Identification of Critical Nodes and Links in a Supply Chain by Robust Optimization

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***Abstract*—The impact of a supplier or transportation link breakdown in a supply chain can strongly differ depending on which nodes/links are affected. While the breakdown of producers of rarely needed products or backup suppliers might result in no or only minor repercussions, the breakdown of central suppliers or transportation links, also called critical nodes/links, can be fatal and may cause a severe delivery delay or even a complete production failure of certain product lines. Therefore, it is of high importance for a company to identify its critical nodes/links in the supply chain and take precautionary actions such as organizing additional backup suppliers or alternative ways of transportation. In this paper, we describe a novel method to identify critical nodes and links in a supply chain based on robust optimization, which has the advantage that supply chain risks are considered, and also precise risk cost estimates regarding the possible breakdown of each supplier node are provided. Finally, we demonstrate this method on an example supply chain and discuss its distribution of critical nodes and links.**

Keywords*—*supply chain management; critical nodes; critical links; robust optimization; supply chain risks.

I. Introduction

According to Craighead et al.[1], node criticality is defined as the relative importance of a given node or set of nodes within a supply chain (see Figure 1). A breakdown of a critical node has typically severe implications, such as serious delay or even a complete collapse of the production process for certain product lines, which can result in non-fulfillment of customer demand. Consequently, the affected company suffers lost revenue and faces a potential non-delivery contract penalty. Thus, it is of great importance to identify the critical nodes in the supply chain and mitigate their possible breakdown risks by implementing precautionary measures such as identifying backup suppliers.

The concept of critical nodes can also be transferred to important transportation links. A link in a supply chain denotes a certain transport mode (e.g., airplane, truck, or ship transportation) and a route between two suppliers or between a supplier and a customer. Analog to the definition of critical nodes, a critical link denotes a link that is of high importance for the total supply chain. Critical links should therefore be secured by identifying alternative means of transportation.

The rest of the paper is structured as follows. Related work is given in the upcoming section (Section II). The employed optimization model is given in Section III. In Section IV, we describe how the node criticality is assessed and discuss the obtained results. Finally, we conclude the paper with Section V where we summarize our contribution and give potential future work.

II. Related work

Zhang and Han [3] propose to use network centrality (especially degree and betweenness centrality) as indicators for the criticality of a node in a supply chain.

Gaura et al. [4] assess the criticality of a certain network node by determining the decrease in network efficiency when this node is removed from the network. The network efficiency is measured by the normalized sum of the reciprocal of graph distances between any two nodes in the network. Prior to applying their approach, nodes with low clustering indices are removed from the network, wherefore the authors termed their approach clustering-based.

The approaches described so far assess a node criticality alone by topological network measures. In contrast, Falasca et al. [2] propose to also consider throughput through the network, but fail to suggest a concrete measure. Sebouhi et al. [5] consider a node as critical, if the throughput through this node as determined by solving a linear optimization problem, exceeds a certain predefined threshold. However, this measure does not take into account the use of backup suppliers as we do here, which can de facto reduce node criticality of alternative suppliers.

There are also some existing approaches to identify critical links. Scott et al. [6] introduce the so-called Network Robustness Index (NRI), "for evaluating the critical importance of a given highway segment (i.e., network link) to the overall system as the change in travel-time cost associated with rerouting all traffic in the system should that segment become unusable." Note that the NRI only takes into account costs that are directly transportation-related but disregards repercussions of item non-delivery for downstream production processes as we considered in our proposed method.

III. Employed optimization model

Our approach is based on robust optimization, which itself is based on stochastic optimization, which again is based on a deterministic optimization model.

We describe each of these three models subsequently in the following sections starting with the most basic one.

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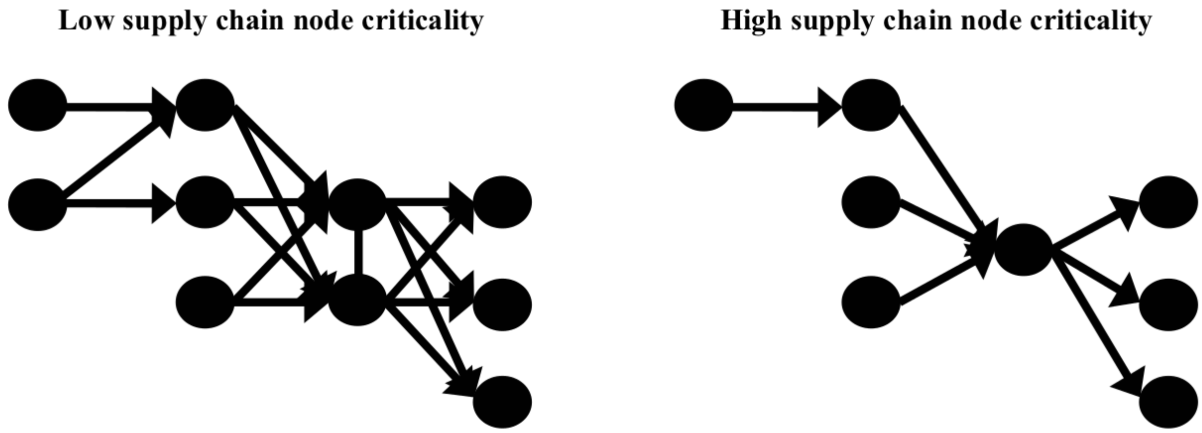

Fig. 1. Supply chain with (right) and without a critical node (left) [2].

A. Deterministic optimization model

The deterministic model disregards any potential risk for the supply chain and determines the minimum costs of the so-called "happy flow", which denotes the best-case situation that no supply chain disruption occurs. Since such a model contains no stochastic part, it can be computed very efficiently. Note that we use due to the computational complexity of the stochastic and robust model, for all of our 3 optimization models a single period of 12 months, over which we aggregate the total customer demand.

The following constants must be specified beforehand:

- d_{jz}: demand at node j for product z
- c_{ij}: cost to move one kg over one km from i to j
- pc_{iz}: cost to produce one item of product z at supplier i
- a_{xz}: number of items of product x to produce one amount of product z
- cap_{iz}: production capacity of product z on node i
- in_{iz}: initial number of items of product z contained in the inventory of supplier i
- ic_{iz}: inventory cost for storing z at location i
- $dist_{ij}$: geographical distance between node i and j
- $weight_z$: weight of product z

The following decision variables are to be determined by the optimizer:

- T_{ijz}: number of items z that are moved from location i to j
- IT_{il}: internal transfer of item l from inventory at location i
- P_{iz}: number of items z produced at supplier i
- WT_{iz}: number of items z removed from the warehouse of supplier i

Model constraints:

- $d_{jz} \leq \sum_i T_{ijz}$: demand of item z at location j is met
- $\sum_z a_{lz} P_{iz} = P_{il} + IT_{il} + \sum_k T_{kil}$: number of items l required to build items z at location i
- $P_{iz} \leq cap_{iz}$: supplier at node i can at most produce cap_{iz} items for product z
- $P_{iz} + WT_{iz} \geq \sum_j T_{ijz}$: produced + removed from the inventory of supplier $i \geq$ number of items transported from supplier i
- $IT_{iz} + WT_{iz} \leq in_{iz}$ for each item z and supplier i: inventory contents cannot become negative

The following objective is used:
Minimize $costs_{total}$ with:

$$\begin{aligned} costs_{total} := & \sum_{ijz} T_{ijz} c_{ijz} dist_{ij} weight_z \\ & + \sum_{iz} (P_{iz} pc_{iz} + in_{iz} ic_{iz}) \end{aligned} \tag{1}$$

B. Stochastic optimization model

The stochastic model takes supply chain risks into account and computes the expected value of the supply chain costs ($\mathbb{E}(C)$) determined over all generated risk scenarios. In a stochastic optimization setting, the set of risk scenarios describes the potential hazards for the whole supply chain. Hence, the nine scenarios from our case company's supply network are used as input for the stochastic optimization approach, which are given in Table I.

The stochastic optimization model determines the minimal supply chain costs under these risks and estimates the supply network resilience of the entire supply chain. Note that certain inventory costs as well as production surges are currently still disregarded in our model but may be considered for future work. We have expanded our initial deterministic optimization model as follows. First, each decision variable is assigned an additional index denoting the associated risk scenario. For instance: P_{izs} denotes the number of item z produced at location i in risk scenario s. Furthermore, an additional decision variable named $Missed_{jzs}$ has been included to denote the shortfall of a produced item z at location j for risk scenario s with respect to the actual demand. To represent the effect of a

TABLE I
SUPPLY CHAIN DISRUPTION RISK SCENARIOS FOR OUR EXAMPLE SUPPLY CHAIN.

Number	Risk Scenario
1	Product line simplification of supplier 1 - supplier no longer delivers the component due to strategy change
2	Product line simplification of supplier 2 - supplier no longer delivers the component due to strategy change
3	Covid19 pandamic
4	Cyber attack
5	Transport disruption
6	Supplier disruption due to export restrictions
7	Delivery problems of a certain part from supplier 3
8	Delivery problems of a certain part from supplier 4
9	Happy Flow - no disruptions

missed demand, we define a variable (per item) non-delivery penalty term pen_{jz}. The penalty is invoked when the demand for item j and location z cannot be met ($Missed_{jzs} > 0$). The non-delivery penalty comprises lost revenue and a possible contract penalty. As a result, the demand constraint changes as follows: $d_{jz} \leq Missed_{jzs} + \sum_i P_{izs} T_{ijzs}$ for every scenario s and the objective function becomes:

$$\begin{aligned} &\text{Minimize } \mathbb{E}(C) + \sum_{jzs} pen_{jz} Missed_{jzs} \\ &\qquad \text{(Missed demand is penalized.)} \\ &\mathbb{E}(C) := \sum_{s} p_s C_s = \sum_{ijzs} p_s T_{ijzs} c_{ijz} dist_{ij} weight_z \\ &\qquad + \sum_{izs} p_s (P_{izs} pc_{iz} + in_{iz} ic_{iz}) \end{aligned} \tag{2}$$

where C_s denotes the total supply chain cost and p_s specifies the probability of occurrence of risk scenario s. We use this cost estimate as objective in the optimization problem.

C. Robust optimization model

The robust model introduces an additional constant σ that specifies the risk affinity of the decision-maker [7] [8]. Large values of σ cause a considerable increase in risk costs accounting for the unsureness about the actual costs. Thus, a risk-averse decision-maker would select a rather high σ, whereas a risk-tolerant decision-maker would select a small value or drop this term altogether. Thus, the objective function changes to:

$$\text{Minimize } \mathbb{E}(C) + \sigma \mathbb{V}(C) + \sum_{jzs} (pen_{jz} Missed_{jzs}) \tag{3}$$

Since the computation of the variance requires quadratic programming, we decided to approximate it by the absolute variance [7] [9]:

$$\mathbb{V}_{abs}(C) := \sum_{s} p_s |C_s - \mathbb{E}(C)| \tag{4}$$

The absolute variance can be modeled by linear programming as follows. First, we introduce additional non-negative decision variables : $\phi(s)^+$ und $\phi(s)^-$ with the following two constraints

$$\begin{aligned} \phi_s^+ &\geq p_s(C_s - \mathbb{E}(C)) \\ \phi_s^- &\geq p_s(\mathbb{E}(C) - C_s) \end{aligned} \tag{5}$$

The objective function is then given by:

$$\text{Min. } \mathbb{E}(C) + \sum_{s} \sigma(\phi_s^+ + \phi_s^-) + \sum_{jzs} (pen_{jz} Missed_{jzs}) \tag{6}$$

ϕ_s^+ captures the part of the variance, where the costs exceed their expected value, whereas ϕ_s^+ captures the remaining part, where the costs fall below their expected value. It can be shown that for the absolute variance, both parts must coincide. Thus:

$$\phi_s := \phi_s^+ = \phi_s^- \tag{7}$$

With this, the constraints in (5) simplify to [9]:

$$\phi_s \geq p_s(C_s - \mathbb{E}(C)) \tag{8}$$

and the objective function changes to

$$Minimize\ \mathbb{E}(C) + \sum_{s} \sigma \cdot 2\phi_s + \sum_{jzs} (pen_{jz} Missed_{jzs}) \tag{9}$$

IV. ASSESSING NODE CRITICALITY

Thus far, we have explained our robust optimization model, which is the basis for our proposed node criticality assessment. In particular, the robust optimization method as described above estimates the supply chain‘s risk costs that are composed of the expected total supply chain costs considering several disruption risk scenarios and their variance. A large variance implies that the supply chain costs can vary strongly depending on the occurred risk scenarios. In this case, there is high uncertainty about the incurring costs and therefore the overall supply chain risk is quite high. In contrast, low variance means that the supply chain costs do not deviate much across the scenarios. In this case, the overall supply chain risk remains small. The risk costs are leveraged in our approach for identifying the critical nodes of the supply chain.

By using risk costs instead of ordinary deterministic costs, we obtain more accurate criticality assessments of the nodes. Consider for example the case, that an important supplier S is backed up by a second supplier, which is threatened by probable bankruptcy. In a deterministic setup, the supplier S would be assigned a low criticality because of the provided backup supplier. However, in case supply chain risks are considered, the criticality of supplier S remains high due to the foreseeable default of the backup supplier.

In our approach, a supplier node is considered critical, if its complete breakdown causes a high increase in risk costs of the supply chain, which can be estimated by our robust optimization approach. In contrast, a node is considered uncritical, if the total risk costs of the supply chain do not change in case the associated supplier breaks down and can no longer produce or deliver any goods. Therefore, we consider the criticality of a node being proportional to the overall risk

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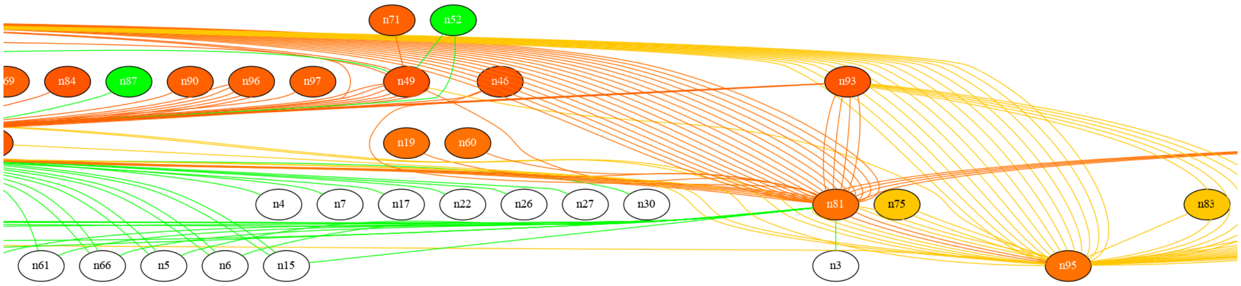

Fig. 2. Part of our example supply chain, where supplier nodes and transportation links are colored according to their criticality.

costs increase of the supply chain when the node in question is removed.

A node in the supply chain network can represent either a supplier or a customer, while the edges represent transportation links either between two suppliers or between a supplier and a customer. We consider in the following an example supply chain with 40 customers, 80 suppliers, 200 components and products, 200 transportation links, and 400 product demands. Due to its size, we only depict a part of the total supply chain in Figure 2, which has similar characteristics in terms of critical links and nodes as the total supply chain.

Each supplier node in this network is colorized according to its criticality. Suppliers are colored green if the risk costs of the supply chain are not increased by its potential breakdown, they are colored yellow if the supply chain risk costs are increased by a certain threshold factor f_1 (we use 30%), and red if the costs were increased by a second larger threshold factor f_2 (we use 60%) or more. Note that the exact values of factors f_1 and f_2 can vary depending on the corporate branch and the degree of competition. For costs increases between 0 and f_1, we interpolate the RGB color values linearly between green (red=0, green=255, blue=0) and yellow (red=255, green=255, blue=0), for costs increased between f_1 and f_2, we interpolate the color values between yellow and red (red=255, green=0, blue=0). Customers are not associated with any production risks and therefore their associated graph nodes are not colored and instead visualized by unfilled circles. The entire process is illustrated in the form of pseudocode in Figure 3.

Like critical nodes, we also visualize critical links in the supply chain. Analog to the node case, they are colored in green if uncritical, in yellow if somewhat critical, and in red if critical. Again, mixtures of the colors red and yellow as well as green and red are possible. In case there are several transportation modes available between two connected nodes, we consider only the most critical mode for the visualization. Note that a link originating from an uncritical supplier node must also be uncritical. However, the opposite does not hold. A link originating from a critical supplier node, can be considered uncritical, if alternative (backup) transportation modes are available.

The most critical node in our example supply chain would increase the risk costs by 50% in case of failure. Furthermore, by far the largest part of the suppliers is considered rather

```
procedure GET_RISK_COSTS_COLOR(nodes, costs_hf)
    Input nodes: list of total supply chain nodes
    Input costs_hf: "happy flow" costs
    red := (255, 0, 0)
    green := (0, 255, 0)
    yellow := (255, 255, 0)
    hm := {} # associated risk costs of a node
    hm_color := {} # associated RGB values for a node
    for n ∈ nodes do
        if type(n)==Supplier then
            costs := obj_value(opt(nodes\{n}))
            hm[n] := costs
            if costs < (1 + f1)costs_hf then
                w := (costs − costs_hf)/(f1 · costs_hf)
                hm_color[n] := w·yellow+(1−w)·green
            else if costs < (1 + f2)costs_hf then
                df := f2 − f1
                dcosts := costs − (1 + f1)costs_hf
                w := dcosts/(df · costs_hf)
                hm_color[n] := w · red + (1 − w) · yellow
            else hm_color[n] := red
            end if
        end if
    end for
    return hm, hm_color
end procedure
```

Fig. 3. Identification of risk costs and node criticality for all suppliers.

critical by our chosen definition of f_2, which is caused by the fact that backup suppliers are missing in most cases. The remaining suppliers are to the same part either non-critical (visualized in green) or somewhat critical (visualized in yellow). In contrast, the distribution of links is much more balanced. Almost 56% of the links are regarded as critical, the rest is either somewhat critical or uncritical. In particular, transportation links leading to a customer are all considered uncritical due to existing alternative transportation modes, while most of the inter-supplier links are critical. Optimally, the decision-maker should supply backup suppliers / transportation modes for all critical nodes and links so that all critical nodes / links become somewhat critical or uncritical.

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V. Conclusion

We described a method for identifying critical nodes in a supply chain based on robust optimization. In contrast to other state-of-the-art methods, our method is very precise, since it not only considers network topology but also network throughput as well as possible supply chain disruption risks. Furthermore, our method provides a concrete risk costs estimate for the breakdown of each supplier and transportation link. For future work, we are planning to identify critical supplier groups, i.e., collections of suppliers being located in geographical or political neighborhoods (and therefore vulnerable to similar supply chain risks) that are causing major disruptions to the supply chain if they are failing simultaneously.

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