

# Selection of Wavelet Decomposition Levels for Vibration Monitoring of Rotating Machinery

Hocine Bendjama, Daoud Idiou, Kaddour Gherfi, Yazid Laib

Welding and NDT Research Centre  
(CSC)

Algiers, Algeria

e-mails: hocine\_bendjama@daad-alumni.de, ddidiou@yahoo.com, gkaddour2@yahoo.fr, yaziddl@yahoo.fr

**Abstract**—The vibration signal of a rotating machine always carries the dynamic information of the machine. Its analysis is very useful for the condition monitoring and fault diagnosis. Many signal analysis methods are able to extract useful information from vibration data. In this paper, bearing fault diagnosis is performed using Wavelet Transform (WT) and Parseval's theorem. The WT is used to decompose the original signal into several signals in order to obtain multiple data series at different resolutions. The fault can be detected from a given level of resolution. For this purpose, Parseval's theorem is used as an evaluation criterion to select the optimal level. Associated to envelope analysis, it allows clear visualization of fault frequencies. Vibration signals from a pilot scale are used to demonstrate the usefulness of the proposed method. The results of the application in inner and outer races bearing diagnosis are satisfactory.

**Keywords**-vibration; fault diagnosis; wavelet transform; Parseval's theorem; bearing.

## I. INTRODUCTION

Fault diagnosis is extremely important task in process monitoring. During the two past decades, various monitoring methods have been developed, such as dynamics, vibration, tribology and non-destructive techniques [1][2]. The vibration signal analysis is essential in improving condition monitoring and fault diagnosis of rotating machinery, because it always carries the dynamic information of the system. Effective utilization of the vibration signals depends upon the effectiveness of the applied signal processing techniques. A wide variety of techniques have been introduced such as: time domain and frequency domain [3][4]. Unfortunately, they are not suitable for non-stationary signal analysis [5]. In order to solve this problem, Wavelet Transform (WT) has been developed. The WT, also called time-frequency analysis, is a kind of variable window technology, which uses a time interval to analyze the frequency components of the signal. This makes the application of the WT for non-stationary signal processing an area of active research over the past decade. An overview of the WT used in vibration signal analysis was provided in [6][7][8].

The original signal using WT can be decomposed into approximations and details versions with different frequency bands by using a successive low-pass and high-

pass filtering. The decomposed levels will not change their information in the time domain [9]. However, useful information can be contained in some sub-bands. So, the fault can be detected from a given level of resolution. This is based on a choice of an indicator to determine the optimal level where failure can occur. The selection of the most reliable indicator has been studied by several authors. Prabhakar et al. [10] selected the periodic impulses of bearing faults in time domain based on low and high frequency nature of decomposed levels. Similar analyses were carried out by Purushotham et al. [11] in order to extract the periodic impulses from the time signals using discrete wavelet transform at Mel-frequency scales. Chinmaya and Mohanty [12] used the sidebands of the gear meshing frequencies as an evaluation criterion for gear faults diagnosis. Djebala et al. [13] analyzed the vibration of faults inducing periodical impulsive forces by selecting the kurtosis as indicator.

In this work, the measured vibration signals are decomposed using the Daubechies wavelet. Clearly, useful information is contained in some decomposition levels. In order to extract useful information, the energy distribution is established by Parseval's theorem. The latter is used as principal criterion to select the optimal level of resolution. The proposed method is evaluated using the vibration measurements obtained from accelerometer sensors. The aim of this method is to provide a solution of bearing fault diagnosis.

The remainder of this paper is structured as follows. Section II presents the experimental rig used. Section III describes the fault diagnosis method. Results and discussion are presented in Section IV. Finally, the main conclusions are outlined in Section V.

## II. EXPERIMENT DATA ACQUISITION

Vibrations caused by defective bearing elements account for the vast majority of problems with rotating machinery. Each element such as inner race or outer race has a characteristic rotational frequency. With a fault on a particular element, an increase in the vibration energy at this element rotational frequency may occur. The monitoring of these elements has a primary importance for the correct operation of the machine.

The experimental measurements presented in this paper are entirely based on the vibration data obtained from the

Case Western Reserve University Bearing Data Centre [14]. As shown in Figure 1, the motor is connected to a dynamometer and torque sensor by a self-aligning coupling. The vibration signals were collected from an accelerometer mounted on the motor housing at the drive end of the motor. The vibration data was obtained from the experimental system under the four different operating conditions: (1) normal condition; (2) with inner race fault; (3) with outer race fault; and (4) with ball fault. The data is sampled at a rate of 12 kHz and the duration of each vibration signal was 10 seconds. More details about experimental setup were reported in [14].

The bearings used in this study are deep groove ball bearings manufactured by SKF. Faults were introduced to the test bearings using electro-discharge machining method. The defect diameters of the three faults were the same: 0.018, 0.036, and 0.053 mm. The motor speed during the experimental tests is 1797–1720 rpm. Each bearing was tested under the four different loads: 0, 1, 2, and 3 horse power (hp).

In order to evaluate the proposed method, the data measured under 0-load (0 hp) at rotation speed of 1797 rpm (30 Hz) including the faults on the inner and outer races were used. The original signal is divided into segments of samples that each sample covered 4096 data points.

Figures 2a, 2b and 2c represent respectively a vibration signal collected at 1797 rpm from the normal state, inner race fault and outer race fault.

The fault frequency can be calculated from the geometry of the bearing and element rotational speed. Frequencies associated with defective inner and outer races are as follows:

$$f_{IR} = (n/2)f_r(1 + (d/D)\cos\alpha) \tag{1}$$

$$f_{OR} = (n/2)f_r(1 - (d/D)\cos\alpha) \tag{2}$$

where,  $f_r$  is the rotational frequency,  $d$  the ball diameter,  $D$  the pitch diameter,  $n$  the number of balls and  $\alpha$  the contact angle.

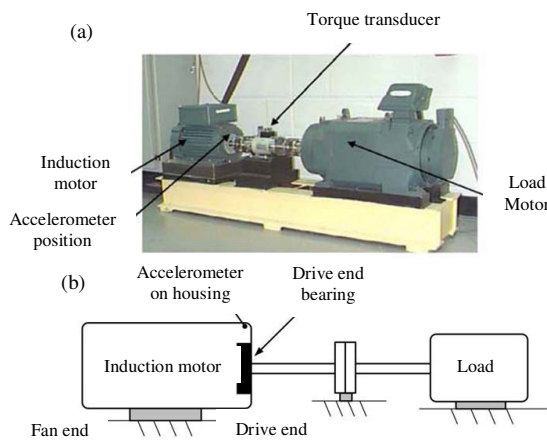


Figure 1. (a) Bearing test rig and (b) its schematic description [15].

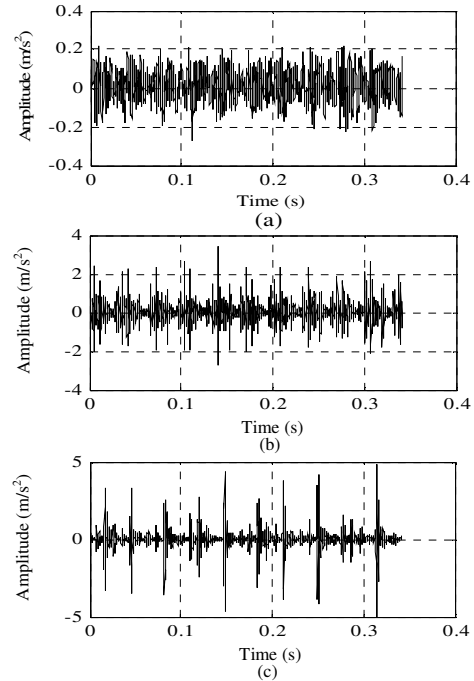


Figure 2. Vibration signals of: a) normal state, b) inner race fault and c) outer race fault.

The fault frequencies of inner race and outer race are calculated, respectively, according to (1) and (2), which are 162 Hz and 107 Hz.

### III. FAULT DIAGNOSIS METHOD

In this section, a diagnosis method, which consists of two approaches, namely, the WT and Parseval’s theorem, is described to monitor the bearing inner and outer races.

#### A. Wavelet Transform

The WT is one of the most important methods in signal analysis. It is a time-frequency analysis technique. Due to its strong capability in time and frequency domain, it is applied recently by many researchers in rotating machinery. The WT decomposes a signal in both time and frequency in terms of a wavelet, called mother wavelet (3). The mother wavelet must be compactly supported and satisfied with the admissibility condition (4).

$$\psi(t) = 1/(\sqrt{a})\psi((t-b)/a) \tag{3}$$

$$\int_{-\infty}^{+\infty} |\hat{\psi}(w)|^2 / |w| dw < \infty \tag{4}$$

where  $\hat{\psi}(w)$  is the Fourier transformation of  $\psi(t)$ .

Two variations of the WT exist: Continuous Wavelet Transform (CWT) and Discrete Wavelet Transform (DWT). They are described below: let  $s(t)$  be the original signal, the CWT of  $s(t)$  is defined as:

$$CWT(a,b)=\frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} s(t)\psi^*((t-b)/a) dt \quad (5)$$

where \* denotes complex conjugate, *a* and *b* are the dilation (scaling) and translation (shift) parameters, respectively.

The DWT is derived from the discretization of the CWT by discrete values of *a* and *b*. The DWT is given by:

$$DWT(j,k)=\frac{1}{\sqrt{2^j}} \int_{-\infty}^{\infty} s(t)\psi^*((t-2^j k)/2^j) dt \quad (6)$$

where *a* and *b* are replaced by  $2^j$  and  $2^j k$ , *j* is an integer.

The DWT can be regarded as a multiresolution analysis technique [16], as illustrated in Figure 3. The DWT analyzes the signal at different scales or resolutions. It employs two sets of functions, called scaling functions and wavelet functions [16][17], which are associated with low pass (L) and high pass (H) filters, respectively. The discrete signal is convolved with L and H, resulting in two vectors *A1* and *D1* on a first level. The vector *A1* is called approximation and the vector *D1* is called detail. The application of the same transform on the approximation *A1* causes it to be decomposed further into approximation *A2* and detail *D2* on a second level. Finally, the signal is decomposed at the expected level.

The selection of the appropriate wavelet is very important in signals analysis. There are many functions available can be used, such as Haar, Daubechies, Meyer, and Morlet functions [18][19]. In the present study, we use the Daubechies wavelet to identify the inner and outer races bearing frequencies.

**B. Parseval's Theorem**

The Parseval's theorem refers to the result that the sum of square of a function is equal to the sum of the square of its transform.

In the wavelet domain, the Parseval's theorem can be defined as the energy of a function in the time domain is equal to the sum of all energy concentrated in the different decomposition levels. This can be described by [20]:

$$\sum_1^N |s(t)|^2 = \sum_1^N |A_m(t)|^2 + \sum_1^m \sum_1^N |D_m(t)|^2 \quad (7)$$

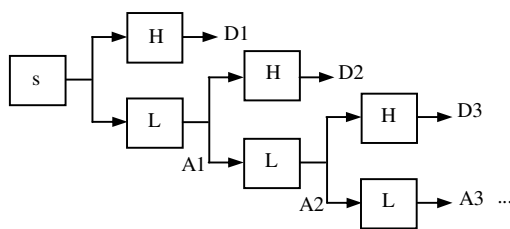


Figure 3. Principal of DWT decomposition.

where *N* is the number of samples and *m* is the maximum level of wavelet decomposition. The left-hand term of (7) represents the total energy of the signal *s(t)*, the first and the second term on the right denote respectively, the total energy of the approximation in the level *m* and the total energy of the detail from level 1 to *m*.

The time domain information will not be lost when the signal is decomposed. In order to extract the maximum information in the different resolution levels, the energy distribution of the approximation and the detail of the signal is calculated. It is given by:

$$P_a = \frac{\|A\|^2}{N_m} \quad (8)$$

$$P_d = \frac{\|D_m\|^2}{N_m} \quad (9)$$

where  $\| \|$  denotes the norm operator.

**IV. MONITORING RESULTS**

The proposed method is applied to the diagnosis of the SKF bearing with inner race fault and outer race fault. The motor runs at a speed of 1797 rpm (30 Hz).

The multiresolution analysis is applied by using the Daubechies wavelet of order 4 (db4). Here, level 4 decomposition is employed to extract approximations and details coefficients from vibration signals. The result of db4 decomposition is given in Figures 4 and 5, respectively.

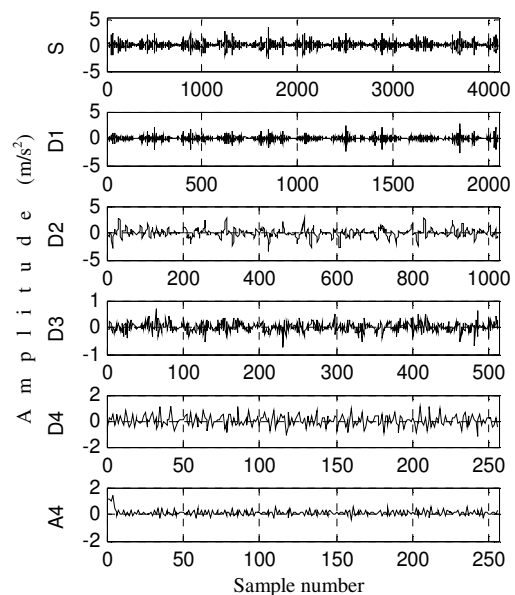


Figure 4. Wavelet decomposition of inner race fault.

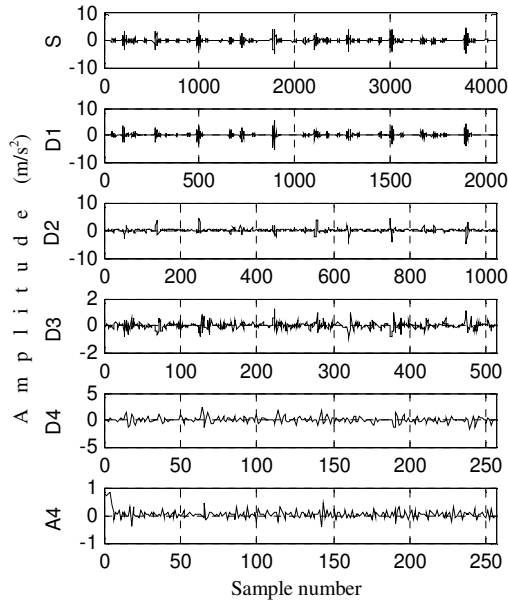


Figure 5. Wavelet decomposition of outer outer fault.

The objective of the proposed method is to demonstrate the effectiveness of the energy distribution as principal criterion for selecting the optimal decomposition level. The level having the largest value indicates the desired level.

The energy distribution of each level is shown in Figure 6. The decomposition levels 1 to 4 represent the detailed version and the levels 5 stand for the approximated version of the signal. The figure shows the obvious difference between levels. From this figure, it can be seen that the energy distribution using db4 occurs in the second level for each fault. So, our choice is attached to the detail *D2*.

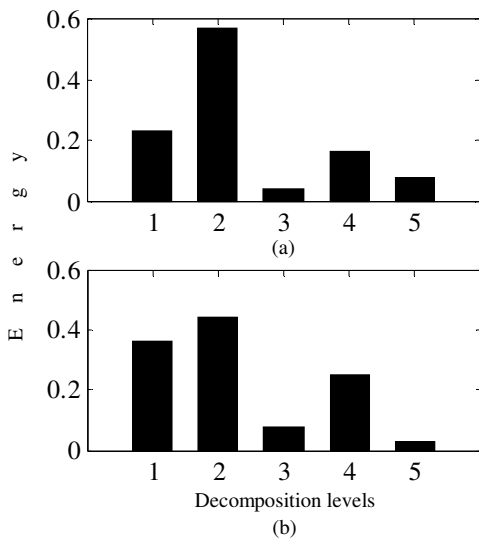


Figure 6. Energy distribution: a) inner race and b) outer race.

In order to diagnose the inner race fault and the outer race fault from the selected level, we use envelope analysis. Figures 7a and 8a show respectively the selected decomposition level (*D2*) of inner race fault and outer race fault. It is clear that this level shows the shocks generated by the considered faults.

Figures 7b and 8b illustrate respectively the envelope spectrum of *D2* of inner race fault and outer race fault. The frequency spectra clearly show many frequency components, at the rotation frequency (30 Hz), also at the characteristic frequencies of the inner race (162 Hz) and the outer race (107 Hz) and their harmonics, which indicates a defective bearing.

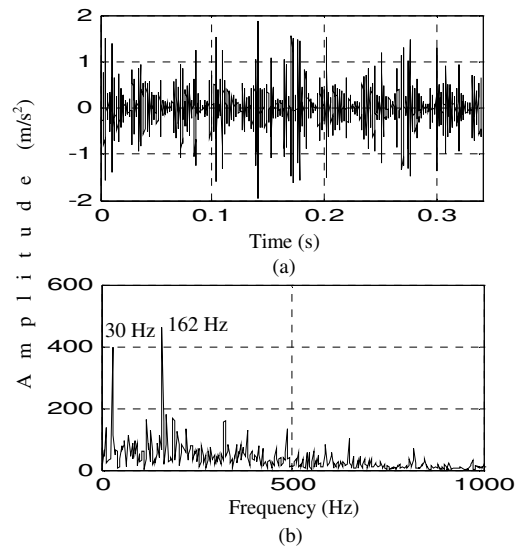


Figure 7. (a) Selected level of inner race fault and (b) its envelope spectrum.

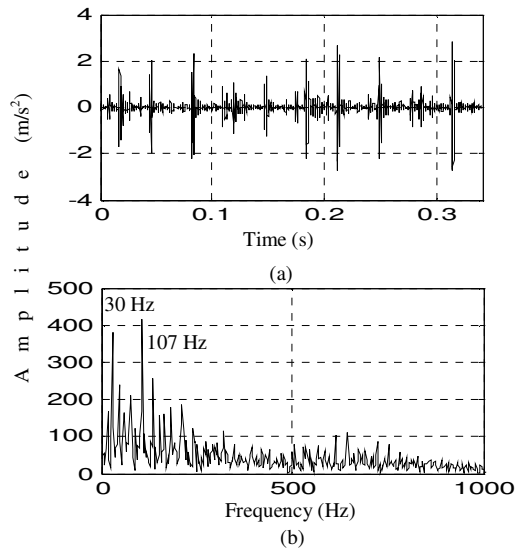


Figure 8. (a) Selected level of outer race fault and (b) its envelope spectrum.

## V. CONCLUSION AND FUTURE WORK

This paper presented a method for improving the bearing fault diagnosis based on WT and Parseval's theorem. It is adapted to obtain multiple data series at different resolutions by wavelet decomposition and calculate the energy distribution using Parseval's theorem in order to select the optimal decomposition level, for a possible diagnosis. A case study on SKF bearing diagnosis with defective inner race and outer race has shown that this method can greatly improve the accuracy of diagnosis. Hence, the proposed method is a successful approach for vibration monitoring. It remains to test its application on a signal containing other types of faults.

## REFERENCES

- [1] J. Altmann, Application of discrete wavelet packet analysis for the detection and diagnosis of low speed rolling-element bearing faults, Ph.D. thesis, Monash University, Melbourne, Australia, 1999.
- [2] H. Yang, Automatic fault diagnosis of rolling element bearings using wavelet based pursuit features, Ph.D. thesis, Queensland University of Technology, Australia, 2004.
- [3] S. Seker and E. Ayaz, "A study on condition monitoring for induction motors under the accelerated aging processes," *IEEE Power Engineering*, vol. 22, no. 7, 2002, pp. 35-37.
- [4] K. Shibata, A. Takahashi, and T. Shirai, "Fault diagnosis of rotating machinery through visualisation of sound signal," *Mechanical Systems and Signal Processing*, vol. 14, 2000, pp. 229-241.
- [5] J. D. Wu and C. H. Liu, "Investigation of engine fault diagnosis using discrete wavelet transform and neural network," *Expert Systems with Applications*, vol. 35, 2008, pp. 1200-1213.
- [6] G. K. Singh and S. A. S. Al Kazzaz, "Isolation and identification of dry bearing faults in induction machine using wavelet transform," *Tribology International*, vol. 42, 2009, pp. 849-861.
- [7] H. Bendjama, S. Bouhouche, and M. S. Boucherit, "Vibration monitoring for fault diagnosis in rotating machinery using wavelet transform," *Proc. of the International Conference on Advanced Computer Theory and Engineering*, ASME Press, Dec 2011, pp. 167-170, ISBN: 978-0-7918-5993-3.
- [8] R. Yan, R. X. Gao, and X. Chen, "Wavelets for fault diagnosis of rotary machines: A review with applications," *Signal Processing*, vol. 96, 2014, pp. 1-15.
- [9] Z. L. Gaing, "Wavelet-based neural network for power disturbance recognition and classification," *IEEE Transactions on Power Delivery*, vol. 19, 2004, pp. 1560-1568.
- [10] S. Prabhakar, A. R. Mohanty, and A. S. Sekhar, "Application of discrete wavelet transform for detection of ball bearing race faults," *Tribology International*, vol. 35, 2002, pp. 793-800.
- [11] V. Purushotham, S. Narayanan, and S. A. N. Prasad, "Multi-fault diagnosis of rolling bearing elements using wavelet analysis and hidden Markov model based fault recognition," *NDT&E International*, vol. 38, 2005, pp. 654-664.
- [12] K. Chinmaya and A. R. Mohanty, "Monitoring gear vibrations through motor current signature analysis and wavelet transform," *Mechanical Systems and Signal Processing*, vol. 20, no. 1, 2006, pp. 158-187.
- [13] A. Djebala, N. Ouelaa, and N. Hamzaoui, "Detection of rolling bearing defects using discrete wavelet analysis," *Meccanica*, vol. 43, 2008, pp. 339-348.
- [14] K. A. Loparo, Bearings vibration data set, Case Western Reserve University, Available from: <http://www.eecs.cwru.edu>, 2003, [retrieved: January, 2010].
- [15] Y. Huang, C. Liu, X. F. Zha, and Y. Li, "A lean model for performance assessment of machinery using second generation wavelet packet transform and Fisher criterion", *Expert Systems with Applications*, vol. 37, 2010, pp. 3815-3822.
- [16] S. G. Mallat, "A theory for multiresolution signal decomposition: the wavelet representation," *IEEE Trans Pattern Anal Machine Intelligence*, vol. 11, no. 7, 1989, pp. 674-693.
- [17] N. Lu, F. Wang, and F. Gao, "Combination method of principal component and wavelet analysis for multivariate process monitoring and fault diagnosis," *Industrial & Engineering Chemistry Research*, vol. 42, 2003, pp. 4198-4207.
- [18] C. K. Chui, An introduction to wavelets, Academic Press, New York, 1992.
- [19] I. Daubechies, "Orthonormal bases of compactly supported wavelets," *Communication on Pure and Applied Mathematics*, vol. 41, 1988, pp. 909-996.
- [20] A. M. Gaouda, M. M. A. Salma, M. R. Sultan, and A.Y. Chikhani, "Power quality detection and classification using wavelet-multiresolution signal decomposition," *IEEE Transaction on Power Delivery*, vol.14, no.4, 1999, pp. 1469-1476.