

# Performance Study in Multi-rate Switching Networks with Additional Inter-stage Links

Michał Stasiak

*Chair of Communications and Computer Networks  
Poznan University of Technology  
ul. Polanka 3, room 110, 60-965 Poznan, Poland  
E-mail: michal.m.stasiak@doctorate.put.poznan.pl*

Piotr Zwierzykowski

*Chair of Communications and Computer Networks  
Poznan University of Technology  
ul. Polanka 3, room 231, 60-965 Poznan, Poland  
E-mail: piotr.zwierzykowski@put.poznan.pl*

**Abstract**—The paper presents the results of a simulation study on multi-rate three-stage Clos switching networks with additional inter-stage links. Switching networks in which a system of overflow links was provided for the first, second and the third stage are investigated. Additionally, different structures of connections between inter-stage links are discussed. The investigation has proved that an application of overflow systems is followed by a substantial decrease in the internal blocking probability in the switching network. An appropriate capacity of overflow links can even result in a virtually complete elimination of internal blocking. This paper proves that the best results are obtained with the introduction of overflow links to the first stage of the network, both in the case of point-to-point and point-to-group selection.

**Keywords**-switching networks, inter-stage links

## I. INTRODUCTION

Switching networks form a base for the operation of many systems and devices, including that of exchanges or routers that constitute nodes in telecommunications networks. It is their effectiveness in performance that is decisive in enabling the maximum amount of traffic that a network can carry. Most of switching networks used in network nodes are blocking networks. This means a loss to performance with regard to part of traffic offered to network nodes. Constructions that eliminate the phenomenon of blocking in the switching network (the so-called non-blocking networks) are also known [1], [2], [3], [4], [5]. These networks, however, require a large number of switches, which translates directly into a substantial increase in the financial outlays for the construction of such switching networks. There exist techniques for reducing blocking states in switching networks. The three fundamental techniques include: the application of additional inter-stage (overflow) links [6], call rearrangement or repacking and re-switching [5]. Call repacking and re-switching do not interfere with the actual architecture of a switching network. These are based on the introduction of special algorithms for executing procedures related to setting up connections in a required way. Due to computational complexity, solutions of this type tend to overload controlling devices while setting up connections to a much higher degree than the most commonly used random

algorithms and sequential algorithms, which is a drawback and a considerable limitation. The application of overflow links is accompanied with a slight interference into the physical architecture of a switching network and is based on an application of switches with an appropriately higher number of inputs and outputs. In the past, switching networks with overflow links were used on an industrial scale in analogue telephone exchanges manufactured by Pentaconta [6]. The exchange of the Pentaconta system was composed of two-stage switching networks in which switching devices of the first stage were connected with one another by special inter-stage links. The application of overflow links made it possible to reduce substantially (by several percent) the internal blocking probability of the exchange [7]. Later, a possibility of introducing overflow links in electronic exchanges was also considered [8], [9]. No analysis of the effectiveness of the introduction of overflow traffic has been carried out so far for multi-stage switching networks with multi-rate traffic. This paper presents the results of a simulation study of multi-rate three-stage Clos networks to which different architectures of overflow systems were applied.

The paper has been divided into 5 sections. Section 2 introduces the reader to the issues of modelling blocking and non-blocking switching networks. Section 3 discusses the architecture and the routing algorithms in the proposed switching structures with inter-stage links, while Section 4 presents the obtained results of the study. A conclusion follows in Section 5 of the paper.

## II. BLOCKING AND NON-BLOCKING SWITCHING NETWORKS

Consider the Clos three-stage switching network presented in Figure 1. The switching network is composed of  $n$  symmetrical switches  $n \times n$  of links in every stage. Each link has the capacity of  $f$  BBUs (Basic Bandwidth Units [10]). The switching network is offered multi-rate traffic composed of a number of traffic streams called traffic classes. Each traffic class is defined by parameters: call intensity, service intensity and the number of BBUs required

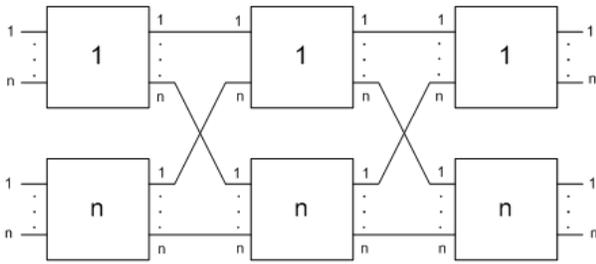


Figure 1. Three-stage Clos network v(m,n,r)

to set-up a connection. Our further assumption is that a stream of offered traffic of each class can be either an Erlang or Engset stream [11]. The Basic Bandwidth Unit is defined as the greatest common divisor of all bit rates allocated to calls of particular classes:

$$R_{BBU} = GCD[R_1, \dots, R_M], \tag{1}$$

where  $R_i$  is the bit rate allocated to each call of class  $i$ . The number of BBUs corresponding to a call of class  $i$  is defined by the following formula:

$$t_i = \left\lceil \frac{R_i}{R_{BBU}} \right\rceil, \tag{2}$$

The capacity of one free link of a switch can be also determined in BBUs:

$$f = \left\lfloor \frac{C}{R_{BBU}} \right\rfloor, \tag{3}$$

where  $C$  is the total bitrate in one link. The process of a determination of the parameters of individual call classes and the capacity of the component elements of the system in BBUs, on the basis of Equations (1)–(3), is called *bandwidth discretization* [10]. The process makes the analysis and modelling of multi-stage switching networks and many other switching systems much more easy [14], [15].

Two basic types of selections are commonly used in switching networks: point-to-point selection and point-to-group selection [12], [13]. With the point-to-point selection, the controlling algorithm determines the input stage switch at which a call of class  $i$  appeared. Then, the controlling algorithm defines a switch for the output stage that has at least one free outputs, i.e. an output link that has at least  $t_i$  free BBUs. In the next step, the algorithm tries to find a free connection path between these switches. If this is not possible, then the call is lost due to internal blocking. If the controlling algorithm indicates the absence of just one switch in the last stage with a free output, then the call is lost as well due to internal blocking. With the application of the point-to-group selection, the outputs of the switches of the last stage are divided into link groups called directions. The controlling algorithm for the network first determines the

input switch at which a call of a given class appears. Then, the controlling algorithm chooses an output switch that has a free output in the required direction. In the next step, the controlling algorithm tries to set up a connection between a given switch of the first stage and a selected switch of the outgoing stage. If there is no possibility of setting up a connection path between these switches, then the controlling algorithm determines another switch of the last stage of the network that has a free output in the demanded direction. If, again, the connection between the switches of the first and the last stages cannot be set up, then the controlling algorithm determines next switch with free outputs in the direction and another attempt at setting up a connection is made. If, during one of such attempts, a connection can be successfully set up, then the controlling algorithm proceeds to execute the connection. If, in turn, all possible attempts fail to set up a connection, then the connection is lost due to internal blocking. The occupancy of all links in a given direction is followed by a loss of a call due to external blocking.

There are two basic methods for dividing outputs of the last stage in direction [12], [14]. The first method assumes that all outputs for a given switch represent just one direction. The other method is based on a selection of one (or more) output links with the same number in each of the switches of the last stage. This paper considers the latter method for the execution of the output direction.

A measure for the number of events of blocking is the blocking probability, which can be defined as a ratio of the number of blocked connections to the total number of demands for setting up connections in the switching network. The total blocking probability  $E_T$  is the sum of the internal blocking probability  $E_{in}$  and the external blocking probability  $E_{ex}$ :

$$E_T = E_{in} + E_{ex}. \tag{4}$$

In non-blocking networks, internal blocking probabilities are equal to zero. Thus, in non-blocking networks, the following equality takes place:

$$E_T = E_{ex}. \tag{5}$$

Equation (5) means that in non-blocking networks the blocking event occurs solely in output groups of the network and depends exclusively on their capacity and offered traffic. Hence, the internal structure of a non-blocking network is "transparent" for offered traffic and the whole network can be treated - from the traffic engineering perspective - as a single-stage system, i.e. as an output group of the network. Structures of non-blocking networks are constructed based on strictly defined rules and theorems [1], [2], [3], [5]. A transformation of a given structure of a blocking network into a structure of a non-blocking network may be based on an increase in the number of switches of appropriate stages or, additionally, an increase in the number of stages in a

switching network. Special control algorithms that manage the procedures of setting up connections are also known. These algorithms lead eventually to a non-blocking property of the network [5]. From the economic point of view, non-blocking networks are far more expensive than blocking networks. Therefore, in engineering practice most solutions employ much cheaper blocking networks in which the phenomenon of internal blocking is handled in such a way as to guarantee its maximum limitation. Switching networks with a low value of the internal blocking probability (that deliver at least one order of magnitude lower than the external blocking probability) are called quasi-non-blocking networks. One of the most effective ways of diminishing the internal blocking in the switching network is the application of overflow links. Switching networks with overflow links are described in the following section.

### III. SWITCHING NETWORKS WITH ADDITIONAL INTER-STAGE LINKS

Consider any example of a stage of the switching network shown in Figure 1, in which additional inter-stage links have been introduced. Figure 2 presents two ways of setting up overflow connections in a given stage of the switching network. In the first case (Figure 2a), the presented system of overflow links (called system I) is based on an application of switches with one additional input and one additional output in a given stage. An overflow link connects a given switch with another switch, whereas the last switch is connected with the first switch. The second example (Figure 2b) shows a system of overflow links (further on called system II). In this system, switches with two additional inputs and outputs are used. Each switch is connected by one inter-stage link with a neighbouring switch. The second overflow link leads to the next switch in a row. Thus,  $i$ -switch of a given stage is connected with  $i + 1$  switch and  $i + 2$  switch of the same stage. Each of the presented two overflow systems can be applied in each stages of a three-stage Clos network. If System I is applied to the second stage of the network, then thus obtained structure will be labelled as System 1.2 (the number of system, the number of the stages).

The algorithm for setting up a connection always tries to set up a connection without overflow links first. When the connection cannot be executed, an attempt is made to effect the connection with the help of overflow links. Assume that the 1.3 structure is introduced to the switching network with point-to-point selection, i.e. a system with overflows in the third stage. If in a given state of the network a point-to-point connection with a given output of the second switch of the third stage cannot be set up, then the controlling algorithm tries to set up this connection with the first switch of the third stage that has a connection (through the overflow link) with the second switch of the third stage.

The following general algorithm for setting up connections is applied to the considered switching networks with inter-

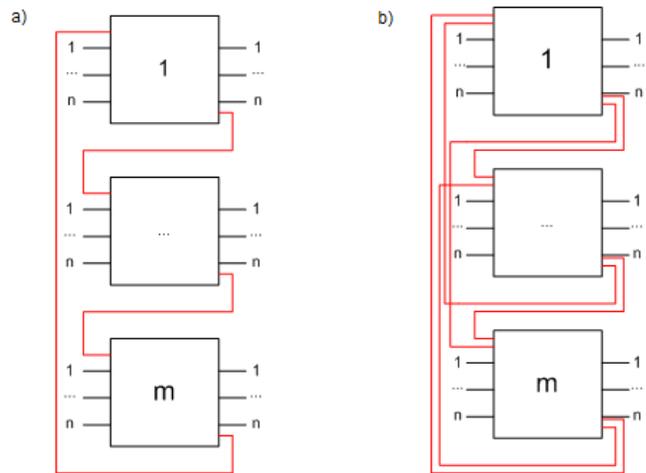


Figure 2. Overflow systems in a selected stage of the Clos network.

stage links. In the first step, the controlling device determines the input on which a new call appeared, as well as a required output in a given direction (point-to-point selection), or a demanded direction (point-to-group selection). In the next step an attempt is made to set up a point-to-point connection (or, alternatively, a point-to-group connection) without the application of overflow links. If setting up of a connection without overflow links fails to be executed, the controlling device proceeds to step three, in which all possible connection paths executed by the switches of a given stage to which the considered switch has access to through free overflow links are checked. If there are any free connection paths that use overflow links, then, on the basis of the random algorithm, one path is chosen and a connection of given type (point-to-point or point-to-group) is set up. Otherwise, a call is rejected. In order to diminish the amount of the volume of occupied resources and the complexity of the controlling algorithm for setting up a connection, one overflow link is used at the maximum. This means that if an attempt at setting up a connection with the use of overflow links is failed, then no attempts with the application of next overflow links will be made. It is assumed then, that in each attempt to set up a connection only one overflow link can be used.

### IV. NUMERICAL RESULTS

To determine traffic effectiveness of switching networks with additional inter-stage links, a simulation tool that enables to determine traffic characteristics of the networks with the required confidence interval was developed for the purposes of the study. An event planning method was used to construct the simulator [16]. Thus constructed simulator allows you to study three-stage Clos switching networks with point-to-point selection and point-to-group selection. Either of the overflow links systems described in Section 2 can be introduced to each of the stages. The network is

offered a mixture of Erlang traffic of various classes (a call stream of each of the traffic classes is a Poisson stream, whereas the service time of each class of calls is described by an exponential distribution).

The investigation involved a simulation of a three-stage Clos switching network composed of symmetrical switches  $4 \times 4$  links, each with the capacity of  $f=30$  BBUs. The number of switches of each of the stages is equal to 4 switches. A network is offered three classes of calls that demand respectively  $t_1=9$  BBUs,  $t_2=6$  BBUs, and  $t_3=2$  BBUs. It is assumed that appropriate traffic classes are offered in the following proportions:  $a_1 : a_2 : a_3 = 2 : 3 : 6$ . The simulations were carried out for such a number of series (each series includes 100,000 calls of the class with the highest demands) that guarantees 99% confidence interval defined on the basis of the  $t$ -Student distribution. This interval in the simulation experiments in question is lower by at least one order of magnitude than the average value of the simulation result. The results of the simulation are presented in relation to the volume of traffic offered to one BBU of an output link ( $a$ ). Figure 3 shows a comparison of the internal point-to-point blocking probabilities for the calls of class  $i$  in the networks 0.0 (without overflow links) and 1.1 (overflow system I in the first stage of the network).

An introduction of a simple system of overflow links leads to a decrease in the internal blocking probability in a switching network.

Figure 4 shows the changes in the internal blocking probability of a call of class 1 in the network 1.1 in relation to the changes in the capacity of the overflow link for  $a=1$  Erl/BBU. The figure indicates the exponential nature of these changes. With the capacity of the overflow link equal to the capacity of a link used in the switching network ( $f=30$ ), the internal blocking probability does not continue to decrease. It emerges from the study that for  $a < 1$  Erl/BBU, the internal blocking probability stabilizes earlier  $f < 30$ . These results are of practical significance since the conclusion that follows indicates that an application of overflow links with identical capacity as the remaining links of a switching network are sufficient for constructing effective quasi-non-blocking switching network.

Figure 4 presents the changes in the internal, external and the total blocking probability for calls of class 1 in relation to the capacity of overflow links in the network 1.1. Along with the increase of the capacity of overflow links, the internal blocking probability decreases. A decrease in the probability leads to an increase in the external blocking probability. This phenomenon can be explained in the following way. A decrease in the internal probability blocking is equivalent to an increase in call access of a given class to output groups (directions) of the switching network. Increased traffic at the outputs of the switching network results in an increase in the external blocking probability. Since the decrease in the internal blocking probability is higher than the increase in

the external blocking probability, then, as a result, we obtain a decrease in the total blocking probability.

A system of overflow links can be applied in any stage of the three-stage Clos network. A choice of a stage of the network and an overflow system in the switching network has a substantial influence on the internal blocking probability. Hence, the study included an analysis of System I and System II used in each of the stages of the switching network. Relevant investigations were carried out for the point-to-point selection and the point-to-group selection. Figure 5 shows exemplary juxtaposition and comparison of the point-to-point internal blocking probability for different structures of switching networks with overflow in line with the notation adopted in Section 2. The probability is expressed as a percentage of the value of the internal blocking probability in a network without overflow links, with the assumption that the average traffic for a BBU is equal to  $a=1$  Erl. Figure 5 presents the results ordered according to the level of the decrease in the blocking probability.

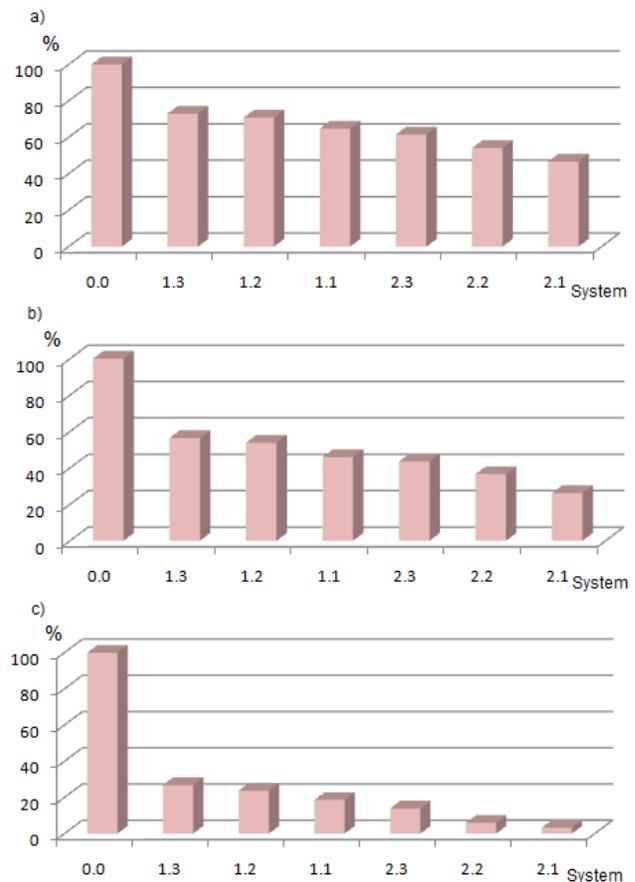


Figure 5. Internal blocking probability in a three-stage Clos network for the point-to-point selection: a) class 1, b) class 2, c) class 3

As the results of the study show, the introduction of overflow links in the first stage of the switching network

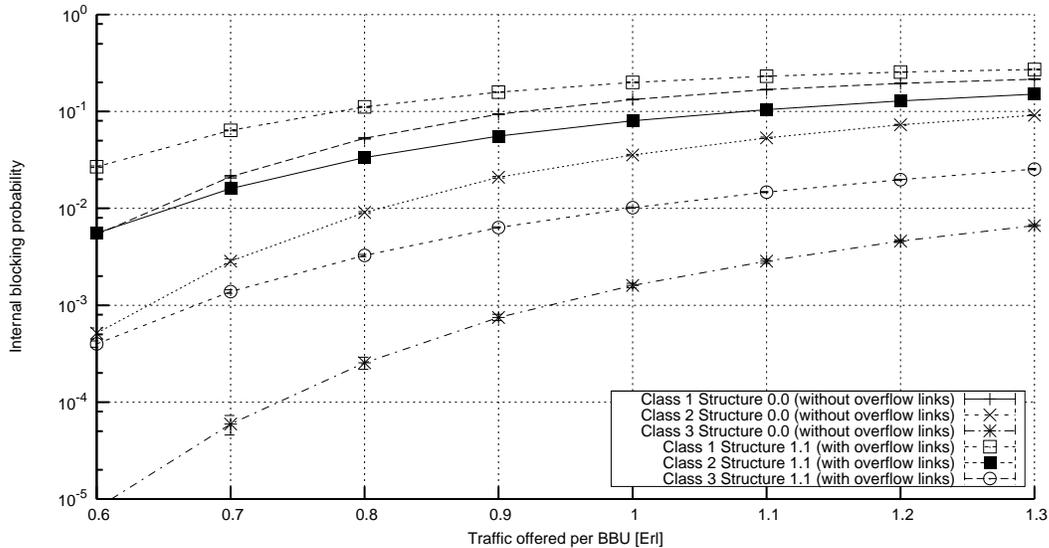


Figure 3. Internal blocking probability in a three-stage Clos network for the point-to-point selection with and without overflow links

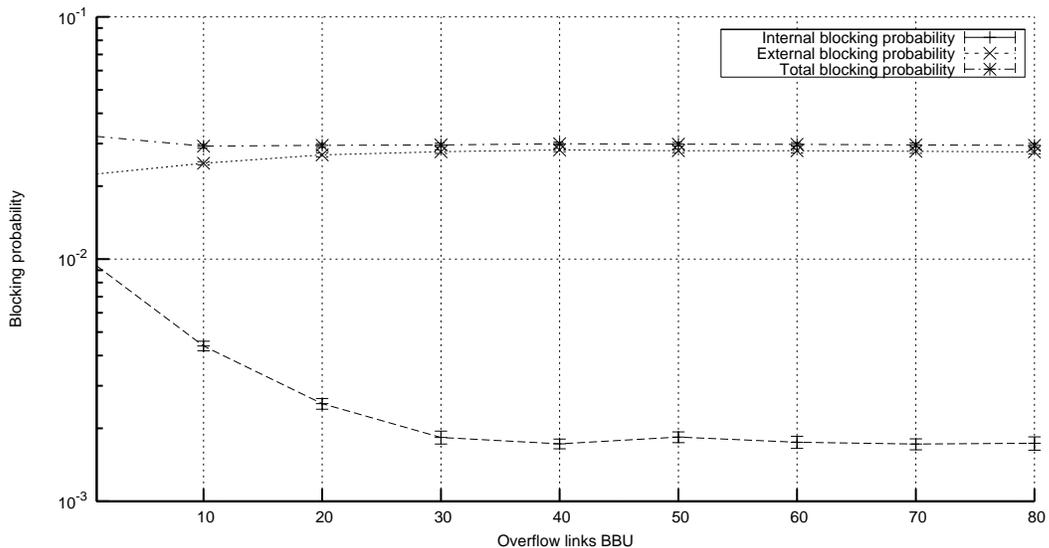


Figure 4. Dependence between internal, external and the total blocking probability and the capacity of overflow links in the network of System 1 for the first traffic class ( $t_1 = 10$  BBUs), point-to-point selection

is the most effective. This phenomenon can be described and interpreted combinatorially. The application of overflow links in the first stage is followed by the highest number of potentially possible connection paths for connections, whereas an increase in the number of potentially possible connection paths in the third stage, resulting from the application of overflow links, is the lowest. The presented results constitute only a small fragment of the more complex study carried out by the authors. Due to the limited space of the paper, the figures show only the results yielded for the point-to-point selection. With the case of the point-to-group selection, however, similar changes occur in the blocking probability and can be compared to those obtained for the

point-to-point selection, though the extent of these changes is slightly lower. The phenomenon is related to the particular way of operation of the point-to-point selection, in which there are more potential connection paths for a point-to-direction (a number of points) to be set up than in the case of setting up a point-to-point connection.

The results presented in the paper were limited to switching networks  $4 \times 4$ . The authors also examined switching networks with a much larger size, i.e.  $16 \times 16$ . The obtained results confirm that an introduction of overflow links in the first stage of the switching network also in that case is the most effective.

## V. CONCLUSIONS

The paper discusses the results of a simulation study of Clos networks with the application of inter-stage links. On the basis of the obtained results it can be unequivocally stated that the most optimal solution is the application of overflow structures in the first stage of the network. All the simulations carried out by the authors confirm the highest percentage decrease in the blocking probability recorded in this particular configuration.

The paper also includes the results of an investigation into the dependencies between the internal blocking probability and the capacity of overflow links. It is then proved that the value of the capacity of an overflow link that is equal to the capacities of links in a switch of the switching network can virtually eliminate internal blocking. It emerges from the studies carried out by the authors that in switching networks with higher capacities the actual operation of the system of overflow links is even more effective, i.e. a lower number of overflow links (with lower capacities) appears to be sufficient to eliminate the phenomenon of internal blocking in the network. Given the low cost of constructing a system of overflow links, i.e. first stage switches with an increased number of input and output links, the application of the solution in practice (in construction of quasi-non-blocking networks) seems to be justified and well-grounded.

## REFERENCES

- [1] C. Clos, A study of non-blocking switching networks, *Bell System Technical Journal*, vol. 32, No. 2, 1953, pp.406-424.
- [2] F.K. Hwang, Three-stage multiconnection network which are nonblocking in the wide sense. *Bell System Technical Journal*, vol. 58, No. 10, 1979, pp. 2183-2187.
- [3] A. Jajszczyk ,On nonblocking switching networks composed of digital symmetrical matrices. *IEEE Transactions on Communications*, vol. 31, No. 1, 1983, pp. 2-9.
- [4] W. Kabaciński, On nonblocking switching networks for multi-channel connections. *IEEE Transactions on Communications*, vol. 43, No. 2/3/4, 1995, pp. 222-224.
- [5] W. Kabaciński, Nonblocking Electronic and Photonic Switching Fabrics, Springer, 2005.
- [6] R. Fortet Ed. ,Calcul d'organe, Systeme Pentaconta, L.M.T Paris, 1961.
- [7] M. Stasiak, Computation of the probability of losses in commutation systems with mutual aid selectors. *Rozprawy Elektrotechniczne*, Journal of the Polish Academy of Science, Vol. XXXII, No. 3, 1986, pp. 961-977 (in polish)
- [8] H. Inose, T. Saito, M. Kato, Three-stage time-division switching junctor as alternate route. *Electronics letters*, vol 2, No. 5, 1966, pp.78-84
- [9] L. Katzschner, W. Lorcher, H. Weisschuh, On a experimental Local PCM Switching Network, *Proc. International Seminar on Integrated System for Speech, Video and Data Communication*, Zurich, 1972, pp. 61-68.
- [10] J. Roberts, V. Mocchi, and I. Virtamo, Eds., Broadband Network Teletraffic, *Final Report of Action COST 242*, Berlin: Commission of the European Communities, Springer, 1996.
- [11] V.B. Iversen, Engineering Handbook, *ITU-D, SG-2*, Geneva, 2002.
- [12] A. Eldin, Automatic telephone exchanges based on the link connection principle, *L. M. Ericsson*, Stockholm, 1969.
- [13] A. Lotze, A. Roder, G. Thierer, PCM- charts, *Published by Institute of Switching and Data Technics*, University of Stuttgart, 1979.
- [14] M. Stasiak, Combinatorial considerations for switching systems carrying multi-channel traffic streams. *Annals of Telecommunications*, vol. 51, No. 11-12, 1996, pp. 611-625.
- [15] M. Stasiak, Effective availability models for switching networks. Published by Poznań University of Technology, Poznań, 2005, pp. 1-260 (in polish)
- [16] J. Tyszer, *Object-Oriented Computer Simulation of Discrete-Event Systems*, Springer, 1999.