Takagi Sugeno Fuzzy Controller for Uncertain Single Link Manipulator

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***Abstract*—This paper presents the design of a fuzzy tracking controller for uncertain Single Link Manipulator (SLM) moving in the vertical plane. A Takagi-Sugeno (TS) fuzzy model of the uncertain nonlinear plant is constructed using sector nonlinearity approach and a set of operations point technique. The controller design for TS fuzzy plant is then carried out using Francis-Isidori-Byrnes (FIB) nonlinear regulation theory and parallel distributed compensation (PDC) technique. MATLAB simulations are performed to validate the designed controller for tracking constant and sinusoidal reference signals.**

Keywords- Uncertain single link manipulator; TS fuzzy model; Parallel distributed compensation; Linear matrix inequalities; Francis-Isidori-Byrnes nonlinear regulation theory; MATLAB/Simulink.

I. INTRODUCTION

Single Link Manipulators (SLM) are popular platforms to study control algorithms. Two types of SLM are often deployed by researchers to validate their control techniques. These include flexible joint and flexible link manipulators which can be made to operate either in horizontal or vertical plane. The vertical plane motion introduces an additional factor of gravity in the model. A variety of linear and nonlinear techniques are found in literature for the control of SLM. The design of H-∞ based Proportional-Derivative-Integral (PID) control is presented in [1] for tip regulation task in SLM. The method considers the model uncertainty as a result of neglecting high frequency modes and computes the gain space for PID controller using H-∞ optimization criterion. Real-time implementation results using a digital signal processor validates the proposed controller which is also found to outperform the Ziegler-Nichols-PID controller in terms of the transient performance and robustness. Back stepping method tuned by Genetic algorithm is used by Ali Sahab and Modabbernia [2] to control SLM. Through a series of virtual control inputs and control Lyapunov functions, convergence of tracking error is shown. A fitness function is formed to minimize the settling time and percentage overshoot. Based on this function, Genetic algorithm finds optimal gains for back stepping controller. The proposed algorithm is shown to perform better than robust control methods for stabilization and reference tracking tasks. An adaptive controller is proposed in [3] to control a SLM which adjusts the position and velocity gains based on the tracking error. The controller demands large bandwidth (as a function of error) during startup to provide fast response and bandwidth decreases as the error converges to zero which helps to eliminate the overshoot in system response. A notion of dynamic pole motion explains the system stability under the presented design scheme. The use of fuzzy logic in controlling a SLM is also addressed [4]-[7]. A two stage fuzzy controller is presented in [4] for tip position tracking in SLM. The first stage employs two fuzzy logic controllers with motor angle and its derivative being the inputs of first 81-rule base controller while the second controller processes tip angle and its derivative using a rule-base containing 49 rules. The outputs from these fuzzy logic controllers form input to a second stage fuzzy logic controller which generates pulse width modulated signal to drive the DC motor. Simulation and experimental results show the superior performance of the proposed controller in comparison to PID controller. The optimization of a fuzzy controller in terms of its scaling gains and membership functions is carried out using Genetic algorithm [5] which uses a weighted combination of conflicting objectives including the fast response and minimal overshoot as a fitness function. In addition, a command shaper is also integrated to modify the reference signal keeping in view the vibration modes. The command shaper is also tuned using genetic algorithm to give the optimal locations and amplitudes of impulses which are then convolved with desired reference signal to generate a modified reference signal.

This paper follows a model-based approach for the design of fuzzy logic controller for stabilization and tracking control of SLM moving in vertical plane. By assuming the parameters to be uncertain, a TS fuzzy plant model is constructed which exactly represents the original nonlinear dynamics in compact region formed from parameter bounds and operating region [8]. Francis-Isidori-Byrnes (FIB) nonlinear regulation theory and Parallel Distributed Compensation (PDC) technique [9][10] is used to design a controller for tracking constant and sinusoidal references. Controller part based on FIB is responsible for directing the system motion towards the steady state manifold and generates steady state input for forcing the system to stay there while PDC part ensures the system stability during convergence to steady state manifold. FIB part is designed after solving time varying matrix differential equations in

 ISBN: 978-1-61208-419-0

terms of fuzzy sets, while PDC part is designed using linear matrix inequality techniques where the existence of a symmetric positive definite matrix for all fuzzy sub-systems proves the system stability. MATLAB simulations are performed to show the effectiveness of the designed controller for SLM. It is found that controller has remained successful in tracking the reference trajectories with good transient performance. The contribution of the paper lies in constructing a fuzzy model for uncertain single link manipulator based on the idea of set of operation point's technique. The stabilization and tracking of the resulting model is achieved using exact output regulation theory.

We start by constructing the TS fuzzy plant model in Section II. Controller design is presented in Section III followed by simulation results in Section IV. Conclusions are drawn in Section V.

II. TS Fuzzy Model Of SLM

The dynamics of a SLM consisting of a rod with a circular disc at one end and moving in the vertical plane can be described by the following differential equation:

$$\left(\frac{1}{3}ml^2+Ma^2+M\left(l+a\right)^2\right)\ddot{\theta}+b\dot{\theta}+g\left(M\left(a+l\right)+\frac{ml}{2}\right)\sin\theta=\tau \tag{1}$$

Where m is the mass of the rod, l is the length of the rod, M is the mass of the circular disc, a is the radius of the disc, b is the coefficient of viscous friction at the pivot, θ is the angle of the link from vertical, g is the acceleration due to gravity and τ is the torque provided by the DC motor for reference tracking purposes. The numerical values of these parameters are listed in Table 1 where mass of the circular disc and the damping coefficient are assumed to be uncertain.

By defining the state vector to be $\mathbf{x}=\begin{bmatrix} x_1=\theta & x_2=\dot{\theta}\end{bmatrix}^T$ the system in (1) can be represented as:

$$\begin{aligned}\dot{\mathbf{x}}&=\begin{bmatrix}0 & 1\\ f_{21}\left(x_1,M\right) & f_{22}\left(b,M\right)\end{bmatrix}\mathbf{x}+\begin{bmatrix}0\\ g_2\left(M\right)\end{bmatrix}u\\ y&=\begin{bmatrix}1 & 0\end{bmatrix}\mathbf{x}\end{aligned} \tag{2}$$

Where u represents the torque to be generated by the motor, $y=x_1$ denotes the system output and the nonlinear functions are given as:

$$f_{21}\left(x_1,M\right)=-\frac{g\left(M\left(a+l\right)+\frac{ml}{2}\right)}{\left(\frac{1}{3}ml^2+Ma^2+M\left(l+a\right)^2\right)}\cdot\frac{\sin x_1\left(t\right)}{x_1\left(t\right)} \tag{3}$$

$$f_{22}\left(b,M\right)=-\frac{b}{\left(\frac{1}{3}ml^2+Ma^2+M\left(l+a\right)^2\right)} \tag{4}$$

$$g_2\left(M\right)=\frac{1}{\left(\frac{1}{3}ml^2+Ma^2+M\left(l+a\right)^2\right)} \tag{5}$$

By assuming the angular displacement of the link to lie in the range $-\frac{\pi}{2}\le x_1\left(t\right)\le\frac{\pi}{2}$, we can define the following compact region covering the parametric uncertainties and operating range as:

$$D=\left\{\left(b,M,x_1\right)\in\mathbb{R}^3:0.01\le b\le 0.05, 0.01\le M\le 0.1,-\frac{\pi}{2}\le x_1\le\frac{\pi}{2}\right\} \tag{6}$$

TS fuzzy model of the system in (2) can be constructed so as to exactly reproduce the plant dynamics over the compact region (6) by finding the extreme values of the nonlinear functions (3)-(5) for this region. This result is based on the following property and will ensure the stabilization of the plant over the compact region by using PDC controller:

Property 1: Let $I_p\subset\mathbb{R}^p$ and $I_q\subset\mathbb{R}^q$ be compact subsets and $I=I_p\times I_q$. Let $f:I\subset\mathbb{R}^t\to\mathbb{R}$ be a continuous function with $t=p+q$. If for some given $p_0\in I_p$, $M=\max_{q\in I_q}\left\{f\left(p_0,q\right)\right\}$, and $m=\min_{q\in I_q}\left\{f\left(p_0,q\right)\right\}$; then $M\le\max_{\left(p_0,q\right)\in I}\left\{f\left(p_0,q\right)\right\}$, and $m\ge\min_{\left(p_0,q\right)\in I}\left\{f\left(p_0,q\right)\right\}$.

The variation of the functions (3)-(5) over the compact region (6) is depicted in Fig. 1 and the extreme values are found to be:

$$\underset{\left(M,x_1\right)\in D}{f_{21,\min}}=-28.0339 \tag{7}$$

$$\underset{\left(M,x_1\right)\in D}{f_{21,\max}}=-14.7767 \tag{8}$$

$$\underset{\left(b,M\right)\in D}{f_{22,\min}}=-2.5949 \tag{9}$$

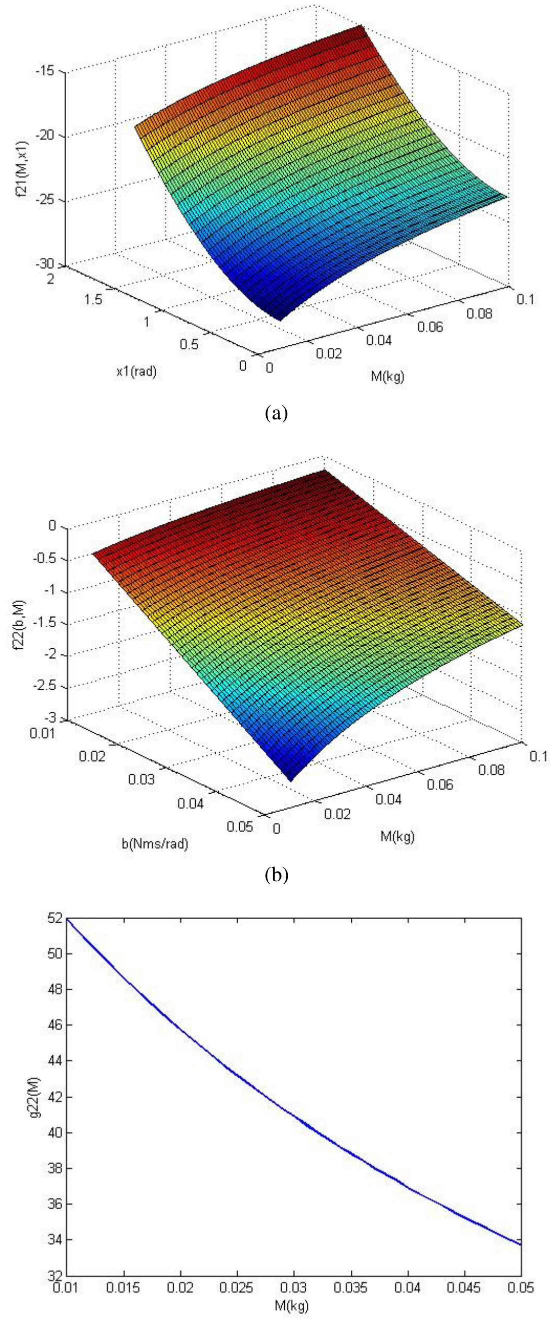
$$\underset{\left(b,M\right)\in D}{f_{22,\max}}=-0.2343 \tag{10}$$

$$\underset{M\in D}{g_{2,\min}}=33.6965 \tag{11}$$

$$\underset{M\in D}{g_{2,\max}}=51.8977 \tag{12}$$

We can now define the following fuzzy sets with universe of discourse being the extreme values in (7)-(12) which will enable us to build the TS fuzzy plant model:

$$\begin{aligned}M_1&=\begin{cases}\dfrac{f_{21}\left(M,x_1\right)-f_{21,\min}}{f_{21,\max}-f_{21,\min}}, & x_1\left(t\right)\ne 0\\ 0 & ,x_1\left(t\right)=0\end{cases}\\ M_2&=1-M_1\end{aligned} \tag{13}$$



(a) (b) (c)

Figure 1. Plot of functins (3)-(5) over compact region (6) (a) $f_{21}\left(x_1^+, M\right)$ (b) $f_{22}\left(b, M\right)$ (c) $g_2\left(M\right)$

$$N_1 = \frac{f_{22}\left(b,M\right) - f_{22,\min}}{f_{22,\max} - f_{22,\min}} \qquad (14)$$
$$N_2 = 1 - N_1$$

$$O_1 = \frac{g_2\left(M\right) - g_{2,\min}}{g_{2,\max} - g_{2,\min}} \qquad (15)$$
$$O_2 = 1 - O_1$$

Based on the fuzzy sets (13)-(15), we define the following plant rules:

Rule 1: IF f_{21} is M_2 AND f_{22} is N_2 AND g_2 is O_2 THEN $\dot{\mathbf{x}} = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1 u$

Rule 2: IF f_{21} is M_2 AND f_{22} is N_2 AND g_2 is O_1 THEN $\dot{\mathbf{x}} = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2 u$

Rule 3: IF f_{21} is M_2 AND f_{22} is N_1 AND g_2 is O_2 THEN $\dot{\mathbf{x}} = \mathbf{A}_3\mathbf{x} + \mathbf{B}_3 u$

Rule 4: IF f_{21} is M_2 AND f_{22} is N_1 AND g_2 is O_1 THEN $\dot{\mathbf{x}} = \mathbf{A}_4\mathbf{x} + \mathbf{B}_4 u$

Rule 5: IF f_{21} is M_1 AND f_{22} is N_2 AND g_2 is O_2 THEN $\dot{\mathbf{x}} = \mathbf{A}_5\mathbf{x} + \mathbf{B}_5 u$

Rule 6: IF f_{21} is M_1 AND f_{22} is N_2 AND g_2 is O_1 THEN $\dot{\mathbf{x}} = \mathbf{A}_6\mathbf{x} + \mathbf{B}_6 u$

Rule 7: IF f_{21} is M_1 AND f_{22} is N_1 AND g_2 is O_2 THEN $\dot{\mathbf{x}} = \mathbf{A}_7\mathbf{x} + \mathbf{B}_7 u$

Rule 8: IF f_{21} is M_1 AND f_{22} is N_1 AND g_2 is O_1 THEN $\dot{\mathbf{x}} = \mathbf{A}_8\mathbf{x} + \mathbf{B}_8 u$

Where

$$\begin{aligned}
\mathbf{A}_1 = \mathbf{A}_2 &= \begin{bmatrix} 0 & 1 \\ -28.0339 & -2.5949 \end{bmatrix} \\
\mathbf{A}_3 = \mathbf{A}_4 &= \begin{bmatrix} 0 & 1 \\ -28.0339 & -0.2343 \end{bmatrix} \\
\mathbf{A}_5 = \mathbf{A}_6 &= \begin{bmatrix} 0 & 1 \\ -14.7767 & -2.5949 \end{bmatrix} \\
\mathbf{A}_7 = \mathbf{A}_8 &= \begin{bmatrix} 0 & 1 \\ -14.7767 & -0.2343 \end{bmatrix} \\
\mathbf{B}_1 = \mathbf{B}_3 = \mathbf{B}_5 = \mathbf{B}_7 &= \begin{bmatrix} 0 \\ 33.6965 \end{bmatrix} \\
\mathbf{B}_2 = \mathbf{B}_4 = \mathbf{B}_6 = \mathbf{B}_8 &= \begin{bmatrix} 0 \\ 51.8977 \end{bmatrix}
\end{aligned} \qquad (16)$$

Using the singleton fuzzification, product inference engine and average defuzzification technique, TS fuzzy plant model can be given as:

TABLE I. PLANT PARAMETERS

Parameter	Value
m	0.2 *Kg*
l	0.5 *m*
a	0.01 *m*
M	[0.01, 0.1] *Kg*
b	[0.01, 0.05] *Nms/rad*
g	9.8 m/s^2

$$\begin{aligned}\dot{\mathbf{x}} &= \sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\left(\mathbf{A}_i\mathbf{x}+\mathbf{B}_i u\right)\\ y &= \sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{C}_i\mathbf{x}\end{aligned} \tag{17}$$

$$\alpha_i\left(\mathbf{z}(t)\right)=\frac{\rho_i\left(\mathbf{z}(t)\right)}{\sum_{i=1}^{8}\rho_i\left(\mathbf{z}(t)\right)} \tag{18}$$

$$\rho_i\left(\mathbf{z}(t)\right)=M_i\left(M,x_1\right)\times N_i\left(b,M\right)\times O_i\left(M\right) \tag{19}$$

Where $\rho_i\left(\mathbf{z}(t)\right)$ and $\alpha_i\left(\mathbf{z}(t)\right)$ are the firing and normalized firing strengths of the 'i^{th}'rule respectively which contains the fuzzy sets M_i, N_i and O_i. This degree of belongingness is determined based on the scheduling vector, $\mathbf{z}(t)=\left[f_{21}\left(M,x_1\right)\quad f_{22}\left(b,M\right)\quad g_2\left(M\right)\right]$.

III. TS FUZZY CONTROLLER DESIGN

TS fuzzy controller for SLM is designed based on Francis-Isidori-Byrnes (FIB) nonlinear regulation theory which guarantees exact tracking through the control law (20) subject to the solution of the differential equations (21):

$$u(t)=-\mathbf{K}\left(\mathbf{x}(t)-\boldsymbol{\pi}\left(\mathbf{w}(t)\right)\right)-\boldsymbol{\gamma}\left(\mathbf{w}(t)\right) \tag{20}$$

$$\begin{aligned}&\frac{\partial\boldsymbol{\pi}\left(\mathbf{w}(t)\right)}{\partial\mathbf{w}(t)}\mathbf{s}\left(\mathbf{w}(t)\right)=\mathbf{f}\left(\boldsymbol{\pi}\left(\mathbf{w}(t)\right),\mathbf{w}(t),\boldsymbol{\gamma}\left(\mathbf{w}(t)\right)\right)\\ &\mathbf{0}=\mathbf{h}\left(\boldsymbol{\pi}\left(\mathbf{w}(t)\right),\mathbf{w}(t)\right)\end{aligned} \tag{21}$$

Where $\boldsymbol{\pi}\left(\mathbf{w}(t)\right)$ is the steady state zero error manifold, $\boldsymbol{\gamma}\left(\mathbf{w}(t)\right)$ is the steady state input, $\mathbf{f}$ is the system dynamics, $\mathbf{s}$ forms the exosystem to be tracked, $\mathbf{h}$ is the tracking error and $\mathbf{K}$ is the stabilizing gain. It is shown in [9] that these nonlinear equations can be exactly solved in terms of fuzzy sets with time varying degree of membership. We introduce the following fuzzy exosystem which will serve the purpose of reference signal generation:

$$\begin{aligned}\dot{\mathbf{w}} &= \sum_{i=1}^{2}\beta_i\left(f_{21}(t)\right)\mathbf{S}_{ji}\mathbf{w}\\ y_{ref} &= \sum_{i=1}^{2}\beta_i\left(f_{21}(t)\right)\mathbf{Q}_i\mathbf{w}\end{aligned} \tag{22}$$

Where $\mathbf{S}_{1i}$ and $\mathbf{S}_{2i}$ denote the constant and sinusoidal reference state matrices respectively while $\mathbf{Q}_i$ is the reference output vector. β_i is the normalized firing strength for the 'i^{th}'rule of fuzzy exosystem.

$$\mathbf{S}_{11}=\mathbf{S}_{12}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\mathbf{w}(0)=\begin{bmatrix}1 & 0\end{bmatrix}^T \tag{23}$$

$$\mathbf{S}_{21}=\mathbf{S}_{22}=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},\mathbf{w}(0)=\begin{bmatrix}1 & 0\end{bmatrix}^T \tag{24}$$

$$\mathbf{Q}_1=\mathbf{Q}_2=\begin{bmatrix}x_{1ref} & 0\end{bmatrix}^T \tag{25}$$

Using (17) and (22), the tracking error can be given as:

$$e(t)=\sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{C}_i\mathbf{x}-\sum_{i=1}^{2}\beta_i\left(f_{21}(t)\right)\mathbf{Q}_i\mathbf{w} \tag{26}$$

The above error will be converged asymptotically to zero using the control law (27):

$$u(t)=-\sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{K}\left(\mathbf{x}(t)-\boldsymbol{\Pi}\left(\mathbf{w}(t)\right)\right)-\boldsymbol{\Gamma}\left(\mathbf{w}(t)\right) \tag{27}$$

Where $\boldsymbol{\Pi}(t)$ and $\boldsymbol{\Gamma}(t)$ are updated as a result of the solution of following time varying matrix equations:

$$\begin{aligned}\dot{\boldsymbol{\Pi}}(t) &= \sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{A}_i\boldsymbol{\Pi}(t)+\sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{B}_i\boldsymbol{\Gamma}(t)-\boldsymbol{\Pi}(t)\sum_{i=1}^{2}\beta_i\left(f_{21}(t)\right)\mathbf{S}_{ji}\\ \mathbf{0} &= \sum_{i=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\mathbf{C}_i\boldsymbol{\Pi}(t)-\sum_{i=1}^{2}\beta_i\left(f_{21}(t)\right)\mathbf{Q}_i\end{aligned} \tag{28}$$

The control objective of tracking the constant and sinusoidal references by SLM leads to the following mappings:

$$\begin{aligned}&x_1(t)=w_1(t)\\ &\dot{x}_1(t)=\dot{w}_1(t)\Rightarrow x_2(t)=w_2(t)\end{aligned} \tag{29}$$

$$\begin{aligned}\dot{x}_2(t)=&M_1\left(x_1(t)\right)\left(f_{21,\max}x_1(t)+f_{22n}x_2(t)+g_{2n}u(t)\right)+\\ &M_2\left(x_1(t)\right)\left(f_{21,\min}x_1(t)+f_{22n}x_2(t)+g_{2n}u(t)\right)\end{aligned} \tag{30}$$

Where f_{22n} and g_{2n} denote the nominal function values. Using (28)-(30), we find the following steady state zero error manifold and steady state input matrices:

$$\mathbf{\Pi}(t)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \tag{31}$$

$$\mathbf{\Gamma}^1(t)=-\frac{1}{g_{2n}}\left[M_1\left(w_1(t)\right)f_{21,\max}+M_2\left(w_1(t)\right)f_{21,\min}\quad f_{22n}\right] \tag{32}$$

$$\mathbf{\Gamma}^2(t)=-\frac{1}{g_{2n}}\left[M_1\left(w_1(t)\right)f_{21,\max}+M_2\left(w_1(t)\right)f_{21,\min}+1\quad f_{22n}\right] \tag{33}$$

Where $\mathbf{\Gamma}^1(t)$ and $\mathbf{\Gamma}^2(t)$ govern the steady inputs for steady state zero error manifold corresponding to reference state matrices $\mathbf{S}_{1i}$ and $\mathbf{S}_{2i}$ respectively. The other part of the control law will ensure the stabilization of the equilibrium point. We will use PDC technique to design the stabilizing controller for SLM model (17) which will share the same fuzzy sets as that of plant to weight the control gains of fuzzy sub-systems. The 'i^{th}' control rule will be defined as:

Con. Rule i: IF f_{21} is M_i AND f_{22} is N_i AND g_2 is O_i
THEN $u_{Ki}(t)=-\mathbf{K}_i\mathbf{x}(t)$

The net stabilizing control gain is found as:

$$u_K(t)=-\sum_{i=1}^{8}\eta_i\left(\mathbf{z}(t)\right)\mathbf{K}_i\mathbf{x}(t) \tag{34}$$

Using (17) and (34), the closed loop dynamics can be given as:

$$\dot{\mathbf{x}}(t)=\sum_{i=1}^{8}\sum_{j=1}^{8}\alpha_i\left(\mathbf{z}(t)\right)\eta_j\left(\mathbf{z}(t)\right)\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{K}_j\right)\mathbf{x}(t) \tag{35}$$

To ensure the closed loop system stability, the following Lyapunov inequality must hold:

$$\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{K}_j\right)^T\mathbf{P}+\mathbf{P}\left(\mathbf{A}_i-\mathbf{B}_i\mathbf{K}_j\right)<0,\forall i,j\le 8 \tag{36}$$

Where $\mathbf{P}$ is a symmetric positive definite matrix. The above inequalities can be cast as LMIs whose solution can return the control gains for fuzzy sub-systems. By pre- and post-multiplying (36) with $\mathbf{P}^{-1}$ and re-defining, $\mathbf{P}=\mathbf{P}^{-1}$ and defining $\mathbf{Q}_i=\mathbf{K}_i\mathbf{P}$, we obtain the following LMIs with the inclusion of decay rate constraint:

$$\begin{aligned}&\mathbf{P}>0\\ &\mathbf{A}_i\mathbf{P}+\mathbf{P}\mathbf{A}_i^T-\mathbf{B}_i\mathbf{Q}_i-\mathbf{Q}_i^T\mathbf{B}_i^T+2\lambda\mathbf{P}<0,\forall i\le 8\\ &\mathbf{A}_i\mathbf{P}+\mathbf{P}\mathbf{A}_i^T+\mathbf{A}_j\mathbf{P}+\mathbf{P}\mathbf{A}_j^T-\mathbf{B}_i\mathbf{Q}_j\\ &-\mathbf{Q}_j^T\mathbf{B}_i^T-\mathbf{B}_j\mathbf{Q}_i-\mathbf{Q}_i^T\mathbf{B}_j^T+4\lambda\mathbf{P}\le 0,\forall i<j\le 8\end{aligned} \tag{37}$$

The solution of LMIs will give $\mathbf{P}$ and $\mathbf{Q}_i$ matrices from which the control gains can be determined as:

$$\mathbf{K}_i=\mathbf{Q}_i\mathbf{P}^{-1},\forall i=1-8 \tag{38}$$

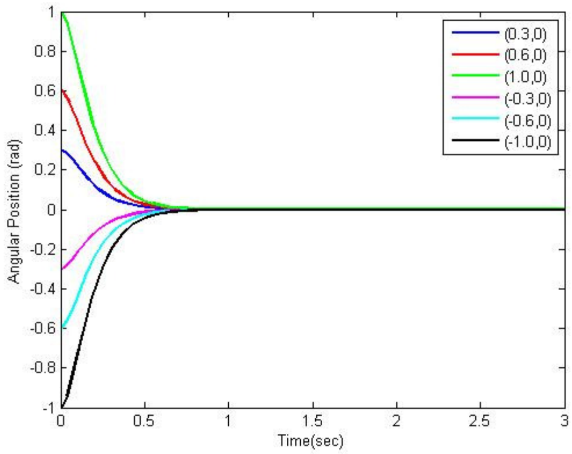
The above set of 37 LMIs (37) is solved using LMI toolbox of MATLAB with $\lambda=1$ and following control gains and symmetric positive definite matrix are found:

$$\begin{aligned}\mathbf{K}_1&=\begin{bmatrix}2.4249 & 0.5923\end{bmatrix}\\ \mathbf{K}_2&=\begin{bmatrix}1.5788 & 0.3869\end{bmatrix}\\ \mathbf{K}_3&=\begin{bmatrix}2.4692 & 0.6691\end{bmatrix}\\ \mathbf{K}_4&=\begin{bmatrix}1.5175 & 0.4275\end{bmatrix}\\ \mathbf{K}_5&=\begin{bmatrix}2.6174 & 0.5716\end{bmatrix}\\ \mathbf{K}_6&=\begin{bmatrix}1.8637 & 0.3917\end{bmatrix}\\ \mathbf{K}_7&=\begin{bmatrix}2.7258 & 0.6453\end{bmatrix}\\ \mathbf{K}_8&=\begin{bmatrix}1.8669 & 0.4385\end{bmatrix}\end{aligned} \tag{39}$$

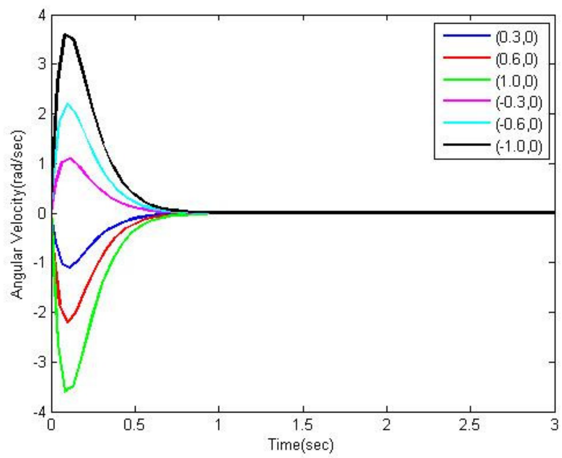
$$\mathbf{P}=\begin{bmatrix}0.5153 & -2.1941\\ -2.1941 & 11.2771\end{bmatrix} \tag{40}$$

IV. SIMULATION RESULTS

The designed controller is simulated in MATLAB/Simulink environment for stabilization and tracking control of SLM. We select M =0.05Kg and b =0.03Nms/rad from the compact region for simulation purpose. The stabilization result is depicted in Fig. 2 for various initial conditions. Note that the reference generator for the stabilization $\mathbf{w}(t)$ has zero initial conditions. It can be seen that controller has remained successful to stabilize the plant. The step response of the controller is shown in Fig. 3. A set of constant reference points are also generated and controller is found to track these set points offering no overshoot, zero steady state error and less than 1sec settling time as evident from Fig. 4. Square wave reference tracking by the controller is shown in Fig. 5. It should be noted that the steady state input for all these reference signals is computed as: $u_{ss}(t)=-\mathbf{\Gamma}^1(t)\mathbf{w}(t)$. Performance of the controller for sinusoidal reference signals is also evaluated. Perfect tracking is achieved as seen from simulation results in Fig. 6, where the tracking error converges to zero within 1sec. Note that the steady state input in this case is generated as: $u_{ss}(t)=-\mathbf{\Gamma}^2(t)\mathbf{w}(t)$. For the purpose of comparison, a pole placement controller is designed for the same transient performance as offered by fuzzy logic controller $\left(T_s=0.6s,\xi=1\right)$. The comparison result in the form of tracking error is depicted in Fig. 7 when both the controllers are made to track the sinusoidal reference signal with angular position varying in the range [-1,1] rad. It can be seen that steady state error exists in case of pole placement controller while fuzzy logic controller exactly tracks the input signal after a transient.

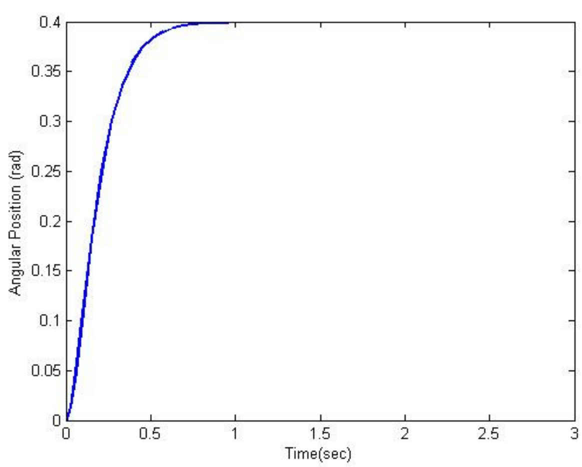


(a)



(b)

Figure 2. Stabilization of SLM for various initial conditions (a) Angular position (b) Angular velocity



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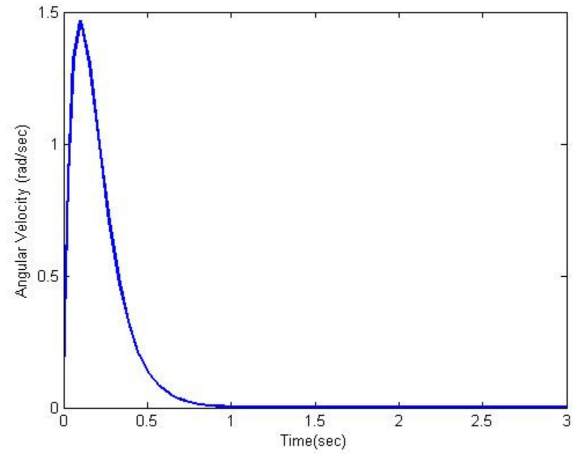
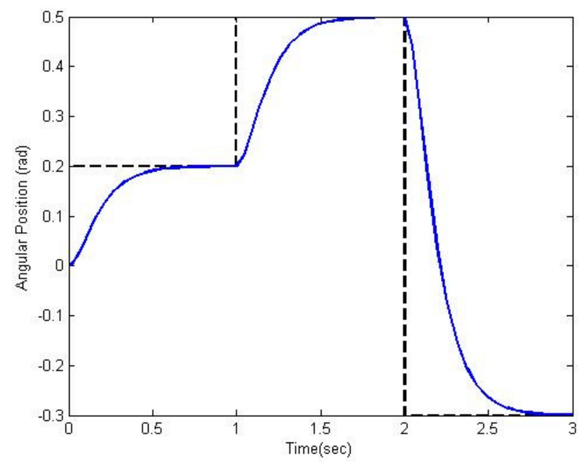
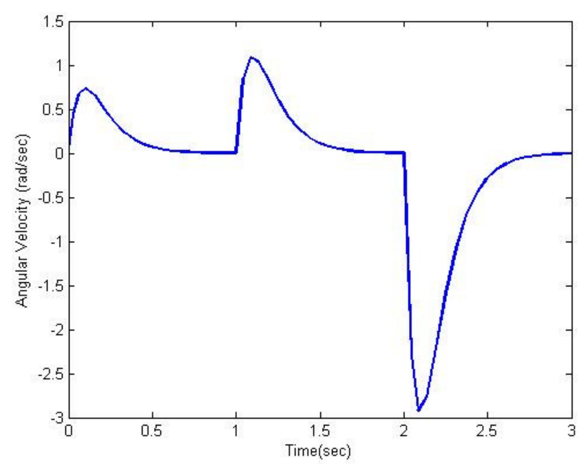


(b)

Figure 3. Step response (a) Angular position (b) Angular velocity

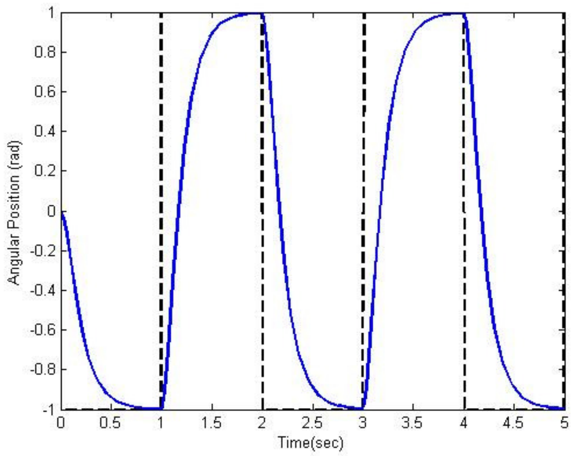


(a)

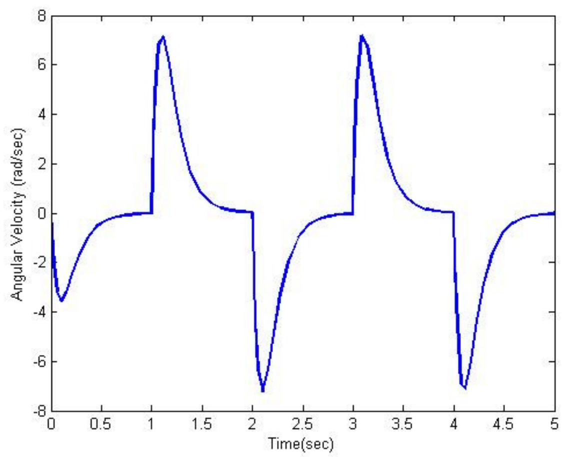


(b)

Figure 4. Reference tracking (a) Angular position (b) Angular velocity

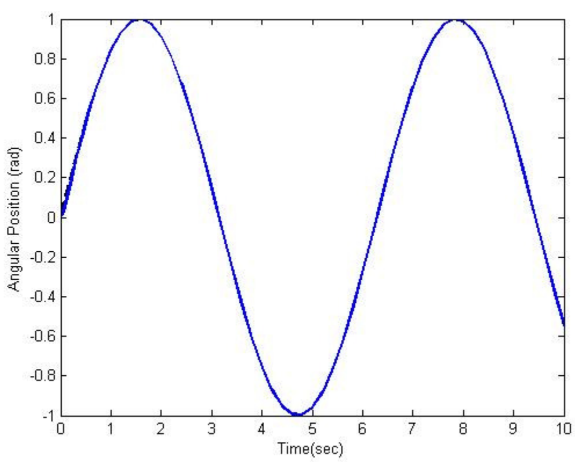


(a)

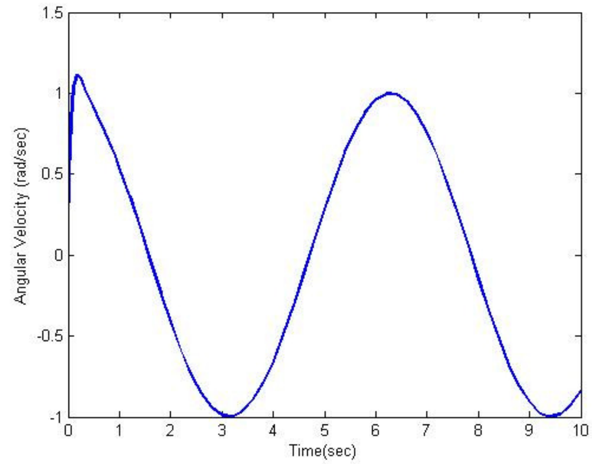


(b)

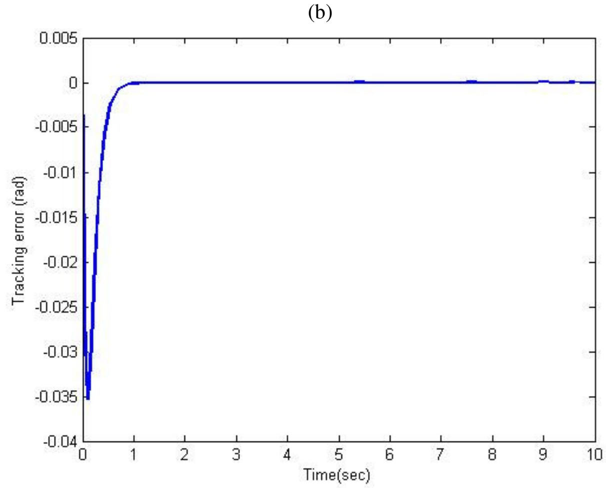
Figure 5. Square wave tracking (a) Angular position (b) Angular velocity



(a)



(b)



(c)

Figure 6. Sine wave tracking (a) Angular position (b) Angular velocity (c) Tracking error

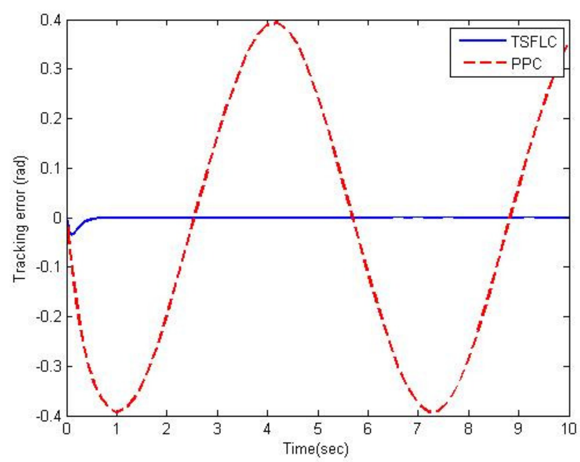


Figure 7. Comparison of TS FLC and PPC for Sine wave tracking

V. CONCLUSIONS

TS fuzzy model of uncertain single link manipulator is derived using set of operations point technique. For the purpose of demonstration, mass of the disc and friction coefficient are assumed as uncertain parameters. For exact output regulation, a PDC controller in conjunction with FIB theory is designed. MATLAB simulations are then performed to validate the designed controller for tracking constant and time varying trajectories. A comparison with pole placement controller is also drawn. Future work involves the design of estimation law for immeasurable scheduling vector.

REFERENCES

[1] M. T. Ho and Y. W. Tu, "Position control of a single link flexible manipulator using H∞ based PID control," IEE Proceedings of Control Theory, vol. 153, no. 5, 2006, pp. 615-622.

[2] A. R. Sahab and M. R. Modabbernia, "Backstepping method for single link flexible joint manipulator using genetic algorithm," International Journal of Innovative Computing, Information and Control, vol. 7, no. 7(B), 2011, pp. 4161-4170.

[3] K. Y. Song, M. M. Gupta and N. Homma, "Design of an error based adaptive controller for a flexible robot arm using dynamic pole motion approach," Journal of Robotics, vol. 2011, pp. 1-9.

[4] I. H. Akyuz, Z. Bingul and S. Kizir, "Cascade fuzzy logic control of a single link flexible joint manipulator," Turkish Journal of Electrical Engineering and Computer Sciences, vol. 20, no. 5, 2012, pp. 713-726.

[5] M. S. Alam and M. O. Tokhi, "Hybrid fuzzy logic control with genetic optimization for single link flexible manipulator," Engineering Applications of Artificial Intelligence, vol. 21, no. 6, 2008, pp. 858-873.

[6] M. A. Ahmad, M. Z. M. Tumari and A. N. K. Nasir, "Composite fuzzy logic control approach to a flexible joint manipulator," International Journal of Advanced Robotic Systems, vol. 10, no. 58, 2013, pp. 1-9.

[7] M. R. Kandroodi, M. Mansouri, M. A. Shoorehdeli and M. Teshnehlab, "Control of flexible joint manipulator via reduced rule based fuzzy control with experimental validation," ISRN Artificial Intelligence, vol. 2012, 2012, pp. 1-8.

[8] M. P. A. Santim, M. C. M. Teixeira, W. A. de Souza, R. Cardim and E. Assuncao, "Design of a Takagi-Sugeno fuzzy regulator for a set of operation points," Mathematical Problems in Engineering, vol. 2012, 2012, pp. 1-17.

[9] J. A. Meda-Campana, J. C. Gomez-Mancilla and B. Castillo-Toledo, "Exact output regulation for nonlinear systems described by Takagi-Sugeno fuzzy models," IEEE Transactions on Fuzzy Systems, vol. 20, no. 2, 2012, pp. 235-247.

[10] H. O. Wang, K. Tanaka, M. F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," IEEE Transactions on Fuzzy Systems, vol. 4, no. 1, 1996, pp. 14-23.