# Graph Coloring Applied to Medical Doctors Schedule

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***Abstract*— Scheduling shifts is a tiresome and time consuming task in any business, and particularly in hospitals where errors are costly, rules are plentiful and changes are rapid. The person performing this function (Rota Organizer) will have to keep track of all the employees concerned, distributing hours fairly and avoiding collisions. Rules regulating working hours and breaks have to be followed and the qualifications of individual employees need to be considered. Hours are spent every day on this task in every ward. The goal of this paper is to solve Doctors Scheduling Problem (DSP) and initialize a fair roster for two wards of Pediatric Department (PD) in Prince Sultan Military Medical City (PSMMC) in Saudi Arabia. So, to find a solution to DSP, we used Graph Coloring which is one of the methods mostly used to solve this problem.**

***Keywords- graph coloring; doctors roster; greedy algorithm.***

## I. INTRODUCTION

In the study of graphs, which are mathematical structures used to model pair wise relations between objects, a “graph” is made up of vertices (or nodes) and lines, called edges, that connect them.

Graphs can be used to model many types of relations and processes in physical, biological, social and information systems. Graph coloring has been studied extensively for the past decades. The surge for studying graph coloring in recent times has resulted in countless real-life problem applications, which include scheduling problems. The research presented in this paper aims to provide an effective procedure for solving Doctors Scheduling Problem (DSP) related to their shifts. DSP is a major problem faced by many hospitals all over the world. We tried in two wards of PSMMC to establish a roster for one month as a beginning to achieve a general procedure for every month. After exploring relevant references, the problem will be described in detail, followed by the formulation of the relevant coloring problem and its solution with recent results.

## II. RELATED WORK

Nurses' rostering problem has been studied in the literature by many researchers, for example [1] [4] [6]. Kumara et al. [6] used graph coloring for scheduling with a specific different algorithm (other than greedy algorithm). [1] [4] used graph coloring to make the schedule by greedy algorithm, but for a few number of nurses. In this paper graph coloring is used to solve doctors' scheduling problem (DSP) related to their shifts for one month in two wards of PSMMC. Realizing that establishing a general procedure to create a roster for every month has not been done yet for any hospitals in Saudi Arabia, this work might lead to a follow up by more research. For the advantages of this procedure to be clear, a needed comparison will be done.

## III. STATEMENT OF THE PROBLEM

In this paper, we consider a scheduling problem for Pediatric Department of Prince Sultan Military Medical City (PSMMC) in Riyadh/ Saudi Arabia. It is considered one of the top governmental hospitals with almost twenty seven departments. There are more than five thousand doctors in the hospital. We communicated with Dr. Nawaf Al khayat (Dr.N.A.,) in PD of PSMMIC; he is a consultant and supervisor of resident doctors training in PSMMIC.

Usually, monthly doctors roster are made manually before the end of each month. Rota organizer has the responsibility to publish next month's roster. Even though making monthly rosters manually requires great effort, it does not resolve all conflicts. Instead, it has created more tedious adjustments to accomplish needed tasks. There are consultants, senior and junior doctors working in this department. This paper is concerned with scheduling shifts for junior and senior doctors in two of the department wards for one month only.

There are three types of doctors: Consultants, Seniors (R3 & R4) and Juniors (R1 & R2), where the number indicates trainee year. In PSMMC pediatric department, which has thirteen wards, there are thirty eight resident doctors; R1, R2, R3 and R4. It has been specified for us to work on scheduling shifts for only two wards: PG (Pediatrics General) and PGICU (Pediatrics General Intensive Care Unit) wards. In addition, we were given the following information:

1. Each working day consists of eight to eight and half hours.
2. Each shift consists of twenty four hours.
3. Every doctor who has participated in a shift will not be given anther shift for the next three days.
4. There are twenty two junior and sixteen senior doctors.
5. In each working month, there are doctors who are

excluded from participating in shifts due to other responsibilities or personal circumstances.

We chose September 2016 to be the month we consider for scheduling. In September, only eighteen doctors are excluded which is the least number of doctors, compared with the remaining months. From the list of names given to us by Dr.N.A., we have listed doctors who will participate in September shifts (S.S.). There are twelve juniors and eight seniors available. In the proposed solution for DSP (Doctors Scheduling Problem), we need to assign doctors to shifts.

## IV. FORMULATION OF A GRAPH COLORING PROBLEM

First, we divided the doctors into four shift groups. Each group has two seniors and four juniors, based on department daily requirement, and the fact that we only have twelve juniors and eight seniors in this month. Since both PG and PGICU involved doctors cannot accomplish needed shifts while meeting restrictions of the department, we had several options.

We chose to involve one doctor outside PG and PGICU, each day of September to complement needed shifts. So we had A, B, C and D groups to cover first four consecutive days which will represent the same groups for the following four days, and so on. This implies a need for additional four juniors {SUB.1, SUB.2, SUB.3, SUB.4}, (SUB.1 refer to first substitute for a junior), to complement the required number of doctors (8 seniors and 16 juniors in condition 3 above).

TABLE I. GROUPS OF SEPTEBER SECHDULE DOCTORS(S.S.D.).

| Group | Doctors |
|---|---|
| A | $R3_1$ , $R4_1$ , $R1_1$ , $R1_5$ , $R2_4$ , $SUB_1$. |
| B | $R3_2$ , $R4_2$ , $R1_2$ , $R2_1$ , $R2_5$ , $SUB_2$. |
| C | $R3_3$ , $R4_3$ , $R1_3$ , $R2_2$ , $R2_6$ , $SUB_3$. |
| D | $R3_4$, $R4_4$, $R1_4$, $R2_3$, $R2_7$, $SUB_4$ |

By using above data, an incident matrix is initiated for S.S.D. It is a 24 × 24 matrix. If any two doctors are in same group then ij-th entry is 1 otherwise it is 0. Now, since the incident matrix is a big matrix, we had to divide it into 4 submatrices (blocks) which include all nonzero entries (for the sake of presenting results in appropriate template).

| | $R3_1$ | $R4_1$ | $R1_1$ | $R1_5$ | $R2_4$ | $SUB_1$ |
|---|---|---|---|---|---|---|
| $R3_1$ | 0 | 1 | 1 | 1 | 1 | 1 |
| $R4_1$ | 1 | 0 | 1 | 1 | 1 | 1 |
| $R1_1$ | 1 | 1 | 0 | 1 | 1 | 1 |
| $R1_5$ | 1 | 1 | 1 | 0 | 1 | 1 |
| $R2_4$ | 1 | 1 | 1 | 1 | 0 | 1 |
| $SUB_1$ | 1 | 1 | 1 | 1 | 1 | 0 |

Frist submatrix $[a^1_{ij}]$.

| | $R3_2$ | $R4_2$ | $R1_2$ | $R2_1$ | $R2_5$ | $SUB_2$ |
|---|---|---|---|---|---|---|
| $R3_2$ | 0 | 1 | 1 | 1 | 1 | 1 |
| $R4_2$ | 1 | 0 | 1 | 1 | 1 | 1 |
| $R1_2$ | 1 | 1 | 0 | 1 | 1 | 1 |
| $R2_1$ | 1 | 1 | 1 | 0 | 1 | 1 |
| $R2_5$ | 1 | 1 | 1 | 1 | 0 | 1 |
| $SUB_2$ | 1 | 1 | 1 | 1 | 1 | 0 |

Second submatrix $[a^2_{ij}]$.

| | $R3_3$ | $R4_3$ | $R1_3$ | $R2_2$ | $R2_6$ | $SUB_3$ |
|---|---|---|---|---|---|---|
| $R3_3$ | 0 | 1 | 1 | 1 | 1 | 1 |
| $R4_3$ | 1 | 0 | 1 | 1 | 1 | 1 |
| $R1_3$ | 1 | 1 | 0 | 1 | 1 | 1 |
| $R2_2$ | 1 | 1 | 1 | 0 | 1 | 1 |
| $R2_6$ | 1 | 1 | 1 | 1 | 0 | 1 |
| $SUB_3$ | 1 | 1 | 1 | 1 | 1 | 0 |

Third submatrix $[a^3_{ij}]$.

| | $R3_4$ | $R4_4$ | $R1_4$ | $R2_3$ | $R2_7$ | $SUB_4$ |
|---|---|---|---|---|---|---|
| $R3_4$ | 0 | 1 | 1 | 1 | 1 | 1 |
| $R4_4$ | 1 | 0 | 1 | 1 | 1 | 1 |
| $R1_4$ | 1 | 1 | 0 | 1 | 1 | 1 |
| $R2_3$ | 1 | 1 | 1 | 0 | 1 | 1 |
| $R2_7$ | 1 | 1 | 1 | 1 | 0 | 1 |
| $SUB_4$ | 1 | 1 | 1 | 1 | 1 | 0 |

Fourth submatrix $[a^4_{ij}]$.

Each block is $a$ $6 \times 6$ submatrix, for first block say $[a^1_{ij}]$ where $1 \leq ij \leq 6$. $1^{st}$ column is for $R3_1$, $2^{nd}$ column is for $R4_1$, …, and last column ($6^{th}$ column) is for $SUB_1$. Rows has been delt with in same way. So $[a^2_{ij}]$, $[a^3_{ij}]$ and $[a^4_{ij}]$ are $6 \times 6$ submatrices in the same way. ($[a_{ij}]$ is a $24 \times 24$ matrix). By using the incident matrix $[a_{ij}]$, a graph is constructed with S.S.D. as vertices. If two doctors are in same group, then they are linked by an edge.

Next, we colored the graph by using Greedy algorithm. We started with red as color number one and took color number two to be yellow. Color number three is gray, also color number four is orange and color number five is

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green. We needed six colors, so the last color is going to be blue.

Now we will start with vertices representing seniors; first senior of each day will be colored in red. The second senior adjacent to the first one will be colored yellow. Since every day there are four juniors adjacent to each other and to the red and yellow seniors, the first one of these juniors every day will be colored gray, the second one is orange, third one is green, and last one is blue. See Figure 1. Above, we have included substitutes in the coloring scheme to organize needed shifts. This means the first day group will be repeated in fifth day, second group will be repeated in sixth day, third group will be repeated in seventh day and fourth group will be repeated in eighth day. So, to finish up the process the cycle will be repeated every four days.

TABLE II. GROUPS AFTER APPLYING GRAH COLORING

| Group | Doctors |
|---|---|
| $G1$ | $R3_4, R4_2, R4_3, R3_1$ |
| $G2$ | $R4_1, R4_4, R3_3, R3_2$ |
| $G3$ | $R1_4, R1_2, R1_1, SUB_3$ |
| $G4$ | $R1_5, R2_5, SUB_4, R2_6$ |
| $G5$ | $R2_4, R1_3, SUB_2, R2_7$ |
| $G6$ | $R2_3, R2_2, R2_1, SUB_1$ |

Accordingly, we formulated a table where S.S.D. doctors in G1,G2,…, G6 should be in the same ward if they are in the same group. Based on restrictions specified in Section 4 concerning the problem of making doctors roster for September 2016 in PD of PSMMC, we have divided doctors into shift groups.

Then, we applied the graph theory to these groups so that each vertex represents a doctor and there is an edge between any two doctors in one shift group, see section 3.1 for details.

After graphing, we used Greedy Algorithm to color. We used six colors for twenty four vertices, where each four vertices have one color.

In the graph, doctors represented by vertices of the same color are grouped as one since they will be working in the same ward. The shift groups (Table I) are then used to form a $24 \times 2$ matrix. The elements of first columns represent numbering of doctors and element of second column represent colors of these doctors which are numbering like that (1 is red, 2 is yellow, 3 is gray, 4 is orange, 5 is green and 6 is blue). We used this matrix to create code by Matlab. The output of this code is distribution in the wards.

## V. RESULTS

Table III shows the first phase of result: distribution of doctors in wards. We note that each element represents the participation of a specific doctor in a shift, i.e.,

$$\begin{cases} 1 & \text{if doctor is alternating.} \\ 0 & \text{if doctor is not alternating.} \end{cases}$$

TABLE III. DISTRIBUTION OF DOCTORS IN WARDS AFTER CONVERTING RESULT FROM MATLAB.

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |

In this paper, we have finalized a roster for September 2016 shifts of two wards PG and PGICU in PD of PSMMC. This roster has taken into consideration departmental restrictions. In doing so, we are hoping to have saved PG and PGICU from getting into misunderstandings, as well as any other relevant troubles. Instead of wasting time and effort to generate doctors roster, the staff can concentrate on other important medical duties.

At the end, we can confirm that steps leading to a roster of September 2016, as detailed in Section 4, can be used and applied for other months, and for nurses, as well as doctors.

TABLE IV. THE FINAL ROSTER FOR THE PD.

| Date | Day | A | B | C | D | E | F |
|---|---|---|---|---|---|---|---|
| 4 Sep. | Sun. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 5 | Mon. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 6 | Tue. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 7 | Wed. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |
| 8 | Thu. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 9 | Fri. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 10 | Sat. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 11 | Sun. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |
| 12 | Mon. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 13 | Tue. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 14 | Wed. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 15 | Thu. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |
| 16 | Fri. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 17 | Sat. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 18 | Sun. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 19 | Mon. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |

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| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | Tue. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 21 | Wed. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 22 | Thu. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 23 | Fri. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |
| 24 | Sat. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 25 | Sun. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 26 | Mon. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 27 | Tue. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |
| 28 | Wed. | $R3_1$ | $R4_1$ | $SUB_1$ | $R2_4$ | $R1_5$ | $R1_1$ |
| 29 | Thu. | $R4_2$ | $R3_2$ | $R2_1$ | $SUB_2$ | $R2_5$ | $R1_2$ |
| 30 | Fri. | $R4_3$ | $R3_3$ | $R2_2$ | $R1_3$ | $R2_6$ | $SUB_3$ |
| 1 Oct. | Sat. | $R3_4$ | $R4_4$ | $R2_3$ | $R2_7$ | $SUB_4$ | $R1_4$ |

## VI. FUTURE WORK

Many hospitals take a long time to prepare a doctors roster which is fair to everybody. Instead of wasting time in generating it, we hope to generalize in future work a software, where minimum data is required to have a roster for any month.


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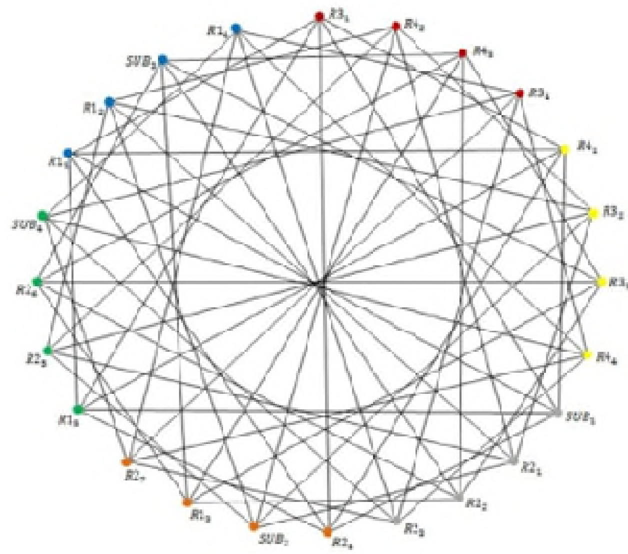

Figure 1. A colored graph of the doctors