Numerical Study for Unsteady Aerodynamics of Multi-Dimensional Freely Falling Plates or Thin Coins

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Abstract—**This paper presents a multi-dimensional high-order gas-kinetic scheme on moving mesh to simulate unsteady flows of freely falling plates or thin coins. The two-dimensional scheme has previously been successfully implemented on freely falling plates, where the rich dynamical behavior, such as fluttering and tumbling motion, was analyzed and the quantitative comparison has been provided between the experimental measurement and numerical computation. In the past several years, a three-dimensional gas-kinetic scheme on moving grid has also been developed to compute fluid problems with moving boundaries. In this work in progress paper, both the modified grid velocity and high-order gas-kinetic scheme are developed to improve the above gas-kinetic method, which is applied for simulating three-dimensional freely falling thin disks.**

Keywords-moving mesh method; gas-kinetic scheme; high-order scheme; freely falling plates or thin coins.

I. INTRODUCTION

It is a fact that not all falling objects travel straight downwards, such as leaves, tree seeds, and paper cards dropped from a table. Obviously, falling leaves and tumbling sheets of paper can get a lift momentarily to float upward against gravity as they flutter (oscillate from side to side) or tumble (rotate and drift sideways) through still air. To explain this complicated natural phenomenon, the knowledge of the instantaneous fluid forces is required. As we know, most objects moving in a fluid encounter unsteady aerodynamic forces. Therefore, the two-dimensional problem of falling plates has been investigated numerically by solving the Navier-Stokes equation following a moving body, where the instantaneous fluid force for the fluttering and tumbling motion has been analyzed [6].

Considering that the motion of freely falling body is usually a three-dimensional problem, further, the falling behavior of a circular thin disk has been investigated experimentally [4] [5]. Three dimensionless ratios have been proposed to characterize the flow:

$$\lambda = \frac{h}{d}, \quad I^* = \frac{\pi \rho_d}{64 \rho_f} \lambda, \quad Re = \frac{Ud}{\gamma},$$

The geometric aspect-ratio parameter λ is small for thin disks, with dimensionless moment of inertia I^* and the Reynolds number *Re,* which are most important for describing the dynamics of the disk motion. The free-fall motion of three-dimensional bodies is more complex due to an extra degree of freedom, so numerical simulation becomes more difficult as well. The gas-kinetic scheme on moving mesh for calculating two-dimensional freely falling plates [1] has been developed to simulate three-dimensional viscous fluids with moving boundaries [2]. In this work in progress paper, both the modified grid velocity distribution and high-order scheme are developed to improve the above gas-kinetic method, which has been applied for simulating three-dimensional freely falling thin disks. The complicated dynamics behavior of three-dimensional free-fall thin coins is obtained numerically.

This paper is organized as follows. Firstly, Section 2 introduces the three-dimensional high-order gas-kinetic scheme with the modified grid velocity. Secondly, Section 3 shows some formulation for the free-fall motion of a circular thin disk in still water with a focus on the planar zigzag motion. Thirdly, in Section 4, the rich fluid dynamics behavior is analyzed by subsequent numerical results which are also compared with experimental data and figures.

II. THREE-DIMENSIONAL HIGH-ORDER GAS-KINETIC SCHEME ON MOVING MESH

Under a generalized coordinate transformation with arbitrary grid velocity, the gas-kinetic Bhatnagar-Gross-Krook (BGK) [8] equation is reformulated in a moving frame of reference. Then, a conservative gas-kinetic scheme is developed for the viscous flow computation in the moving system in Eulerian space. Due to the coupling between the grid velocity and the overall solution algorithm, the Eulerian and Lagrangian methods become two limiting cases in the current method. A fully conservative formulation can be obtained, even in the Lagrangian limit, as shown in [2].

The BGK model of the approximate Boltzmann equation in three-dimensional space can be written as:

$$f_t + uf_x + vf_y + wf_z = \frac{g - f}{\tau} \qquad (1)$$

Here, f is the gas distribution function and g is the equilibrium state approached by f. Both f and g are functions of space *(x, y, z)*, time *t*, particle velocity *(u, v, w),* and internal variable ς . The particle collision time τ is related to the viscosity and heat conduction coefficients. In this paper, we are focusing on the modified grid velocity in the cell interface numerical flux evaluation, while both the gas-kinetic scheme on moving mesh and the high-order gas-kinetic scheme are addressed in [2] and [7], respectively.

We can rewrite the mesh velocity for each point on the cell interface as:

$$U_g = U_g^{fc} + U_g^r \tag{2}$$

in which $\boldsymbol{U}_g^{fc} = \left(\boldsymbol{u}_g^{fc}, \boldsymbol{v}_g^{fc}, \boldsymbol{w}_g^{fc}\right)$ is the grid velocity at the interface barycenter. The interface flux $\boldsymbol{F} - \boldsymbol{Q}\boldsymbol{U}_g$ is written as:

$$F - QU_g = \bar{F} - QU_g^r \tag{3}$$

On the right hand side, the first term is the mean flux with a piecewise constant mesh velocity $\boldsymbol{U}_g^{fc}$, which is the same as the one in [2]. For the second term, it is a correction on each interface point, which reduces the difference between the real flux crossing each interface point and the mean flux $\bar{\boldsymbol{F}}$.

III. MOTION DESCRIPTION FOR FREE-FALL THIN DISK

Previously, the gas-kinetic scheme on moving mesh has been applied successfully to simulate the experiment of two-dimensional falling cards [6]. Figure 1 shows a good agreement between the two-dimensional numerical result and the experimental data in [6]. The work in [4] and [5] experimentally investigates the three-dimensional free-fall motions of a circular thin disk in still water with three different trajectories, namely, zigzag, transition, and spiral,

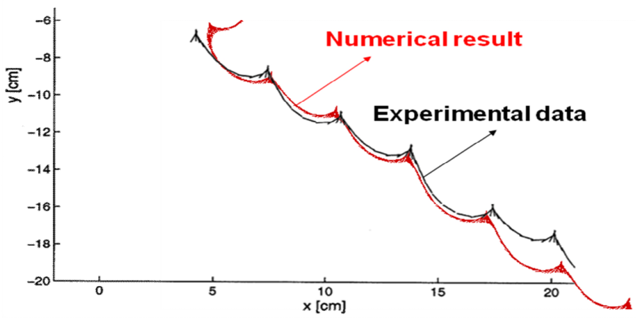

Figure 1. Computed path is compared with the experimental data .

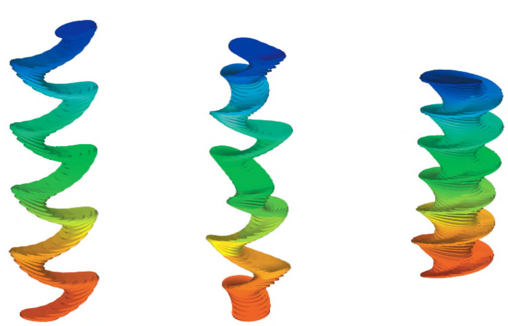

Figure 2. Free-fall motions: zigzag, transition, to spiral.

as in Figure 2. This paper focuses on the numerical simulation for the three-dimensional experiments. At present, we are working on the numerical study for the zigzag motion: namely, the description of moving trajectories and analysis for unsteady aerodynamics.

Consider the two different motions, translation and rotation. The translation acceleration is calculated by Newton law, and rotational angular acceleration is computed by the following formulas:

$$\begin{aligned} d\Omega_1/dt + (I_3 - I_2)\Omega_2\Omega_3/I_1 = 0, \\ d\Omega_2/dt + (I_1 - I_3)\Omega_1\Omega_3/I_2 = 0, \\ d\Omega_3/dt + (I_2 - I_1)\Omega_2\Omega_1/I_3 = 0, \end{aligned} \tag{4}$$

Here, $\boldsymbol{\Omega} = (\Omega_1, \Omega_2, \Omega_3)$ is angular velocity, $\boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3$ are three principal moments of inertia.

IV. CONCLUSIONS

Based on the two-dimensional successful numerical simulation for unsteady aerodynamics of free-fall card motion [1], the three-dimensional high-order moving grid gas-kinetic scheme was developed, and was improved by the modified grid velocity and high order scheme in recent work. At present, the above method is applied to calculate the three-dimensional freely falling thin disks, and the research work of the numerical simulation is in progress. We expect to obtain a good agreement between numerical results and experimental data.

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