Fuzzy LMI Integral Control of DC Series Motor

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***Abstract*—Based on the Takagi Sugeno (TS) fuzzy model of DC series motor, an integral controller is designed for controlling its speed. The nonlinear plant is converted into regional fuzzy models for which the control gains are found through a set of linear matrix inequalities (LMIs). The local fuzzy controllers are blended afterwards through parallel distributed compensation scheme to yield the final control law. MATLAB simulation results are presented to show the effectiveness of the designed controller.**

Keywords-DC series motor;TS fuzzy model;LMI integral controller;Parallel distributed compensation;MATLAB Simulink

I. INTRODUCTION

DC series motor is constructed by placing the field circuit in series wit the armature circuit and is therefore a popular choice for wide variety of applications which require high torques at low speeds such as electric traction applications [1]-[3]. The mathematical model of this motor is nonlinear due to a square law relationship between the torque it produces and the current supplied to it. Another source of nonlinearity is the product of current and speed which constitutes the back EMF. The model can be linearized for small range of operation. However, the dynamic operation requires the nonlinear model for designing the motor controller.

A variety of nonlinear control methods have been reported in literature for the speed control of dc series motors [4]-[15]. The use of feedback linearization technique is reported in [4]. Through nonlinear state transformation, a linear control law is designed to regulate the speed of the motor. A nonlinear observer for speed and load torque is also constructed based on the current measurements. The real time results of the proposed approach are also presented. Another nonlinear control method known as back stepping is employed in [8] to control the speed of dc series motor. By introducing the virtual control inputs in a recursive fashion, the control Lyapunov functions are found. An improved version of this method is reported in [9] for achieving better transient performance.

The model free techniques such as fuzzy logic and neural networks have also been used for the control of dc series motors [10]-[13]. A PID-ANN controller in [10] acquires training data from a conventional PID controller and the trained controller drives the dc motor through a chopper. The controller is also implemented in real time using an 80C51 microcontroller. A rule base fuzzy logic controller is described in [11] which employs speed and current controllers for dc series motor. Both the controllers accept error and change in error from the set points as inputs and generate the firing signals for thyristors after evaluating 49 rules in the rule base. The performance of the designed fuzzy scheme shows improvement compared with classical PI control strategy. The power of fuzzy logic controller for speed of dc series motors is presented in [12] where a simple fuzzy logic controller with a single input and single output is shown to perform better than PI controller.

This paper describes model based fuzzy control of dc series motor. Using Takagi-Sugeno fuzzy modeling approach, the nonlinear terms in the mathematical model are translated as fuzzy sets which yield a number of local linear models. A set of local controllers corresponding to these models are obtained after solving a set of LMIs which also guarantee the global stability of the control scheme. MATLAB simulations are performed to show the validity of the designed controller. The contribution of the paper lies in the design of integral fuzzy disturbance rejection controller for DC series motor with an objective to improve the transient performance of the system when load torque is changing frequently. The proposed controller clearly performs better than the pole placement state feedback controller as evident from the simulation results.

We start by developing the TS fuzzy model of DC series motor in Section II. Controller design is presented in Section III followed by results in Section IV. Conclusions are drawn in Section V.

II. TS FUZZY MODEL

The motion model of DC series motor after neglecting the magnetic saturation in field circuit can be given as [9]:

$$L\frac{di}{dt} = u - Ri - Mi\omega \tag{1}$$

$$J\frac{d\omega}{dt} = Mi^2 - T_L \tag{2}$$

$$T_e = Mi^2 \tag{3}$$

$$E = Mi\omega \tag{4}$$

where, 'i'is the armature (or field) current, 'u'is the terminal voltage, 'ω'is the rotation speed of the motor, 'L'is the net armature and field circuit inductance, 'R'is the net armature and field circuit resistance, 'J'is the inertia associated with both the motor and the load, 'M'is the

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motor constant, ' T_L 'is the load torque, ' T_e 'is the electromagnetic torque, and ' E 'is the back EMF. The motor parameters are adopted from [9] and are listed in Table 1.

Let, the state variables for the system be:

$$x_1 = \omega, x_2 = i, y = x_1 \tag{5}$$

Using (5), the system equations (1)-(2) can be given as:

$$\frac{dx_1}{dt} = \frac{M}{J} x_2{}^2 - \frac{T_L}{J} \tag{6}$$

$$\frac{dx_2}{dt} = -\frac{M}{L} x_2 x_1 - \frac{R}{L} x_2 + \frac{u}{L} \tag{7}$$

We can write (6)-(7) in matrix form as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{M}{J} x_2 \\ -\frac{M}{L} x_2 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} u + \begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix} T_L$$
$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{8}$$

The general form of (8) is given as:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u + \mathbf{E}d$$
$$y = \mathbf{C}\mathbf{x} \tag{9}$$

where, ' $\mathbf{A}$ 'is the system matrix, ' $\mathbf{B}$ 'is the input vector, ' $\mathbf{E}$ 'is the disturbance vector, ' $\mathbf{x}$ ' is the system state vector, ' u 'is the control input which is the motor voltage, ' d 'is the disturbance input which in this case is taken to be the load torque and ' $\mathbf{C}$ 'is the output vector.

$$\mathbf{A} = \begin{bmatrix} 0 & \frac{M}{J} x_2 \\ -\frac{M}{L} x_2 & -\frac{R}{L} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \mathbf{E} = \begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix} \tag{10}$$

It can be seen from (10) that system matrix is state dependent. Thus, we will represent it in TS frame work. Let the state variable ' x_2 ' be taken as premise variable (also known as the scheduling variable) that takes on values in the interval $[x_{2L}, x_{2U}]$. Then we can define the two triangular fuzzy sets namely ' M_1 'and ' M_2 ' that will be centered at x_{2L} and x_{2U} respectively, as shown in Fig. 1. Thus, we have two local subsystems given as:

$$\dot{\mathbf{x}}_i = \mathbf{A}_i \mathbf{x} + \mathbf{B}_i u + \mathbf{E}_i d, \quad i = 1, 2 \tag{11}$$

where,

TABLE I. DC SERIES MOTOR PARAMETERS

Parameters	Values
R	1 Ω
L	0.05 H
M	0.027 H
J	0.5 Kgm^2

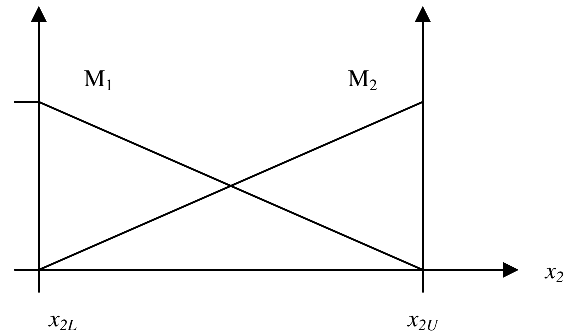


Figure 1. Fuzzy sets for the premise variable

$$\mathbf{A}_1 = \begin{bmatrix} 0 & \frac{M}{J} x_{2L} \\ -\frac{M}{L} x_{2L} & -\frac{R}{L} \end{bmatrix}, \mathbf{B}_1 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}, \mathbf{E}_1 = \begin{bmatrix} -\frac{1}{J} \\ 0 \end{bmatrix}$$
$$\mathbf{A}_2 = \begin{bmatrix} 0 & \frac{M}{J} x_{2U} \\ -\frac{M}{L} x_{2U} & -\frac{R}{L} \end{bmatrix}, \mathbf{B}_2 = \mathbf{B}_1, \mathbf{E}_2 = \mathbf{E}_1 \tag{12}$$

Based on (11)-(12), the two rule TS-fuzzy model for the nonlinear DC series motor model (8) can be constructed as:

Rule 1: IF $x_2(t)$ is M_1 THEN $\dot{\mathbf{x}}_1 = \mathbf{A}_1\mathbf{x} + \mathbf{B}_1 u + \mathbf{E}_1 d$

Rule 2: IF $x_2(t)$ is M_2 THEN $\dot{\mathbf{x}}_2 = \mathbf{A}_2\mathbf{x} + \mathbf{B}_2 u + \mathbf{E}_2 d$

The net TS-fuzzy model is then given as:

$$\dot{\mathbf{x}} = \sum_{i=1}^{2} h_i(x_2(t))(\mathbf{A}_i\mathbf{x} + \mathbf{B}_i u + \mathbf{E}_i d) \tag{13}$$

where, $h_i(x_2(t))$ is the normalized firing strength of i^{th} rule.

$$h_i(x_2(t)) = \frac{w_i(x_2(t))}{\sum_{i=1}^{2} w_i(x_2(t))} \tag{14}$$

where, $w_i(x_2(t))$ is the firing strength of i^{th} rule.

$$w_i(x_2(t)) \geq 0, \sum_{i=1}^{2} w_i(x_2(t)) = 1 \tag{15}$$

III. TS FUZZY CONTROLLER

The control objective here is to design a controller ' u 'that will be able to maintain the reference speed ' ω_{ref} ' by rejecting the jumping load torque disturbances. The employed control structure is shown in Fig. 2. From the integral controller in Fig. 2, we can write the error dynamics as:

$$\dot{\xi} = \omega_{ref} - y = -\mathbf{C}\mathbf{x} + \omega_{ref} \tag{16}$$

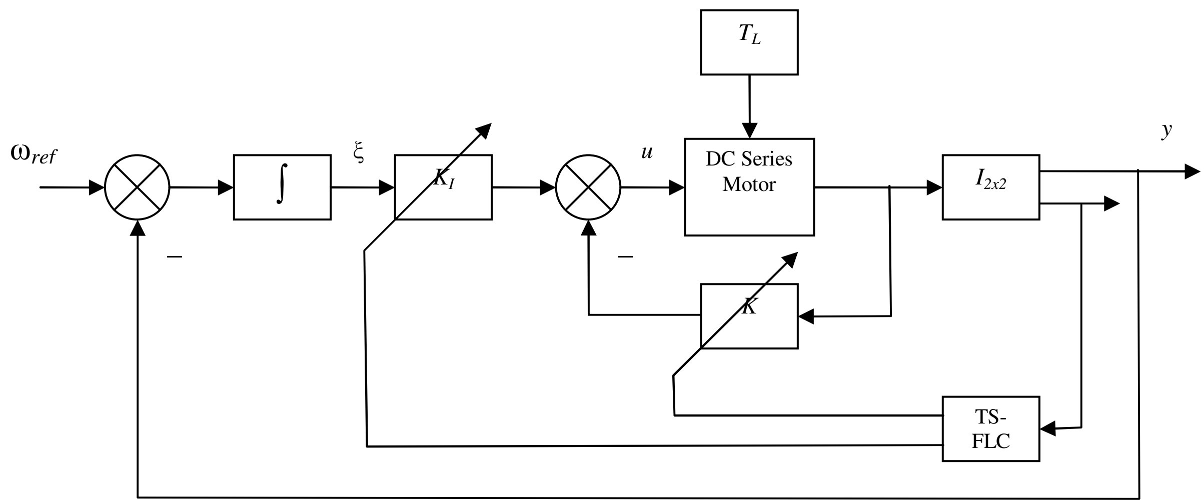


Figure 2. Fuzzy LMI integral controller

Combining (11) and (16), the augmented system dynamics can be given as:

$$\begin{bmatrix} \dot{\mathbf{x}}_i \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ \xi \end{bmatrix} + \begin{bmatrix} \mathbf{B}_i \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{E}_i \\ 0 \end{bmatrix} d + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} \omega_{ref} \tag{17}$$

$$y = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_i \\ \xi \end{bmatrix}$$

The general form of (17) is given as:

$$\dot{\tilde{\mathbf{x}}}_i = \tilde{\mathbf{A}}_i \tilde{\mathbf{x}} + \tilde{\mathbf{B}}_i u + \tilde{\mathbf{E}}_i d + \tilde{\mathbf{F}} \omega_{ref} \tag{18}$$

The control rules for the augmented TS-fuzzy model (17) can be formulated as:Rule 1: IF $x_2(t)$ is M_1 THEN $u(t) = -\tilde{\mathbf{K}}_1 \tilde{\mathbf{x}}(t)$

Rule 2: IF $x_2(t)$ is M_2 THEN $u(t) = -\tilde{\mathbf{K}}_2 \tilde{\mathbf{x}}(t)$

where, $\tilde{\mathbf{K}}_i = \begin{bmatrix} \mathbf{K}^i & -K_I^{\,i} \end{bmatrix}$ is the control gain for i^{th} rule.

The overall TS-fuzzy control law will be given as:

$$u(t) = -\sum_{i=1}^{2} h_i\left(x_2(t)\right) \tilde{\mathbf{K}}_i \tilde{\mathbf{x}}(t) \tag{19}$$

In order to find the control gains '$\tilde{\mathbf{K}}_i$' for the local fuzzy models, we impose certain performance constraints [15]. First, the controller should be able to minimize the effect of jumping load torque disturbances on the speed profile of the DC series motor i.e., 'γ'in the following inequality needs to be minimized:

$$\sup_{\|d(t)\|_2 \neq 0} \frac{\|y(t)\|_2}{\|d(t)\|_2} \leq \gamma \tag{20}$$

Second, the controller should provide good transient performance for disturbance rejection and reference tracking tasks. Thus, the states should decay at a rate 'α' which requires the derivative, $\dot{V}\left(\tilde{\mathbf{x}}(t)\right)$ of quadratic lyapunov function, $V\left(\tilde{\mathbf{x}}(t)\right) = \tilde{\mathbf{x}}^T \mathbf{P} \tilde{\mathbf{x}}$ to satisfy the following inequality:

$$\dot{V}\left(\tilde{\mathbf{x}}(t)\right) < -2\alpha V\left(\tilde{\mathbf{x}}(t)\right) \tag{21}$$

Third, the control input should be realizable in real time, i.e.:

$$\|u(t)\|_2 \leq \mu \tag{22}$$

For the design constraints in (20)-(22), the inequalities in (23)-(25) should hold for a symmetric positive definite matrix, $P > 0$.

$$\begin{bmatrix} -\frac{1}{2}\left(\tilde{\mathbf{A}}_{c,ij}^{\;T} \mathbf{P} + \mathbf{P} \tilde{\mathbf{A}}_{c,ij} + \tilde{\mathbf{A}}_{c,ji}^{\;T} \mathbf{P} + \mathbf{P} \tilde{\mathbf{A}}_{c,ji} \right) & * & * \\ -\frac{1}{2}\left(\tilde{\mathbf{E}}_i + \tilde{\mathbf{E}}_j \right)^T \mathbf{P} & \gamma^2 \mathbf{I} & \mathbf{0} \\ \frac{1}{2}\left(\tilde{\mathbf{C}}_i + \tilde{\mathbf{C}}_j \right) & \mathbf{0} & \mathbf{I} \end{bmatrix} \geq 0 \tag{23}$$

$$\tilde{\mathbf{A}}_{c,ii}^{\;T} \mathbf{P} + \mathbf{P} \tilde{\mathbf{A}}_{c,ii} + 2\alpha \mathbf{P} < 0, \forall i$$

$$\frac{\left(\tilde{\mathbf{A}}_{c,ij} + \tilde{\mathbf{A}}_{c,ji} \right)^T}{2} \mathbf{P} + \mathbf{P} \frac{\left(\tilde{\mathbf{A}}_{c,ij} + \tilde{\mathbf{A}}_{c,ji} \right)}{2} + 2\alpha \mathbf{P} \leq 0, i < j \tag{24}$$

$$\begin{bmatrix} 1 & \mathbf{x}(0)^T \\ \mathbf{x}(0) & \mathbf{P} \end{bmatrix} \geq 0, \begin{bmatrix} \mathbf{P} & \mathbf{P} \tilde{\mathbf{K}}_i^{\,T} \\ \tilde{\mathbf{K}}_i \mathbf{P} & \mu^2 \mathbf{I} \end{bmatrix} \geq 0 \tag{25}$$

where, '$*$'denotes the transposed entry, and

$$\tilde{\mathbf{A}}_{c,ij} = \tilde{\mathbf{A}}_i - \tilde{\mathbf{B}}_i \tilde{\mathbf{K}}_j \tag{26}$$

The inequalities in (23)-(25) are not LMIs. However, they can be recast as LMIs through congruence transformation (which preserves the definiteness property) and intermediate matrix variables. By pre- and post-multiplying (23)-(24) with matrices, $\mathbf{X} = diag\left\{\mathbf{P}^{-1}, \mathbf{I}, \mathbf{I}\right\}$, $\mathbf{Y} = \mathbf{P}^{-1}$,respectively and by defining $\tilde{\mathbf{P}} = \mathbf{P}^{-1}$, $\tilde{\mathbf{Q}}_i = \tilde{\mathbf{K}}_i \tilde{\mathbf{P}}$,the control gains can be determined as:

$$\tilde{\mathbf{K}}_i = \tilde{\mathbf{Q}}_i \tilde{\mathbf{P}}^{-1} \tag{27}$$

By considering, $x_{2L} = 20$, $x_{2U} = 200$, $\gamma = 0.2$, $\alpha = 5$, $\mu = 230$;the following control gains and symmetric positive definite matrix are obtained using LMI toolbox of MATLAB:

$$\tilde{\mathbf{K}}_1 = \begin{bmatrix} 66.8561 & 4.9888 & -350.7599 \end{bmatrix} \tag{28}$$

$$\tilde{\mathbf{K}}_2 = \begin{bmatrix} 57.6915 & 6.0023 & -332.9448 \end{bmatrix} \tag{29}$$

$$\mathbf{P} = \begin{bmatrix} 0.0421 & -0.2594 & 0.0034 \\ -0.2594 & 4.1790 & -0.0006 \\ 0.0034 & -0.0006 & 0.0006 \end{bmatrix} \times 10^3 \tag{30}$$

IV. RESULTS

The designed controller is simulated in MATLAB/Simulink environment for various speed references and load torques. For a reference speed of 80rad/sec and load torque of 50Nm, the simulation results are shown in Fig. 3. With the same load torque applied, the controller is made to track a set of reference speeds ({90,100,110} rad/sec) while motor is already running at 80rad/sec. The results for this case are shown in Fig. 4. Then a sequence of jumping load torques ({60,70,80}Nm) are applied while the motor is running at reference speed of 100 rad/sec with initial load torque of 50Nm. The controller is able to reject the load torque disturbances as evident from Fig. 5.

From the simulation results, the settling time for the controller is found to be 0.72 sec. The deviation from the reference speed is recorded to be 0.36 rad/sec when the load torque disturbance is applied in steady state and is rejected in 0.64 sec by the two rule fuzzy controller. The ripples in the current are limited to $\pm$ 0.4A of the actual value during steady state.

The ripples in the current and voltage can be reduced by employing a larger rule base controller which implies the need of using more membership functions covering the universe of discourse.

The proposed TS FLC is also compared with pole placement controller (PPC). The pole placement controller is designed based on averaged system model for the same transient specifications ($T_s = 0.72s, \xi = 1.0$). The second order desired characteristic equation is therefore formed as: $s^2 + 11.1s + 30.8025 = 0$. The third pole is placed 20 times farther than the dominant closed loop poles. The comparison of TS-FLC and PPC for a reference speed of 50rad/sec is shown in Fig. 7. It can be observed that PPC shows both undershoot and overshoot while tracking the set speed. However, no overshoot is seen in case of TS-FLC. The rejection of load torque disturbances ({60,70,80}Nm) by both the controllers is shown in Fig. 7 while the motor is running at a reference speed of 50 rad/sec. The deviation in speed is found to be less in case of TS-FLC as compared to PPC.

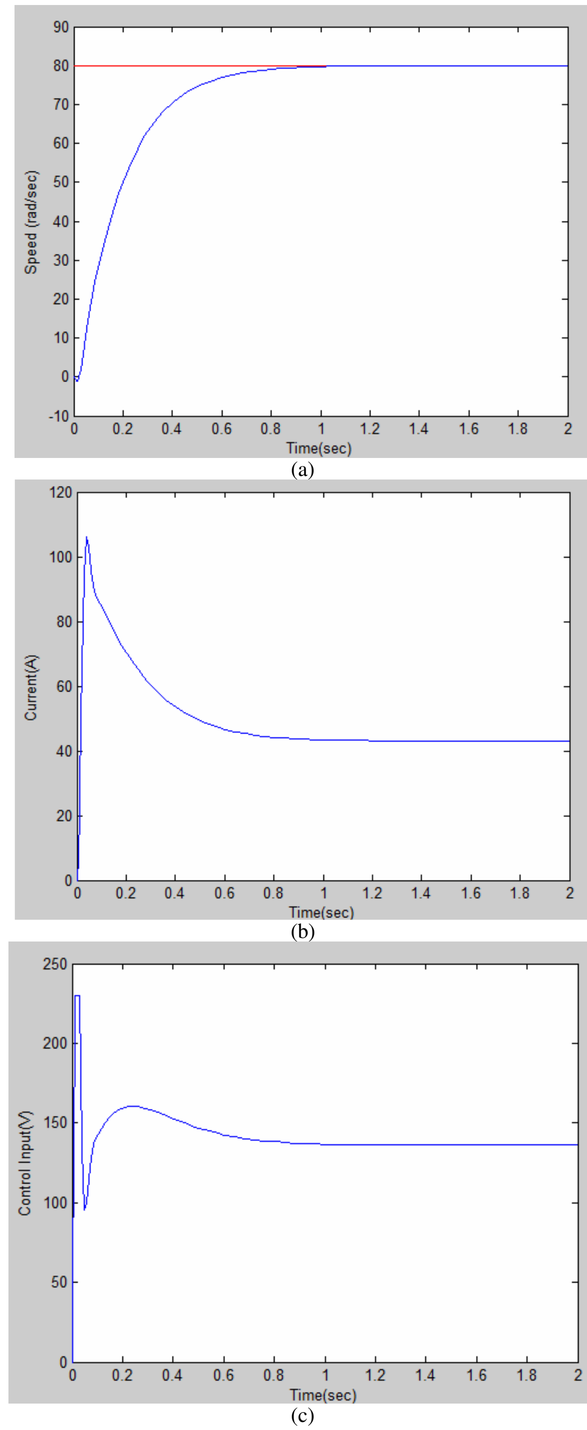
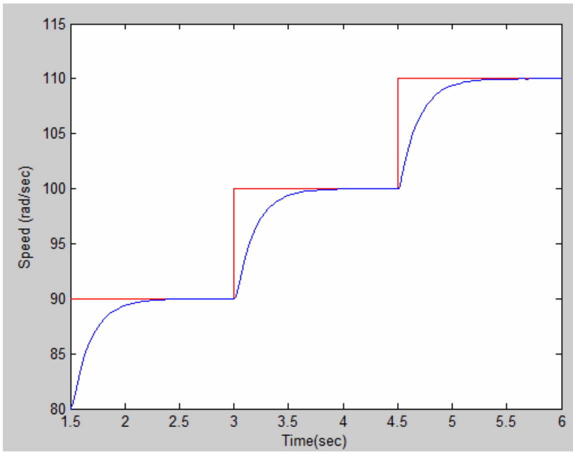
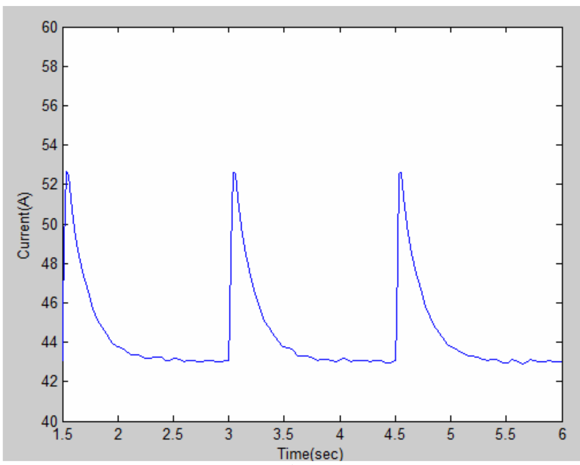


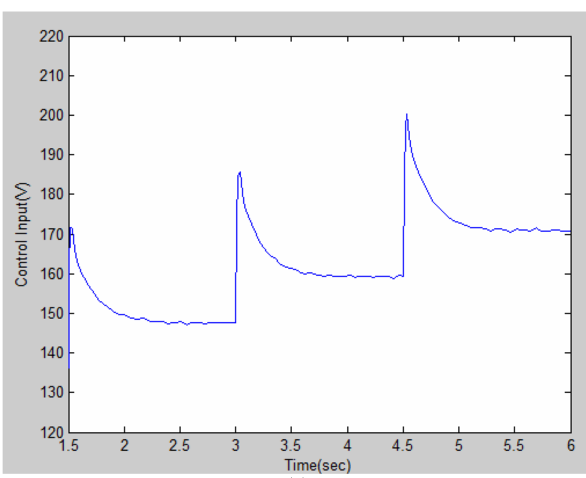
Figure 3. Simulation results for ω_{ref} = 80 rad/s and T_L=50 Nm



(a)

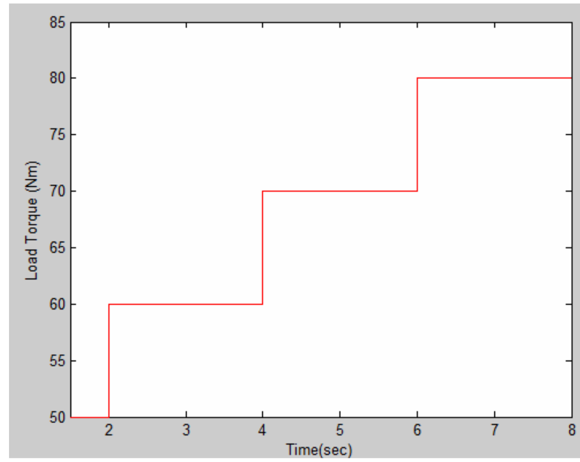


(b)



(c)

Figure 4. Simulation results for ω_{ref} = {90,100,110} rad/s and T_L=50 Nm



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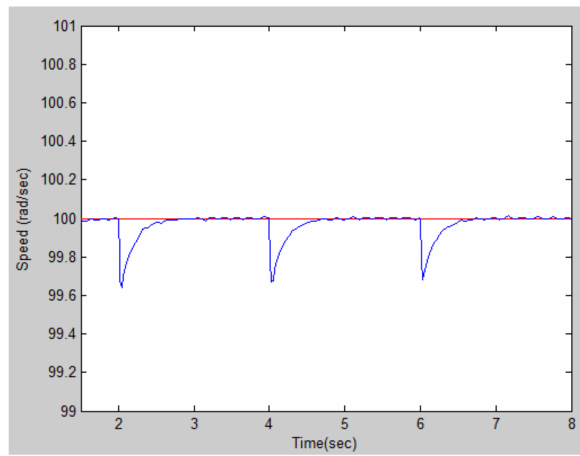

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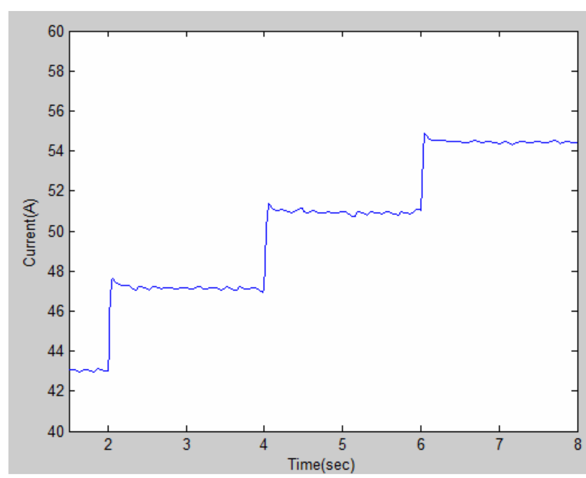

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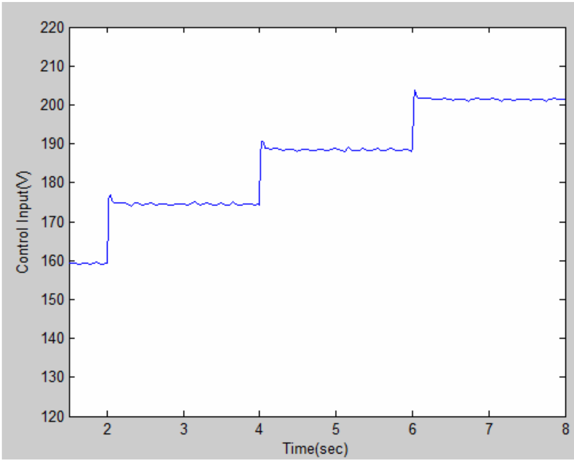

(d)

Figure 5. Simulation results for ω_{ref} = 100 rad/s and T_L= {60,70,80} Nm

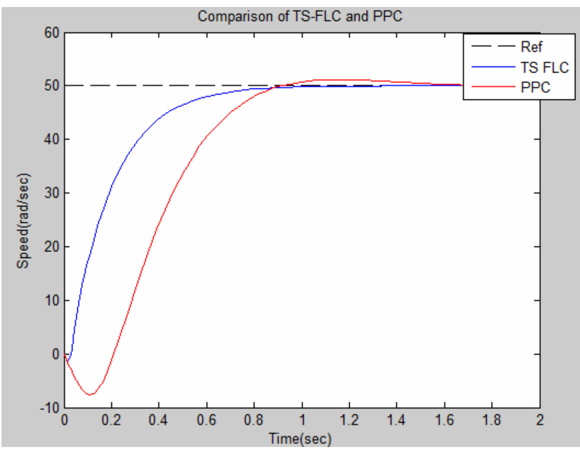

Figure 6. Comparison results for ω_{ref} = 50 rad/s and T_L=50 Nm

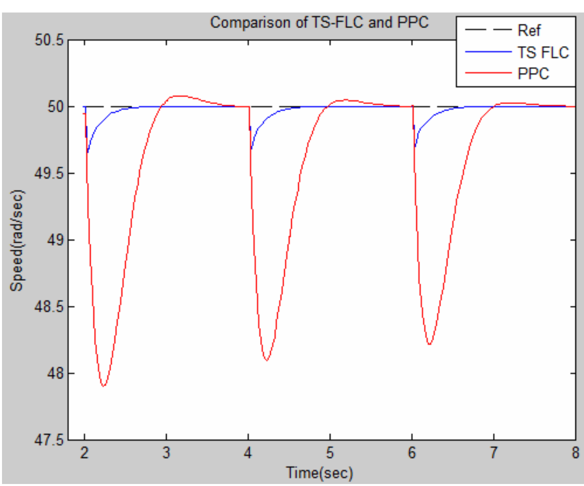

Figure 7. Comparison results for ω_{ref} = 50 rad/s and T_L= {60,70,80} Nm

V. CONCLUSIONS

A simple two rule TS fuzzy model of the DC series motor is constructed followed by a two rule integral controller which shares the same premise membership functions as that of model. The control gains are determined through a set of LMIs which guarantee the closed loop stability of the system for fuzzy regions. The performance of the controller is evaluated through simulation runs in MATLAB environment for various reference speeds and load torque disturbances. Future work involves the design of type-2 fuzzy logic controller.

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