# Switching Networks with Hysteresis Mechanism 

Mariusz Głąbowski, Maciej Sobieraj, Maciej Stasiak, and Joanna Weissenberg<br>Chair of Communication and Computer Networks, Poznan University of Technology<br>ul. Polanka 3, 60-965 Poznan, Poland<br>Email: mglabows@et.put.poznan.pl, maciej.sobieraj@put.poznan.pl, stasiak@et.put.poznan.pl, joanna@weissenberg.pl


#### Abstract

The paper proposes a new analytical method for determining traffic characteristics of multi-stage switching networks implementing threshold mechanism with hysteresis. The proposed method makes it possible to calculate point-togroup blocking probability in switching networks with multirate traffic streams. The basis of the presented method is the effective availability concept. The results of analytical calculations in two switching networks are compared with simulation data.


Keywords-switching networks; multi-rate traffic; threshold mechanism with hysteresis;

## I. Introduction

Due to an enormous increase in the amount of information that the public has access to in an open communications network, attributed to the increasing popularity of multimedia services, the Internet is on the verge of collapse from congestion of available resources (bit rates) [1]. This situation has been triggered by an ever-increasing number of demanded network resources by services that can be differentiated into three traffic classes - considered in traffic theory, i.e. streaming, adaptive and elastic traffic classes [2]. In the streaming service, a demanded amount of resources is fixed and cannot undergo any changes with an increase of the load of the system. The adaptive class allows for decreasing admitted resources to accommodate new calls, with their service time unchanged. As far as the elastic class is concerned, the primary task - regardless of the available resources - is to execute a given service call. Therefore, along with an increase in the load of the system, it is possible to decrease the amount of the resources allocated to new calls with a simultaneous lengthening of the service time.

Any network load balancing solutions adopted by telecommunications operators to counteract the growing congestion require performing appropriate traffic analyses of the operating network systems and their optimal shaping and dimensioning. Until recently, the main emphasis has been put on the traffic analysis of links between telecommunications nodes. Increased capacity of these links and the introduction of technically advanced traffic control mechanisms have effected, however, in a situation in which switching structures of network nodes have again become topical and relevant. One of the most frequently used switching structures, in networks with electronic and optical switching alike, are multi-stage Clos networks. The
literature considers Clos networks both within the context of applicable transmission techniques [1], algorithms for the selection of connection paths [3] and a modification to the structure itself [4]. Nowadays, in order to fully determine the influence of elastic and adaptive services on the effectiveness of a telecommunications network it is thus necessary to work out analytical methods that would enable us to model traffic characteristics of switching networks that perform and execute this type of service.

The present article proposes an analytical method for a determination of point-to-group blocking probability in switching networks carrying multi-rate traffic streams generated by streaming, adaptive and elastic services. For this purpose, a threshold mechanism with hysteresis is proposed, i.e. a system in which the decompression limit ("decompression limit" - the occupancy state below which the amount of assigned resources is equal to the initial values) is lower (often considerably) than the compression limit ("compression limit" - the occupancy state above which the amount of assigned resources decreases). This type of threshold mechanism allows to limit the number of changes in the amount of the assigned resources and improves the operation stability of the switching nodes. In the work on threshold systems hitherto reported and published, the focus is mainly on single resources (links).

The first attempt of elaborating the method devoted to determining blocking probability in switching networks with threshold mechanism was taken in [5]. In [5] only the basic model of threshold mechanism was considered - the threshold mechanism without hysteresis - it is the model assuming that the upper limit (compression limit) is equal to the lower limit (decompression limit). However, such an operation of the threshold mechanism causes frequent changes in the amount of assigned resources for calls, especially when the average load of the system is closed to the threshold state (the upper limit is equal to the lower limit) of the system.

The remaining part of the paper is organized as follows. In Section II the basic assumptions related to switching networks are presented. In Section III the implementation of the threshold mechanisms with hysteresis in the switching networks is proposed. The models of inter-stage links and outgoing links are described in Section IV. The proposed method of determining the blocking probability in


Figure 1. 3-stage switching network
the switching networks implementing threshold mechanism with hysteresis is presented in Section V. The results of analytical modeling of the considered switching networks are compared with simulation data in Section VI. Section VII concludes the paper.

## II. Basic Assumptions

In the paper the switching networks with Clos structure are considered. The model of 3-stage switching network with multi-rate traffic is presented on Fig. 1. Each of inter-stage links has the capacity equal to $f$ BBUs (Basic Bandwidth Units) and outgoing transmission links create link groups called directions. One of typical methods for realization outgoing directions is presented is Fig. 1; each direction $v$ has one outgoing link $v$ from each the last-stage switch. Each switch has $v$ inputs and $v$ outputs.

The switching network is offered $m$ Erlang traffic streams generated by Poisson call streams. Poisson call streams are described by the arrival rates $\lambda_{1}, \ldots, \lambda_{i}, \ldots, \lambda_{m}$ of calls of particular traffic classes. A class $i$ call requires $t_{i}$ BBUs to set up a connection. The holding (service) time for the calls of particular classes has an exponential distribution with the parameters: $\mu_{1}, \ldots, \mu_{i}, \ldots, \mu_{m}$. The intensity $\lambda_{i}$ of Poisson call stream of class $i$ does not depend on the occupancy state of the system. Thus, the mean traffic $A_{i}$ offered to the system by Poisson calls of class $i$ is equal to $\lambda_{i} / \mu_{i}$.

The considered switching network works with point-togroup selection. Following the control algorithm of this selection, the control device of the switching network determines the first stage switch, on the incoming link of which a class $i$ call appears. Then, the control system finds the last-stage switch having an outgoing link with at least $t_{i}$ free BBUs in the required direction. Next, the control device tries to find the connection path between the first-stage and the last-stage switch. Existence of the connection path causes realization of the connection. In opposite case, the control system begins the second attempt to set up a connection. If the connection path cannot be found during the last try $v$
(the number of possible tries equals the number of links in the required direction), the call is lost as the result of the internal blocking. If each last-stage switch does not have $t_{i}$ free BBUs in the required direction, the call is lost because of the external blocking.

## III. Switching Networks with Hysteresis

The losses in the switching networks occur as the result of the internal blocking or the external blocking. In order to shape the dependencies between the blocking probabilities of various traffic classes it is possible to apply the threshold mechanism with hysteresis to the considered switching networks. The threshold mechanism with hysteresis can be considered as a realization of the Call Admission Control (CAC) function.

In the paper the threshold mechanism with hysteresis is applied only to the outgoing links forming the outgoing directions of the switching network. The application of this mechanism allows to adapt the traffic parameters of carried traffic classes to the occupancy state of a system. It should be noticed that the introduction of threshold mechanism to the outgoing links has an indirect impact on traffic characteristics of the internal links due to the decrease of the number of assigned BBUs for calls accepted in pre-threshold area and post-threshold area.

According to the adopted method of switching networks modeling, in the paper it is assumed that the interstage links can be modeled by the full-availability group [6] while the outgoing links (outgoing directions) can be modeled by the limited-availability group [6], presented in the next section of the paper.

## IV. Switching Network's Links with Hysteresis

## A. Model of Inter-stage Links

Let us consider a threshold model with hysteresis of the full-availability group with multi-rate traffic, the socalled FAGTH model (Full-availability Group Threshold with Hysteresis). The capacity of the system is equal to $V$ BBUs. Let us assume that set $\mathbb{T}$ contains the traffic classes, selected from all $m$ traffic classes offered to the switching network, to which the threshold mechanism with hysteresis has been applied. Let us assume further that two thresholds $Q_{1}$ and $Q_{2}$ are introduced $\left(Q_{2}<Q_{1}\right)$ for calls in set $\mathbb{T}$ (the threshold is the defined occupancy state of the system, determined by the number of busy BBUs). In the instance of an increase in the load of the system above the pre-defined first threshold $Q_{1}$ a decrease in the number of assigned BBUs to calls of belonging to set $\mathbb{T}$ ensues, and the average holding time of the calls may be increased (the holding time changes in the case of elastic services and remains unchanged in the case of adaptive services). However, when the load of the system decreases below the second threshold $Q_{2}$, an increase to the default values of the number of demanded BBUs by calls belonging to set $\mathbb{T}$ ensues, and


Figure 2. Fragment of two Markov chains
the average holding time of the call may be decreased. The service processes of the considered system can be presented as two Markov chains presented in Figure 2. Analyzing both the Markov chains of the service process occurring in the considered system, one in each direction, we can observe that the occupancy distribution in the inter-threshold area can be approximated by the following weight distribution:

$$
\begin{equation*}
\left[P_{n}\right]_{Q, V}=P_{I}\left[P_{n}\right]_{Q_{1}, V}+P_{I I}\left[P_{n}\right]_{Q_{2}, V} \tag{1}
\end{equation*}
$$

where $P_{I}$ and $P_{I I}$ are weights which describe the probabilities that the system is in the occupancy states belonging to inter-threshold area as well as the passage direction in inter-threshold area, while the distributions $\left[P_{n}\right]_{Q_{1}, V}$ and $\left[P_{n}\right]_{Q_{2}, V}$ are the occupancy distributions determined for the systems in which single threshold $Q_{1}$ and $Q_{2}$ are introduced, respectively (i.e., in the systems with thresholds without hysteresis). $P_{n}$ determines the probability of occupancy state $n$, i.e., the probability of $n$ BBUs being busy.

Probabilities $P_{I}$ and $P_{I I}$ can be determined on the basis of passage probabilities described for state $n$ of the Markov chain (Fig. 2):

$$
\begin{align*}
P_{I} & =\frac{\sum_{i=1}^{m} A_{i} t_{i, 0}}{\sum_{i=1}^{m} A_{i} t_{i, 0}+w}  \tag{2}\\
P_{I I} & =\frac{n}{\sum_{i=1}^{m} A_{i} t_{i, 0}+w} \tag{3}
\end{align*}
$$

where $t_{i, 0}$ is the number of BBUs assigned to class $i$ calls in the first threshold area (in the occupancy states lower than $Q_{1}$ when the load of the system increases - Fig. 2a) and $w$ is determined by (4).

In Equations (2) and (3) it is assumed that the probability that the system is being in the given occupancy state depends on the sum of effective traffics ( $\sum_{i=1}^{m} A_{i} t_{i, 0}$ ) offered to the system and on the occupancy state of the system ( $\sum_{i=1}^{m} y_{i}\left(n+t_{i, 0}\right) t_{i, 0}$ ), where $y_{i}$ determines the average number of class $i$ calls being serviced in state $n$. It means, that the probability $P_{I}$ - the increase in the load of the system to the state of $n$ BBUs being busy - is directly proportional to the sum of effective traffics offered to the system. However, probability $P_{I I}$ - the decrease in the load of the system to the state of $n$ BBUs being busy - is directly proportional to the sum of effective traffics serviced in state $n$ (Fig. 3). The values of these probabilities, determined for


Figure 3. Fragment of Markov chain for state $n$
the states belonging to inter-threshold area, differ from each other slightly. Consequently, in order to determine the values $P_{I}$ and $P_{I I}$ state $w$ between states $Q_{1}$ and $Q_{2}$ has been selected:

$$
\begin{equation*}
w=\left\lfloor\left(Q_{1}-Q_{2}\right) / 2\right\rfloor+1 \tag{4}
\end{equation*}
$$

In order to determine the weighted threshold distribution let us determine the occupancy distributions in singlethreshold (without hysteresis) systems, i.e., the distributions $\left[P_{n}\right]_{Q_{1}, V}$ and $\left[P_{n}\right]_{Q_{2}, V}$. First, let us consider the system with single threshold equal to $Q_{1}$ introduced for traffic classes from set $\mathbb{T}$. According to [2], [7]:

$$
\begin{equation*}
n\left[P_{n}\right]_{Q_{1}, V}=\sum_{i=1}^{m} \sum_{q=0}^{1} \sigma_{i, q}^{I}\left(n-t_{i, q}\right) A_{i, q} t_{i, q}\left[P_{n-t_{i, q}}\right]_{Q_{1}, V}, \tag{5}
\end{equation*}
$$

where:

$$
\begin{align*}
& \sigma_{i, 0}^{I}(n)= \begin{cases}1 & \text { for } \quad i \notin \mathbb{T}, \\
1 & \text { for } \quad i \in \mathbb{T} \wedge n \leq Q_{1}, \\
0 & \text { for } \quad i \in \mathbb{T} \wedge n>Q_{1},\end{cases}  \tag{6}\\
& \sigma_{i, 1}^{I}(n)=\left\{\begin{array}{lll}
0 & \text { for } & i \notin \mathbb{T}, \\
0 & \text { for } & i \in \mathbb{T} \wedge n \leq Q_{1}, \\
1 & \text { for } & i \in \mathbb{T} \wedge n>Q_{1} .
\end{array}\right. \tag{7}
\end{align*}
$$

For all states higher than $Q_{1}$ (Fig. 2a), the number of assigned BBUs for class $i$ calls decreases from $t_{i, 0}$ to $t_{i, 1}$.

The mean number of given class calls being serviced in particular occupancy states (threshold area $q$ ) of the system (the so-called reverse transition rates) can be determined on the basis of the local equilibrium equations:

$$
n_{i, q}^{I}(n)=\left\{\begin{array}{c}
A_{i, 0} \sigma_{i, 0}^{I}\left(n-t_{i, 0}\right)\left[P_{n-t_{i, 0}}\right]_{V} /\left[P_{n}\right]_{V}  \tag{8}\\
\text { for } q=0 \wedge n \leq Q_{1}+t_{i, 0}, \\
A_{i, 1} \sigma_{i, 1}^{I}\left(n-t_{i, 1}\right)\left[P_{n-t_{i, 1}}\right]_{V} /\left[P_{n}\right]_{V} . \\
\text { for } q=1 \wedge n>Q_{1}+t_{i, 1} .
\end{array}\right.
$$

Subsequently, let us consider now the other system in which for classes from set $\mathbb{T}$ also a single threshold, equal to $Q_{2}$ has been introduced. Analogously as in the case of
the first system:

$$
\begin{equation*}
n\left[P_{n}\right]_{Q_{1}, V}=\sum_{i=1}^{m} \sum_{q=0}^{1} \sigma_{i, q}^{I I}\left(n-t_{i, q}\right) A_{i, q} t_{i, q}\left[P_{n-t_{i, q}}\right]_{Q_{1}, V} \tag{9}
\end{equation*}
$$

where:

$$
\begin{align*}
\sigma_{i, 0}^{I I}(n) & = \begin{cases}1 & \text { for } \quad i \notin \mathbb{T}, \\
1 & \text { for } \quad i \in \mathbb{T} \wedge n \leq Q_{2}, \\
0 & \text { for } \\
i \in \mathbb{T} \wedge n>Q_{2},\end{cases}  \tag{10}\\
\sigma_{i, 1}^{I I}(n) & =\left\{\begin{array}{lll}
0 & \text { for } \quad i \notin \mathbb{T}, \\
0 & \text { for } & i \in \mathbb{T} \wedge n \leq Q_{2}, \\
1 & \text { for } & i \in \mathbb{T} \wedge n>Q_{2} .
\end{array}\right. \tag{11}
\end{align*}
$$

When the load of the system decreases (the system in occupancy states below state $Q_{2}$ ) (Fig. 2b), the assigned number of BBUs for calls from set $\mathbb{T}$ increases from $t_{i, 1}$ to the initial value $t_{i, 0}$.

The mean number of class $i$ calls being serviced in particular states of the system with threshold $Q_{2}$ can be determined analogously to the system with threshold $Q_{1}$ :

$$
n_{i, q}^{I I}(n)=\left\{\begin{array}{c}
A_{i, 0} \sigma_{i, 0}^{I I}\left(n-t_{i, 0}\right)\left[P_{n-t_{i, 0}}\right]_{V} /\left[P_{n}\right]_{V}  \tag{12}\\
\text { for } q=0 \wedge n \leq Q_{2}+t_{i, 0}, \\
A_{i, 1} \sigma_{i, 1}^{I I}\left(n-t_{i, 1}\right)\left[P_{\left.n-t_{i, 1}\right]_{V} /\left[P_{n}\right]_{V}} \quad \text { for } q=1 \wedge n>Q_{2}+t_{i, 1} .\right.
\end{array}\right.
$$

The blocking probability for class $i$ calls can be determined as the sum of the probabilities of the states in which the system cannot admit a new class $i$ call:

$$
E_{i}=\left\{\begin{array}{cl}
\sum_{n=V-t_{i, 0}+1}^{V}\left[P_{n}\right]_{Q, V} & \text { for } i \notin \mathbb{T},  \tag{13}\\
\sum_{n=V-t_{i, 1}+1}^{V}\left[P_{n}\right]_{Q, V} & \text { for } i \in \mathbb{T},
\end{array}\right.
$$

where the occupancy distribution $\left[P_{n}\right]_{Q, V}$ is calculated on the basis of (1).

## B. Model of Outgoing Links

Since the outgoing directions of the considered switching networks are modeled as the limited availability groups let us consider now the so-called threshold model of limitedavailability group with hysteresis, i.e., LAGTH (Limited Availability Group Threshold with Hysteresis) model. Let us remind that the limited availability group is the model of systems consisting of $v$ identical separated transmission links. Each link has the capacity equal to $f$ BBUs [6]. Thus, the total capacity of the system $V$ is equal to $V=v f$ BBUs. The system services a call - only when this call can be entirely carried by the resources of an arbitrary single link.

In the paper it is assumed the occupancy distribution in the limited-availability group with the threshold mechanism with hysteresis can be calculated on the basis of the appropriately modified generalized Kaufman-Roberts recursion [8]. For taking into consideration the influence of the specific
structure of the limited-availability group on the process of determination of occupancy distribution using the generalized Kaufman-Roberts recursion the using of conditional coefficients of passing $\sigma_{i, \mathrm{LAG}}(n)$, was proposed. The value of parameter $\sigma_{i, \mathrm{LAG}}(n)$ can be determined as follows [6]:

$$
\begin{equation*}
\sigma_{i, \mathrm{LAG}}(n)=\frac{F(V-n, v, f, 0)-F\left(V-n, v, t_{i}-1,0\right)}{F(V-n, v, f, 0)} \tag{14}
\end{equation*}
$$

where $F(x, v, f, t)$ is the number of arrangement of $x$ free BBUs in $v$ links, calculated with the assumption that the capacity of each link is equal to $f$ BBUs and each link has at least $t$ free BBUs:

$$
\begin{align*}
& F(x, v, f, t)= \\
& \sum_{r=0}^{\left\lfloor\frac{x-v t}{f-t+1}\right\rfloor}(-1)^{r}\binom{v}{r}\binom{x-v(t-1)-1-r(f-t+1)}{v-1} . \tag{15}
\end{align*}
$$

Let us observe that in the case of the considered LAGTH model the operation of the threshold mechanism introduces additional dependence between the traffic stream in the system and the current state of the system. This dependence can be taken into consideration by the introduction of the threshold coefficient of passing $\sigma_{i, q}(n)$ to each of the thresholds $q$. The threshold mechanism is introduced to the group regardless of its structure, what allows us to carry on with a product-form determination of the total coefficient of passing in the limited-availability group $\sigma_{i, q, \text { Total }}(n)$ :

$$
\begin{align*}
& \sigma_{i, q, \text { Total }}^{I}(n)=\sigma_{i, q}^{I}(n) \cdot \sigma_{i, q, \mathrm{LAG}}^{I}(n),  \tag{16}\\
& \sigma_{i, q, \text { Total }}^{I I}(n)=\sigma_{i, q}^{I I}(n) \cdot \sigma_{i, q, \mathrm{LAG}}^{I I}(n) . \tag{17}
\end{align*}
$$

For determining basic traffic characteristics in considered system we can used, appropriately adapted, FAGTH method. The modification of FAGTH method consists in regarding the coefficients of passing defined by (16) and (17) during the occupancy distribution calculation.

## V. Analytical Model of Switching Networks with Hysteresis

## A. Effective availability in switching networks

Multi-service switching networks were the subject of many analysis [6], [9]-[11]. At present, for blocking probability determination in the considered switching networks the well-proven methods of the so-called effective availability [6], [11], [12] are usually applied. According to the main idea of the effective availability methods [6], the calculation of blocking probability in switching networks with multirate traffic comes down to the calculation of the blocking in an equivalent network carrying a single-rate traffic. Each link of equivalent network is treated as a single-channel link with a fictitious load $e_{l}(i)$ equal to blocking probability for a class $i$ stream in a link of real switching network between section $l$ and $l+1$. This probability can be calculated on
the basis of the blocking probability in the full-availability group implementing threshold algorithm with hysteresis (Formula (13)).

The effective availability in a real $z$-stage switching network is equal to the effective availability in an equivalent switching network and can be determined by the formula derived in [6]:

$$
\begin{align*}
d(i)=\left[1-\pi_{z}(i)\right] & v \\
& +\pi_{z}(i) \eta Y_{1}(i)+  \tag{18}\\
& +\pi_{z}(i)\left[v-\eta Y_{1}(i)\right] w_{z}(i) \sigma_{z}(i)
\end{align*}
$$

where:
$d(i)$ - the effective availability for the class $i$ traffic stream in an equivalent network,
$\pi_{z}(i)$ - the probability of non availability of a given last stage switch for the class $i$ connection. $\pi_{z}(i)$ is the probability of an event where the class $i$ connection path cannot be set up between a given first-stage switch and a given laststage switch. Evaluation of this parameter is based on the channel graph of the equivalent switching network,
$v$ - the number of outgoing links from the first stage switch, $Y_{1}(i)$ - the average value of the fictitious traffic served by the switch of the first stage:

$$
\begin{equation*}
Y_{1}(i)=v e_{1}(i), \tag{19}
\end{equation*}
$$

$\eta$ - a portion of the average fictitious traffic from the switch of the first stage which is carried by the direction in question; if the traffic is uniformly distributed between all $h$ directions, we obtain $\eta=1 / h$,
$w_{z}(i)$ - the fictitious traffic carried by a single input of the equaivalent switching network, equal to the blocking probablity $e_{z}(i)$ of class $i$ calls in the corresponding link in the real switching network, calculated according to FAGTH method,
$\sigma_{z}(i)$ - the so-called secondary availability coefficient [6] which is the probability of an event in which the connection path of the class $i$ connection passes through directly available switches of intermediate stages:

$$
\begin{equation*}
\sigma_{z}(i)=1-\prod_{r=2}^{z-1} \pi_{r}(i) \tag{20}
\end{equation*}
$$

## B. Distribution of available links

In effective availability methods, the second element essential for determining blocking probability in switching networks is so-called distribution of available links [6]. This distribution determines the probability $P(i, s)$ of an event in which each of arbitrarily chosen $s$ links can carry the class $i$ calls:

$$
\begin{equation*}
P(i, s)=\sum_{n=0}^{V}\left[P_{n}\right]_{V} P(i, s \mid V-n) \tag{21}
\end{equation*}
$$

where $\left[P_{n}\right]_{V}$ is the occupancy distribution in limitedavailability group with hysteresis (LAGTH method) and $P(i, s \mid x)$ is the conditional distribution of available links, which determines the probability of such an arrangement of
$x=V-n$ free BBUs in $s$ links that each of $s$ arbitrarily chosen links has at least $t_{i}$ free BBUs required for set up a connection for class $i$ call, while in each of the remaining $v-s$ links the number of free BBUs is lower than $t_{i}$ [6]:
$P(i, s \mid x)=\frac{\binom{v}{s} \sum_{w=s t_{i}}^{\Psi} F\left(w, s, f, t_{i}\right) F\left(x-w, v-s, t_{i}-1,0\right)}{F(x, v, f, 0)}$
In Equation (22) $\Psi=s f$, if $x \geq s f, \Psi=x$, if $x<s f$, $F(x, v, f, t)$ is the number of arrangement of $x$ free BBUs in $v$ links, calculated with the assumption that capacity of each link is equal to $f$ BBUs and each link has at least $t$ free BBUs.

## C. Point-to-Group Blocking Probability

Total blocking probability $E(i)$ for the class $i$ call is a sum of external $E_{\text {ex }}(i)$ and internal $E_{\text {in }}(i)$ blocking probabilities. Assuming the independence of internal and external blocking events, we obtain:

$$
\begin{equation*}
E(i)=E_{\mathrm{ex}}(i)+E_{\text {in }}(i)\left[1-E_{\mathrm{ex}}(i)\right] . \tag{23}
\end{equation*}
$$

The phenomenon of the external blocking occurs when none of outgoing links of the demanded direction of the switching network can service the class $i$ call (i.e., does not have $t_{i}$ free BBUs). The external blocking probability is equal to the blocking probability in the limited-availability group modeling the outgoing direction of the switching network and can be determined on the basis of LAGTH method.

The internal blocking probability in the considered method is determined under the following conditions:

- there are $s$ links in the require direction which can carry a class $i$ call
- there are $d(i)$ last-stage switches available for the given first-stage switch on the incoming links of which a class $i$ call appears.
For the switching network in the state described by the listed assumptions the internal point-to-group blocking phenomenon appears when all links (of the considered direction) belonging to the $d(i)$ available last-stage switches have not sufficient number of free BBUs for the class $i$ call:

$$
\begin{equation*}
E_{\text {in }}(i)=\sum_{s=1}^{v-d(i)} \frac{P(i, s)}{1-P(i, 0)}\left[\binom{v-s}{d(i)} /\binom{v}{d(i)}\right] \tag{24}
\end{equation*}
$$

where $v$ is the number of links in the given direction.

## VI. Numerical results

In order to evaluate the accuracy of the proposed analytical method of point-to-group calculation in switching networks implementing threshold mechanism with hysteresis, the results of analytical modeling has been compared with data of simulation experiments. The simulations were carried out for a typical three-stage Clos switching network considered both in electronic and optical switching (Figure 1).


Figure 4. Point-to-group blocking probability; switching network structure: $v=4, f=30$ BBUs, $V=120$ BBUs; traffic structure: class 1: $t_{1,0}=1$ $\mathrm{BBU}, \mu_{1,0}^{-1}=1$, class $2: t_{2,0}=6 \mathrm{BBUs}, \mu_{2,0}^{-1}=1, t_{2,1}=3 \mathrm{BBUs}$, $\mu_{2,1}^{-1}=2$, class 3: $t_{3,0}=10 \mathrm{BBUs}, \mu_{3,0}^{-1}=1, t_{3,1}=6 \mathrm{BBUs}, \mu_{3,1}^{-1}=$ $1.667 ; Q_{1}=90$ BBUs, $Q_{2}=60$ BBUs; $\mathbb{T}=\{2,3\}$.


Figure 5. Point-to-group blocking probability; switching network structure: $v=4, f=35$ BBUs, $V=140$ BBUs. traffic structure: class 1: $t_{1,0}=1$ $\mathrm{BBU}, \mu_{1,0}^{-1}=1$, class $2: t_{2,0}=5$ BBUs, $\mu_{2,0}^{-1}=1, t_{2,1}=3 \mathrm{BBUs}$, $\mu_{2,1}^{-1}=1.667$, class $3: t_{3,0}=8$ BBUs, $\mu_{3,0}^{-1}=1, t_{3,1}=6 \mathrm{BBUs}$, $\mu_{3,1}^{-1}=1.33$, class $4: t_{4,0}=10$ BBUs, $\mu_{4,0}^{-1}=1, t_{4,1}=8$ BBUs, $\mu_{4,1}^{-1}=1.25, Q_{1}=105$ BBUs, $Q_{2}=70$ BBUs; $\mathbb{T}=\{2,3,4\}$

The research was conducted for the multi-service switching networks to which independent classes of Erlang traffic streams were offered. The result of simulation are shown in the charts with $95 \%$ confidence intervals that have been calculated according to the $t$-Student distribution for the five series with $1,000,000$ calls of each class. The research was carried out for two different structures of switching networks servicing three (Figure 4) and four (Figure 5) traffic classes.

## VII. Conclusion

In the paper, the threshold mechanism with hysteresis for switching networks servicing a multi-rate traffic was proposed. The proposed mechanism ensures a substantial
decrease in blocking probabilities of certain traffic classes in the access to switching network resources. The analytical methods of calculations of the point-to-group blocking probabilities in the multi-service switching networks with threshold mechanism with hysteresis is also presented. The method is based on the concept of the effective availability and ensure fair accuracy of calculations.

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