^{4k-1} (-1)^{\lfloor \frac{u}{2} \rfloor} p_{\frac{T_s}{2}}\left(t - iT_c - \frac{uT_s}{2}\right), & \text{for BOC}_{\cos}(kn, n) \end{cases} \quad (2)$$

is the square wave sub-carrier, where T_s is the sub-carrier pulse duration of $T_c/2k = 1/(2kn \times 1.023 \text{ MHz})$, $p_{T_s}(t)$ is the unit rectangular sub-carrier pulse waveform over $[0, T_s)$, and $\lceil x \rceil$ is the smallest integer not less than x . In this paper, focusing on the problem of ambiguity due to side-peaks, we assume that there is a pilot channel for acquisition [18] so that no data modulation is present during acquisition (i.e., $d_{\lfloor iT_c/T \rfloor}(t) = 1$ for all i), and do not consider the effect of the secondary code. Then, considering that the PRN code period T is generally much larger than the PRN code chip period T_c and the out-of-phase autocorrelation of a PRN code is designed to be as low as possible for easy signal acquisition, we can obtain the correlation (normalized to the signal power) between the received and locally generated BOC signals as [19]

$$\begin{aligned} R_{\sin}^k(\tau) &= \frac{1}{P} \int_0^T (b(t-\tau) + w(t))b(t)dt \\ &\simeq \sum_{u=0}^{2k-1} \left(N \sum_{v=0}^{2k-1} (-1)^{u+v} \Lambda_{T_s}(\tau - (u-v)T_s) + w_{\sin}^u \right) \end{aligned} \quad (3)$$

for $\text{BOC}_{\sin}(kn, n)$ and

$$R_{\cos}^k(\tau) \simeq \sum_{u=0}^{4k-1} \left(N \sum_{v=0}^{4k-1} (-1)^{\lfloor \frac{u}{2} \rfloor + \lfloor \frac{v}{2} \rfloor} \Lambda_{\frac{T_s}{2}}(\tau - (u-v)\frac{T_s}{2}) + w_{\cos}^u \right) \quad (4)$$

for $\text{BOC}_{\cos}(kn, n)$, where τ is the phase difference between the received and locally generated BOC signals, N is a correlation length and would be generally equal to or less than the PRN code period (normalized to T_c), $w(t)$ is the additive white Gaussian noise (AWGN) process with mean zero and one-sided noise power spectral density N_0 ,

$$w_{\sin}^u = \frac{1}{\sqrt{P}} \int_0^T \sum_{i=-\infty}^{\infty} (-1)^u c_i p_{T_s}(t - iT_c - uT_s) w(t) dt, \quad (5)$$

$$w_{\cos}^u = \frac{1}{\sqrt{P}} \int_0^T \sum_{i=-\infty}^{\infty} (-1)^{\lfloor \frac{u}{2} \rfloor} c_i p_{\frac{T_s}{2}}\left(t - iT_c - \frac{uT_s}{2}\right) w(t) dt, \quad (6)$$

and

$$\Lambda_x(\tau) = \begin{cases} x - |\tau|, & |\tau| \leq x, \\ 0, & \text{otherwise} \end{cases} \quad (7)$$

is the triangular function of height x and area x^2 . After denoting the triangular functions and noise terms in (3) and (4) as

$$R_{\sin}^{k,u}(\tau) = N \sum_{v=0}^{2k-1} (-1)^{u+v} \Lambda_{T_s}(\tau - (u-v)T_s) + w_{\sin}^u \quad (8)$$

and

$$R_{\cos}^{k,u}(\tau) = N \sum_{v=0}^{4k-1} (-1)^{\lfloor \frac{u}{2} \rfloor + \lfloor \frac{v}{2} \rfloor} \Lambda_{\frac{T_s}{2}}\left(\tau - (u-v)\frac{T_s}{2}\right) + w_{\cos}^u, \quad (9)$$

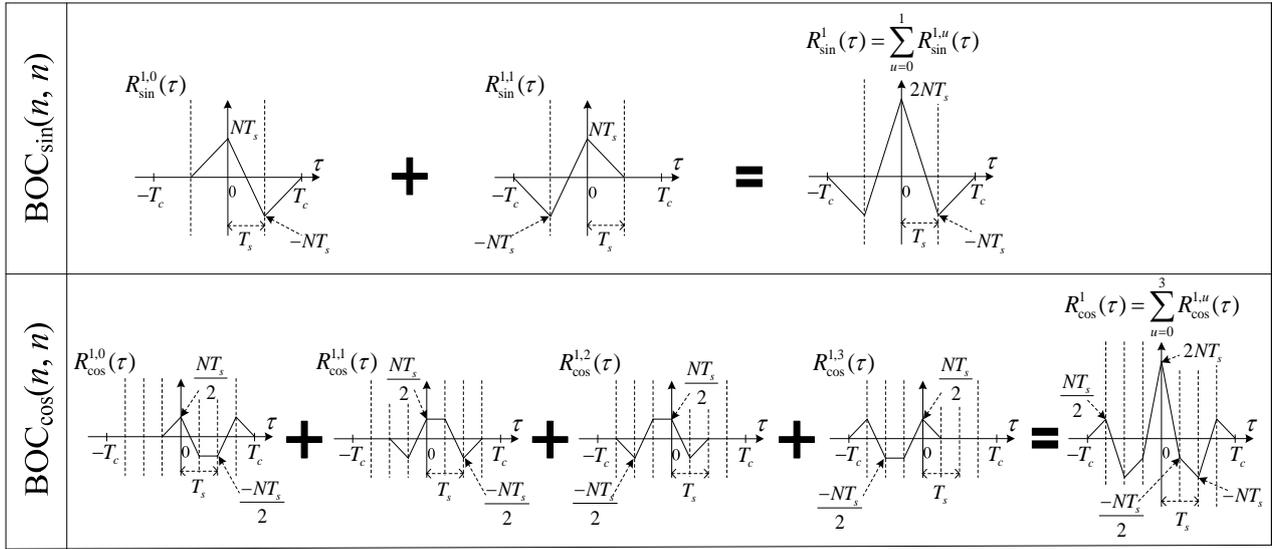
respectively, we can re-write $R_{\sin}^{k,u}(\tau)$ and $R_{\cos}^{k,u}(\tau)$ as

$$\begin{aligned} R_{\sin}^{k,u}(\tau) &= N \sum_{v=0}^{2k-1} (-1)^{u+v} \Lambda_{T_s}(\tau - (u-v)T_s) + w_{\sin}^u \\ &= \sum_{l=0}^{N-1} \sum_{v=0}^{2k-1} (-1)^{u+v} \int_{(2kl+u-1)T_s}^{(2kl+u)T_s} p_{T_s}(t - lT_c - uT_s) \\ &\quad \cdot p_{T_s}(t - \tau - lT_c - vT_s) dt + w_{\sin}^u \\ &= \sum_{l=0}^{N-1} \frac{1}{P} \int_{(2kl+u-1)T_s}^{(2kl+u)T_s} r(t)b(t) dt, \end{aligned} \quad (10)$$

and similarly,

$$R_{\cos}^{k,u}(\tau) = \sum_{l=0}^{N-1} \frac{1}{P} \int_{(\frac{4kl+u}{2}-\frac{1}{2})T_s}^{(\frac{4kl+u}{2})T_s} r(t)b(t) dt, \quad (11)$$

where $r(t) = b(t-\tau) + w(t)$. From (10) and (11), we can see that $R_{\sin}^{k,u}(\tau)$ and $R_{\cos}^{k,u}(\tau)$ are sub-correlations making up the correlations (3) and (4), respectively, and which are shown for $k=1$ in the absence of noise in Fig. 1. From the figure, we can see that the main-peaks of the sub-correlations are coherently combined through the summation of the sub-


 Figure 1. BOC autocorrelation and the associated sub-correlations for $\text{BOC}_{\sin}(n, n)$ and $\text{BOC}_{\cos}(n, n)$.

correlations, thus forming the sharp main-peak of the BOC autocorrelation, and on the other hand, the sub-peaks of the sub-correlations are irregularly spread around the main-peaks, and thus, the summation of the sub-correlations results in the multiple side-peaks of the BOC autocorrelation. In the next section, we propose a novel unambiguous acquisition scheme, removing the side-peaks completely through a recombination of the sub-correlations.

III. PROPOSED SCHEME

Fig. 2 shows the unambiguous correlation functions of the proposed scheme for $\text{BOC}_{\sin}(n, n)$ and $\text{BOC}_{\cos}(n, n)$ as an example. From the figure, we can observe that $R_{\sin}^{k,0}(\tau)$ and $R_{\sin}^{k,2k-1}(\tau)$ and $R_{\cos}^{k,0}(\tau)$ and $R_{\cos}^{k,4k-1}(\tau)$ are symmetric with respect to $\tau = 0$ and have only a single overlapped peak at $\tau = 0$ for $\text{BOC}_{\sin}(kn, n)$ and $\text{BOC}_{\cos}(kn, n)$, respectively. Thus, if the two sub-correlations are summed, a main-peak with a larger magnitude (than that of the main-peak of a sub-correlation) is obtained without increasing the magnitudes of the side-peaks, and on the other hand, the difference between the two sub-correlations yields side-peaks only, whose magnitudes and positions are the same as those of the side-peaks in the sum of the two sub-correlations. Thus, the difference between the two sub-correlations might be used to remove the side-peaks in the sum of the two sub-correlations, leaving only the main-peak. This observation is the key motivation of the proposed scheme.

Since the side-peaks in the sum and difference of the two sub-correlations are out-of-phase and in-phase at $\tau < 0$ and $\tau > 0$, respectively, however, we cannot remove the side-peaks in the sum of the two sub-correlations completely through the subtraction between the sum and difference of

the two sub-correlations. To align the phases of the side-peaks in the sum and difference of the two sub-correlations, thus, we use the sum of the absolute values of the two sub-correlations, obtaining the side-peaks with the same slopes as those of the side-peaks in the absolute difference of the two sub-correlations. Fig. 2 shows that the subtraction of the absolute difference of the two sub-correlations from the sum of the absolute values of the two sub-correlations yields an unambiguous correlation function with a single main-peak and no side-peak.

Since the unambiguous correlation function is generated by using only two sub-correlations out of $2k(4k)$ sub-correlations, the height of the main-peak is limited to $2NT_s(NT_s)$ for $\text{BOC}_{\sin}(kn, n)$ ($\text{BOC}_{\cos}(kn, n)$) while the height of the BOC autocorrelation is $2kNT_s$. Considering that each sub-correlation has only a small portion of the total energy, we multiply the unambiguous correlation function with the BOC autocorrelation to obtain an unambiguous correlation function with higher main-peak, allowing it possible to make use of more signal energy.

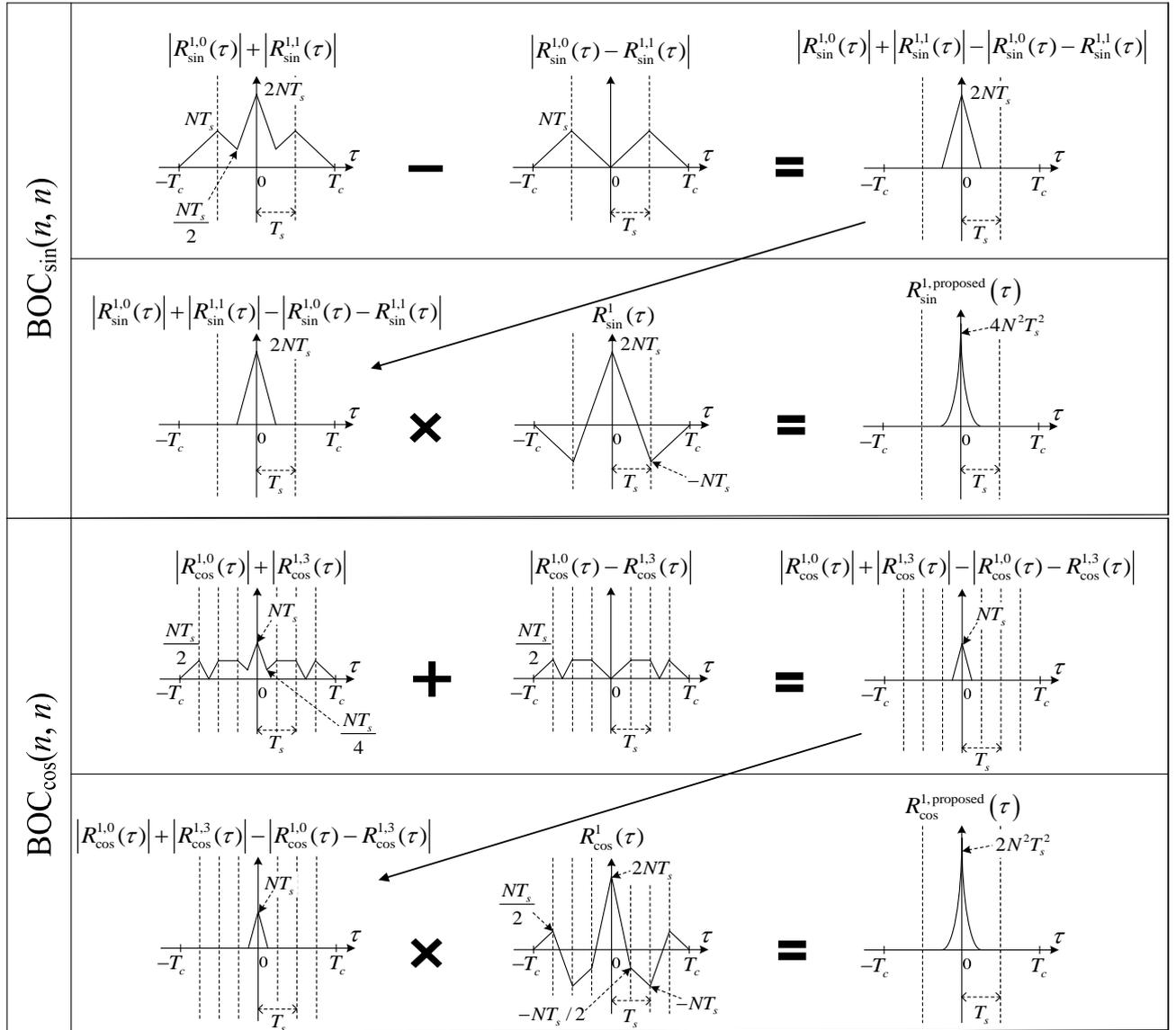
From the above discussions, the proposed unambiguous correlation function can be expressed as

$$R_{\sin}^{k,\text{proposed}}(\tau) = R_{\sin}^k(\tau) \left(\left| R_{\sin}^{k,0}(\tau) \right| + \left| R_{\sin}^{k,2k-1}(\tau) \right| - \left| R_{\sin}^{k,0}(\tau) - R_{\sin}^{k,2k-1}(\tau) \right| \right) \quad (12)$$

for $\text{BOC}_{\sin}(kn, n)$ and

$$R_{\cos}^{k,\text{proposed}}(\tau) = R_{\cos}^k(\tau) \left(\left| R_{\cos}^{k,0}(\tau) \right| + \left| R_{\cos}^{k,4k-1}(\tau) \right| - \left| R_{\cos}^{k,0}(\tau) - R_{\cos}^{k,4k-1}(\tau) \right| \right) \quad (13)$$

for $\text{BOC}_{\cos}(kn, n)$.


 Figure 2. The unambiguous correlation functions of the proposed scheme for $\text{BOC}_{\sin}(n, n)$ and $\text{BOC}_{\cos}(n, n)$.

IV. NUMERICAL RESULTS

In this section, the proposed unambiguous acquisition scheme is compared with the unambiguous acquisition scheme in [16] called the general removing ambiguity via side-peak suppression (GRASS) in terms of the incorrect acquisition probability and MAT. In the comparisons, we assume the following parameters: a PRN code of $T = 127$ chips, a correlation length of $N = 127$ chips, and a search step size of T_s and $T_s/2$ for the sine-phased and cosine-phased BOC signals, respectively. For the MAT simulation, the penalty time and probability of false alarm are set to $4T$ and 10^{-3} , respectively.

Fig. 3 shows the incorrect acquisition probabilities for the proposed, GRASS, and traditional BOC schemes as a

function of the carrier-to-noise ratio (CNR) for $\text{BOC}_{\sin}(kn, n)$ and $\text{BOC}_{\cos}(kn, n)$ when $k = 1$ and 2, where the incorrect acquisition probability is defined as the probability that any one of correlation values at $\tau \neq 0$ exceeds the main-peak magnitude of the correlation function, and the CNR is defined as P/N_0 (dB-Hz). The performance of the GRASS scheme is not shown for $\text{BOC}_{\cos}(kn, n)$ since it is dedicated to the sine-phased BOC signals only. As shown in the figure, the proposed scheme yields an improvement over the GRASS scheme, and the improvement becomes larger as the value of k increases. On the other hand, the performance of the proposed scheme is slightly inferior to that of the traditional BOC scheme at relatively low CNRs (less than about 35 dB-Hz and 38 dB-Hz for

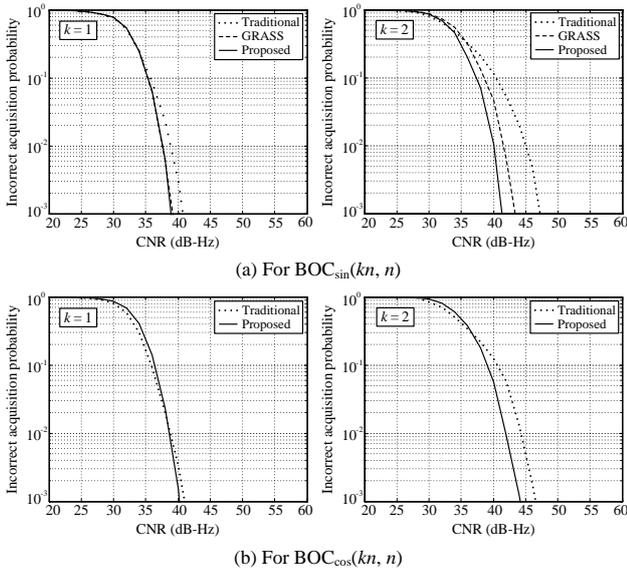


Figure 3. False acquisition probabilities of the proposed, GRASS, and traditional BOC schemes for $BOC_{\sin}(kn, n)$ and $BOC_{\cos}(kn, n)$ when $k=1$ and 2.

$BOC_{\sin}(kn, n)$ and $BOC_{\cos}(kn, n)$, respectively), which can be explained as follows. A noise enhancement arises due to several absolute operations involved in the proposed scheme and its effect becomes more pronounced at relatively low CNRs, and eventually, overwhelms that of the side-peak removal of the proposed scheme, thus degrading the performance of the proposed scheme at relatively low CNRs. However, as the value of CNR increases, the side-peak removal effect would become predominant over the noise enhancement effect, thus resulting in a performance improvement of the proposed scheme over the traditional ambiguous BOC scheme. Furthermore, the improvement increases as the value of k increases.

Fig. 4 shows the MAT performances of the proposed, GRASS, and traditional BOC schemes as a function of CNR for $BOC_{\sin}(kn, n)$ and $BOC_{\cos}(kn, n)$ when $k=1$ and 2. As shown in the figure, the proposed scheme outperforms the GRASS scheme in terms of MAT. Although the traditional BOC scheme has a slightly better performance than that of the proposed scheme at high CNR, the performances of the proposed and traditional BOC schemes are both good at high CNR, and thus, the small performance difference between the two schemes is insignificant at high CNR.

In addition, let us add a brief discussion on the computational complexity of the proposed and traditional schemes. From Fig. 2, we can see that the proposed scheme additionally requires three addition, one multiplication, and three absolute operations compared with the traditional scheme. It also can be seen from (10) and (11) that the sub-correlations can be obtained by collecting the correlations between the received and local BOC signals over the sub-

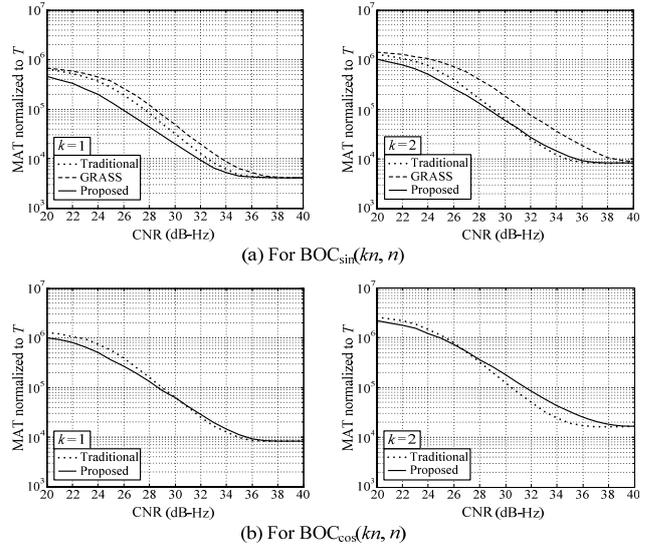


Figure 4. The mean acquisition time of the proposed, GRASS, and traditional BOC schemes for $BOC_{\sin}(kn, n)$ and $BOC_{\cos}(kn, n)$ when $k=1$ and 2.

carrier pulse duration (half duration) for $BOC_{\sin}(kn, n)$ ($BOC_{\cos}(kn, n)$) without any additional operation.

V. CONCLUSION

In this paper, we have proposed a novel unambiguous acquisition scheme for BOC signals in global navigation satellite systems. We have first analyzed the BOC autocorrelation function, showing the fact that it is made up of the sum of several sub-correlations shaped irregularly and which causes the multiple side-peaks of the BOC autocorrelation function. Then, we have proposed the unambiguous acquisition scheme based on a recombination of the sub-correlations. The proposed scheme is applicable to generic $BOC_{\sin}(kn, n)$ and $BOC_{\cos}(kn, n)$ signals, since it exploits the sub-correlations inherent in the BOC autocorrelation, regardless of the type of the BOC signal (i.e., regardless of the value of k). Finally, it has been observed that the proposed scheme removes the side-peaks completely for any sine-phased or cosine-phased BOC signal, and that it offers a performance improvement over the GRASS and traditional BOC schemes in terms of the incorrect acquisition probability and MAT.

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