A Quasi Real-Time Parallel FE Analysis of Masonry Walls

Franco Milicchio
Dept. of Informatics and Automation
Roma Tre University
Rome, Italy
milicchio@dia.uniroma3.it

Giovanni Formica
Dept. of Studies on Structures
Roma Tre University
Rome, Italy
formica@uniroma3.it

***Abstract*—We present a parallel implementation of a non-linear finite element analysis of masonry walls. The implementation is based on a shared-memory architecture, while the mechanical simulation is inspired by a model recently developed for this type of structures. Such a model showed to be both reliable and efficient in predicting collapse mechanisms for safety assessment purposes. Its formulation, as we will explain, favors naturally a parallel implementation because the collapse mechanisms are assumed independently for each Finite Element (FE). Additionally, the non-linear response of the same Element is offered at fast computations, because it is based on average elasto-plastic stress distributions which well simulates the more significant mechanical criticisms, *i.e.*, the frictional toughness along squeezing lines.**

***Keywords*-Finite element methods; Nonlinear systems; Parallel programming.**

I. Introduction

In the field of masonry mechanics, there was an extensive production of research works across the last thirty years. However, comparing the existing literature with other structural typologies such as concrete and steel structures, we face an almost embarrassing bias, finding huge deficiencies in both theory, practice, and numerical simulations. In fact, except for reinforced masonry structures, these inadequacies are still reflected in technical codes and predictive software analysis tools, with dramatic results if we observe recent collapses, such as the Umbrian-Marchigian earthquake in 1997, or in Mexican states of Puebla and Oaxaca in 1999, the infamous Tehuacan earthquake, of magnitude 7, damaging about 1800 historic buildings, among them several temples and convents from early colonial era.

The complexity of simulating the behavior of masonry structures is evident when investigating all available predictive tools. Masonry structures manifest different inhomogeneities that require computational approaches to account for different scales, both in length and in time. Numerical simulations struggle to grasp such features, as they cannot be underestimated when extrapolating crucial information on the overall local and global structural behavior: these procedures are still far from being robust and accurate, yielding acceptable results when dealing with full three-dimensional analysis. Multiscale and algebraic multigrid approaches (see [1], [2] and [3] for a reference) recently emerged for masonry mechanics as a possible research direction, on the basis of simplified, yet complicated, similarities with composite structures. A first step on this track has been recently published in [4]; however, the unstructured nature of general mechanical problems dampen the efficiency of such these methods, unless some ad-hoc solutions is adopted, as proposed in [5], [1], and [6].

The present work presents a parallel implementation, based on a shared-memory architecture, of a non-linear finite element analysis of masonry walls. Fine-grained modeling accurately represent several mechanical features, while coarse scale ones achieve better performances in terms of computational time, while sacrificing precision. An alternative approach has been previously proposed in [4], where a fine-scale model is employed in order to generate a coarse-grained finite element formulation. Our objective is to extend the previous work in order to achieve a quasi real-time simulation, *i.e.*, within a time-frame perceived as "immediate". To the best of our knowledge, no attempt has been previously made in simulating the non-linear behavior of masonry walls; however, several works lie in the field of real-time simulation, for example [7] in the field of visco-elastic materials, and [8] in the computer graphics area.

II. Finite Element Formulation

Let us consider an equivalent continuum model, and let us represent its linear elastic behavior by means of a fine-grained modeling of the overall masonry assembly. Such identification technique is known in literature as "refined Cauchy", although several other alternative approaches have been proposed (cf. [9]).

Then, let the constitutive Cauchy law be $\sigma = E\varepsilon$, where σ and ε represent the 2nd order tensors of stress and linear strain, respectively, and E being the 4th order elastic tensor. Our reference finer scale model, initially proposed in [10], is comprised on an assembly of bricks, considered as rigid bodies of dimensions $h \times b \times s$ (*i.e.*, height, width, and thickness), connected to each other by means of a thick mortar joint, modeled as elastic springs, with normal and tangential stiffness, equal to E and G. Therefore, according to the chosen identification technique, both discrete and continuum models possess the same homogeneous strain patterns; the reference elementary volume will be comprised of one brick, and six mortar joints connecting the reference brick to all its neighboring ones. Additionally, as detailed in [9], the rotational field is obtained imposing the momentum balance on the reference elementary volume, thus obtaining the components of the elastic tensor E as:

$$\begin{aligned} E_{1111} &= \frac{1}{2}\frac{b+a}{h+a}\left(\frac{Gs(b+a)}{2a}+2\frac{Es(h+a)}{a}\right) \\ E_{2222} &= \frac{Es(h+a)}{a} \\ E_{1212} &= \frac{\frac{Gs(b+a)}{2a}\left(\frac{Es(b+a)}{2a}+2\frac{Gs(h+a)}{a}\right)}{\frac{1}{2}\frac{b+a}{h+a}\left(\frac{Es(b+a)}{2a}+2\frac{Gs(h+a)}{a}\right)+\frac{Gs(h+a)}{a}} \\ E_{2121} &= E_{1212}\,, \end{aligned}$$

with all the remaining coefficients being zero.

The FE formulation is based on the classical 5β element (see, *e.g.*, [11] and [9]), a quadrilateral assumed stress mixed-form finite element. Let us indicate with (ξ, η) the intrinsic coordinates, and with (x, y) the global ones; the discretized displacement may be therefore expressed as the following:

$$u := (u_x, u_y)^\top = N(\xi, \eta)d\,, \tag{1}$$

with

$$N(\xi, \eta) := \begin{bmatrix} N_1 & 0 & \dots & N_4 & 0 \\ 0 & N_1 & \dots & 0 & N_4 \end{bmatrix}. \tag{2}$$

Functions N_i interpolate the displacement through 2×4 node parameters, assembled in the vector d, with standard bilinear interpolating functions. The masonry wall is then discretized on a regular quadrangular mesh. The FE formulation is locking free [12], its five stress parameters, collected in the vector β, interpolating the stress as follows

$$\sigma := (\sigma_x, \sigma_y, \sigma_{xy})^\top = P(\xi, \eta)\beta\,, \tag{3}$$

with

$$P(\xi, \eta) := \begin{bmatrix} 1 & 0 & 0 & \eta & 0 \\ 0 & 1 & 0 & 0 & \xi \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{4}$$

Finally, the compatibility and equilibrium conditions are expressed as

$$H\beta - Qd = 0 \tag{5}$$
$$Q^\top\beta - p = 0\,, \tag{6}$$

where

$$H := \int_{\Omega_e} P^\top E^{-1} P\,, \qquad Q := \int_{\Omega_e} P^\top DN\,.$$

A. Non-linear Plasticity

The more significant non-linearity is essentially on the frictional behavior; we neglect the coupling with the damage process. More sophisticated numerical simulations, based on fine-scale models and experimental evidences, show that the frictional resistance plays an important role in the structural response under cyclic loading conditions [10], [4], [13]. Within an elastoplastic model, a Mohr-Coulomb criterion is employed in order to characterize the inelastic part of the structural response. The key idea, based on the microplane modeling, is reported in several works such as [14], [15]. The proposed model holds small-strain elastoplasticity and thermodynamical frameworks, being plastic deformation the only dissipative mechanism.

The frictional criterion is then described by the following condition:

$$|\tau_n| - c - \mu\sigma_n \le 0 \tag{7}$$

where τ_n and σ_n are the shear and normal stress, respectively, acting on a plane with normal vector n, while c and μ are the cohesion and static friction coefficient, respectively. Since we are in a context of non-associated plasticity, we can assume the increments $\dot{\varepsilon}_p$ of plastic deformation to be only in the shear direction, *i.e.*:

$$\dot{\varepsilon}_p = \dot{\gamma}\frac{\tau_n}{|\tau_n|} \tag{8}$$

where $\dot{\gamma} \ge 0$ is the increment of the plastic multiplier.

The yield surface $f[\sigma]$ is completed by two conditions bounding normal tension and compression, thus providing the following elastic domain $\mathcal{D}_e$:

$$\mathcal{D}_e := \{\sigma \,:\, f[\sigma] \le 0\}\,, \quad \text{with}$$

$$f[\sigma] := \begin{cases} -c - \mu\sigma_n + |\tau_n| \\ -\sigma_{yt} + \sigma_n \\ \sigma_{yc} + \sigma_n \end{cases} \tag{9}$$

where σ_{yt} and σ_{yc} are the tension and compression yield normal stresses, respectively, and we define compressive stress as negative.

Within the representation of the element stress field, and by means of the Haar-Kármán principle, we usually get the admissible stress field by controlling the stress level by (9) at some Gauss points of each element, and then numerically integrating on the same element. This is a standard way for FE formulations in elastoplasticity, we anyway tested in our numerical implementation.

We follow an alternative approach, which is less computationally expensive, yet accurate enough, as we will show. Following [16], we reformulate the elastoplastic response of the assumed stress FE by adopting a kinematic approach. Such approach defines a discrete number of possible mechanisms, corresponding to the plastic deformations that the

 ISBN: 978-1-61208-101-4

Element exhibits. More specifically, within each element we consider a set $\mathcal{S}_e$ of possible "bands" (*i.e.*, lines in the 2D element), along which the plastic deformation can take place. This corresponds to fix a discrete number of possible directions $\dot{\varepsilon}_{pn}$, depending on the assumed band with normal $n \in \mathcal{S}_e$. To define $\mathcal{S}_e$, we consider the possible collapse mechanisms of a generic assembly of bricks contained in an element. We choose five bands with their three associated plastic mechanisms, as depicted in Figure 1.

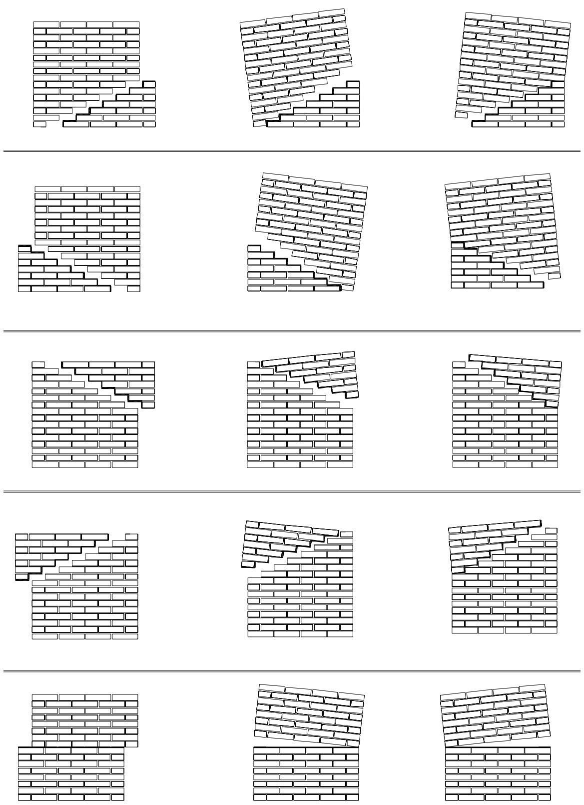

Figure 1. Possible mechanisms that a single FE represents.

Therefore, by imposing the yield conditions in (9) for each band we define for each FE an envelop of planes in the space of the discretized stress components (3).

III. Numerical Solver

A. Overview

Our solver relies on the numerical properties exhibited by non-linear elasto-plastic media. As suggested by the currently available literature, our solver employs a path-following algorithm in order to retrieve the non-linear equilibrium path. The path-following technique [17], recovers the equilibrium path, arising from a non-linear structural response s, subject to a load p varying with a scalar term λ, by means of the following alternative system:

$$\begin{cases} r(u(\xi), \lambda(\xi)) = s(u(\xi)) - \lambda(\xi) p \\ g(u(\xi), \lambda(\xi)) = \xi \,, \end{cases} \tag{10}$$

where g is a known constraining surface, and r represents the equilibrium error.

The system expressed in (10) is then solved by means of a predictor-corrector iterative scheme, starting with an initial solution ($u^0 = 0$, $\lambda^0 = 0$). The solution is attained at convergence on the k-th step in the j-th iteration, providing the ensuing equilibrium point ($u^{k+1} = u_{j*}$, $\lambda^{k+1} = \lambda_{j*}$); the predictor is a trial solution obtained by extrapolation of previous solutions, while the employed corrector is based on an iterative Newton-Raphson scheme:

$$\begin{aligned} r_j &= r(u_j, \lambda_j) = s(u_j) - \lambda_j p \\ \dot{\lambda} &= (u^\top p)^{-1} u^\top r_j \\ \dot{u} &= K^{-1} r_j + \dot{\lambda} u \\ u_{j+1} &= u_j + \dot{u} \\ \lambda_{j+1} &= \lambda_j + \dot{\lambda} \end{aligned}$$

The structural response $s(u_j)$ is evaluated by means of a predictor-corrector scheme, as detailed in [4]. This solution employs the Haar-Kármán principle, *i.e.*:

$$\Phi(\sigma) := 1/2 \int_\Omega (\bar{\sigma} - \sigma)^\top E^{-1} (\bar{\sigma} - \sigma) = \min \,, \tag{11}$$

for all equilibrated stress $\bar{\sigma}$.

B. Parallel Implementation

The solver described in the previous section was implemented on a shared-memory architecture[1], and in the following we will outline the main components along with a speed-up measurement.

First, we highlight the fact that initial and terminal operations in FE analysis, *i.e.*, the stiffness matrix assembly and the output variable update processes, are inherently parallelizable. The non-linear plastic analysis, however, is comprised of parts that are not parallel in the strict sense. A comprehensive diagram of the overall solver architecture is pictured in Figure 2.

Matrix assembly and stress update, being the latter modeled with an incremental algorithm, are carried out in parallel by means of the *reduction* operation. The non-linear structural response of the masonry wall can be easily carried out in a parallel fashion.

The structural response $s(u)$ needed to solve the system (10), is evaluated by means of the Haar-Kármán principle (11), *i.e.*, formulating it as a quadratic programming problem, as expressed in equation (11). The Haar-Kármán principle is local to each element, and may be locally solved

[1]The software was implemented in C++ following the OpenMP 3.0 specification; benchmarks were conducted on an Intel Core 2 Duo processor at 2.9 GHz, with 4 GB of RAM.

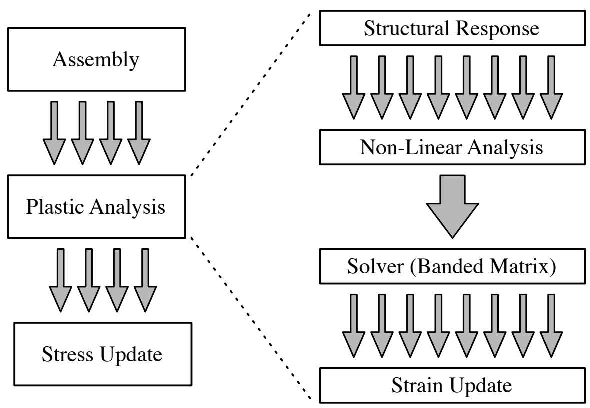

Figure 2. The numerical solver architecture (left), and a detailed plastic analysis diagram (right). Multiple arrows indicate a parallel execution.

Table I
BENCHMARKS FOR THE PROPOSED PARALLEL FE SOLVER.

Threads	1	2	4	8	16	32
Time	2.75	1.68	1.04	0.97	0.65	0.43
Speed-up	1	1.64	2.64	2.83	4.22	6.37

iteratively, adopting the Goldfarb-Idnani method [18]: this allows us to parallelize this process, up to the actual solution of the system of linear equations arising from (10). As pictured in Figure 2, the aforementioned system is solved serially, while the ensuing strain update, and the subsequent output variables, are naïvely parallelized.

IV. RESULTS

The chosen test bed for our parallel implementation is the Pavia's test, an experimental test performed in the University of Pavia [19]. The horizontal displacement employed in the test is pictured in Figure 4, with the overall time of analysis equal to 20 seconds. We mention in passing that a quasi real-time process involves updates of the outcomes with a minimal frequency of 20 Hz circa (cf., [7], [20], and [8]).

Results are reported in Table I, where we reported the number of threads employed in the test bed, the overall time for the nonlinear analysis, and the speed-up [21]; the latter quantity has been calculated as $S := T_i T_1^{-1},\ i = 1, 2, \ldots$, where i indicates the number of threads involved in the analysis.

As expected by known theoretical results (cf. [22] and [23]), we are obtaining a sub-linear speed-up, detailed in Table I, and pictured in Figure 5. In order to better analyze the results, we recall that quasi real-time requires updates at 20 Hz, *i.e.*, 0.05 seconds. Comparing the total analysis time of 20 seconds, with the actual analysis process, we obtain that our FE formulation allows us to obtain a quasi-RT update with a number of threads equal or above four.

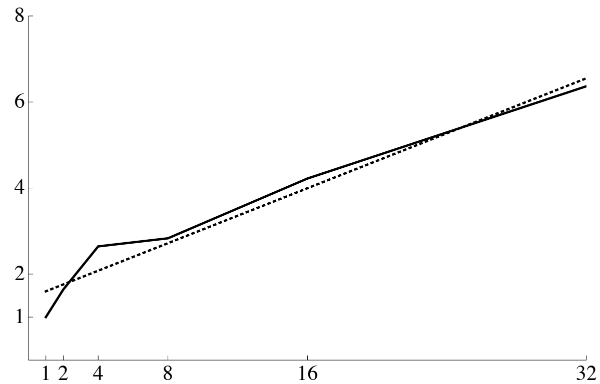

Figure 3. Graph of the speed-up value plotted against the number of processes (in solid black); graph of the linear fit (dotted).

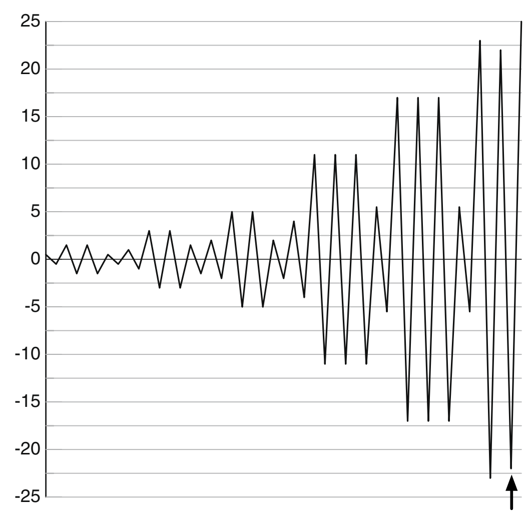

Figure 4. Graph of the horizontal displacement, measured in *mm*, with respect to time. The arrow marks the last input cycle.

V. CLOSING REMARKS

We proposed in this manuscript a parallel implementation of a FE formulation for the analysis of masonry walls involving non-linear plasticity. Such implementation, based on a shared-memory architecture, allows us to obtain results in a quasi real-time fashion.

The proposed FE formulation relies on a fine-grained approach, where a detailed model is employed in the formulation of coarser elements, grasping the non-linear mechanic behavior and obtaining considerably better performances compared to a naïve finer modeling approaches.

Coupling this novel multiscale approach to non-linear plasticity, with a parallel implementation of the analysis process, we are able to hold quasi real-time performances. A future direction of research will investigate all the possible issues affecting performances, clarifying the optimal number of threads on specific architectures, and comparing standard solvers with our custom solution.

 ISBN: 978-1-61208-101-4

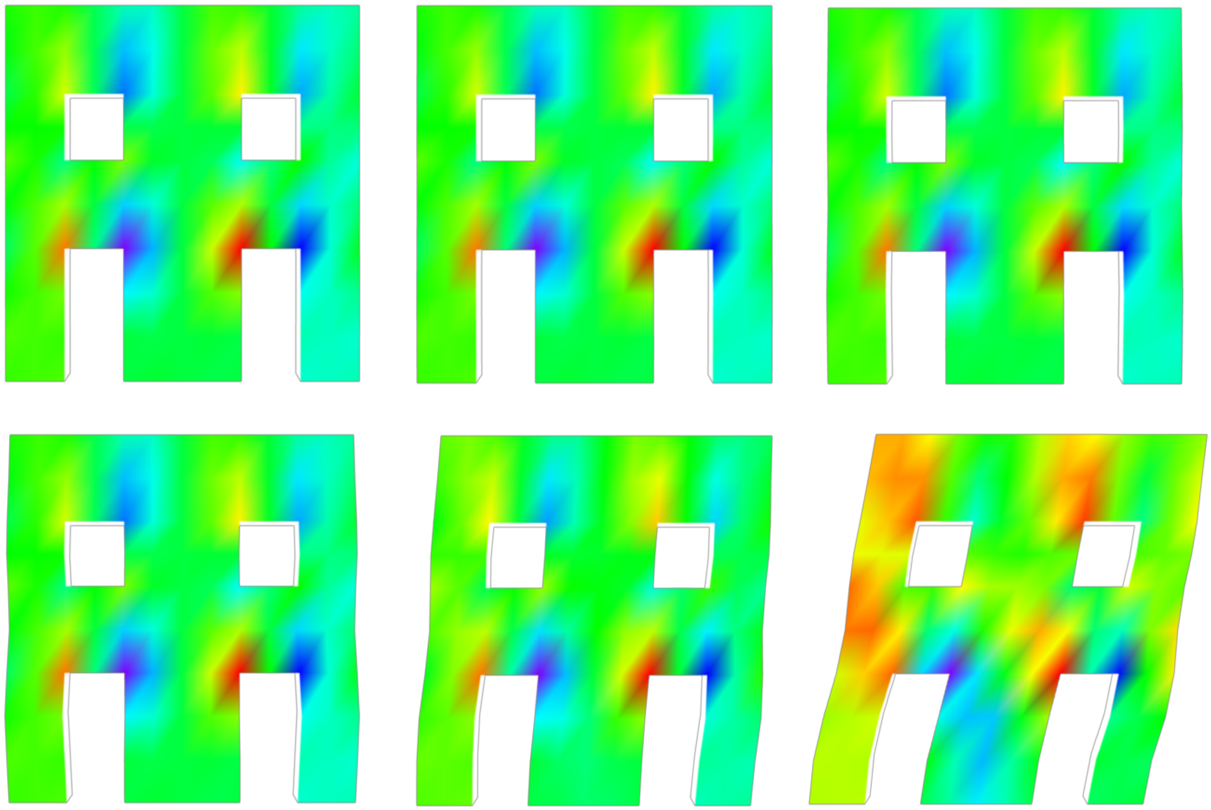

Figure 5. Rendering of the stress σ_{xy} for the last input cycle in the analysis, indicated with an arrow in Figure 4.

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