On the Regularization of a Low-Complexity Recursive Least-Squares Adaptive Algorithm

Cristian-Lucian Stanciu, Cristian Anghel, Camelia Elisei-Iliescu, Laura-Maria Dogariu, Ionuţ-Dorinel Fîciu, and Constantin Paleologu
Department of Telecommunications, Politehnica Bucharest, Romania
Emails: cristian@comm.pub.ro, canghel@comm.pub.ro, camelia.elisei@romatsa.ro, ldogariu@comm.pub.ro, ionut.ficiu22@gmail.com, pale@comm.pub.ro

***Abstract*—The Recursive Least-Squares (RLS) family of adaptive algorithms can be an attractive choice for the identification of unknown acoustic systems, which have hundreds, or even thousands, of coefficients. The RLS have also been combined with Line Search Methods (LSMs) in order to obtain versions without numerical stability issues, and to decrease the corresponding arithmetic complexity. Despite the superior tracking speeds associated with the RLS-LSM methods (with respect to more consecrated algorithms), they remain vulnerable to Double-Talk (DT) situations, when the corresponding update process becomes inaccurate. This paper describes a variable regularization technique for the RLS-LSM general algorithm, which is designed to mitigate DT scenarios by adjusting the contents of the RLS correlation matrix. Simulation results demonstrate the proposed theoretical model in the stereophonic acoustic echo cancellation configuration.**

***Index Terms*—*Recursive Least-Squares (RLS)*; *Line Search Methods (LSMs)*; *Double-Talk (DT)*; *Variable Regularization (VR)*.**

I. Introduction

The identification of unknown acoustic echo paths using adaptive algorithms implies the estimation of impulse responses equivalent to hundreds of millisenconds. Most of the signal processing solutions rely on the Least-Mean-Square (LMS) adaptive methods [1], which have acceptable arithmetic complexities and poor performances when working with highly correlated signals, such as speech.

In this context, the Recursive Least-Squares (RLS) adaptive algorithms are possible alternatives, having superior tracking speeds [2]. However, most RLS versions require prohibitive amounts of resources on most modern chips and manifest numerical stability issues. The complexity problem is even worse when considering the Stereophonic Acoustic Echo Cancellation (SAEC) scenarios, where user terminals employ two microphones, respectively two loudspeakers, to create the impression of audio directionality. The associated setup needs to estimate four acoustic paths, corresponding to each loudspeaker-to-microphone pair.

In [3], Liu et al. combined the exponentially weighted RLS with Line Search Methods (LSMs) in order to approach the corresponding set of normal equations by solving an auxiliary system. The solution avoids the numerical stability issues, and allows the use of less complex LSM variants, which exploit the statistical properties of the input signal. Moreover, for the SAEC setup, the Widely Linear (WL) model was employed in [4] in order to simplify the handling and to allow easier developments of extra features, like mechanisms employed to mitigate the effects of Double-Talk (DT) situations.

TABLE I
WL-RLS-LSM ALGORITHM

Step	Action
Init.	Set: $\widetilde{\mathbf{g}}(0) = \mathbf{0}_{2L\times 1};\ \mathbf{r}(0) = \mathbf{0}_{2L\times 1}$ $\mathbf{R}(0) = \Phi\mathbf{I}_{2L},\ \Phi > 0; 0 < \lambda \leq 1$
	For $n = 1, 2, \ldots,$ number of iterations :
1	Update $2L \times 1$ input vector $\widetilde{\mathbf{x}}(n)$
2	Update correlation matrix $\mathbf{R}(n)$ (time shift) $\mathbf{R}^{(:,1)}(n) = \lambda\mathbf{R}^{(:,1)}(n-1) + x^*(n)\widetilde{\mathbf{x}}(n)$
3	$\widetilde{y}(n) = \widetilde{\mathbf{g}}^H(n-1)\widetilde{\mathbf{x}}(n)$
4	$e(n) = d(n) - \widetilde{y}(n)$
5	$\mathbf{g}_0(n) = \lambda\mathbf{r}(n-1) + e^*(n)\widetilde{\mathbf{x}}(n)$
6	$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{g}_0(n) \xrightarrow[N_u]{\text{LSM}} \Delta\widetilde{\mathbf{g}}(n), \mathbf{r}(n)$
7	$\widetilde{\mathbf{g}}(n) = \widetilde{\mathbf{g}}(n-1) + \Delta\widetilde{\mathbf{g}}(n)$

This paper is organized as follows. Section II describes the WL-RLS-LSM adaptive algorithm with Variable Regularization (VR) of the associated correlation matrix. In Section III, simulation results are discussed for the proposed algorithm in the SAEC setup. The paper draws the main conclusions in Section IV.

II. The VR-WL-RLS-LSM Algorithm

The WL-RLS-LSM adaptive filter working in the SAEC configuration benefits from the simplifications provided by the WL model, and employs a single adaptive filter $\widetilde{g}(n)$ with $2L$ complex valued coefficients in order to estimate the four acoustic loudspeaker-to-microphone impulse responses, each with L real valued coefficients. The input information (corresponding to the two loudspeakers) is grouped in a single complex valued signal $x(n)$, respectively the outputs of the unknown echo paths are combined into $y(n)$. The complex valued microphone information, represented by $d(n)$, comprises the contribution of $y(n)$ cumulated with environmental noise. Consequently, the adaptive filter estimates the complex echo as $\widetilde{y}(n)$ and sends to the interlocutor the error $e(n)$, from which the value $\widetilde{y}(n)$ is subtracted. The WL-RLS-LSM general algorithm is presented in Table I, where $\mathbf{I}_{2L}$ denotes the identity matrix, and ^{H} is the Hermitian operator.

 ISBN: 978-1-68558-153-4

The first two steps of the algorithm are dedicated to the updates of the input vector and the correlation matrix, respectively. The output of the filter results in step 3, while the error signal is computed in step 4. Then, the residual component $\mathbf{g}_0(n)$ is evaluated in step 5. In step 6 of the algorithm, the method employs a complex valued LSM to solve an auxiliary system of equations and generate the *solution vector* $\Delta\mathbf{g}(n)$, which is used in step 7 to update the filter estimate.

A wide range of LSMs can be used for step 6 of the algorithm. For example, the complex valued Conjugate Gradient (CG) method has a complexity proportional to $4L^2$ real valued multiplications and attractive tracking capabilities. A low-complexity alternative for the CG are the Dichotomous Coordinate Descent (DCD) iterations [5], [6], which can solve the system in step 6 using only additions and bit-shifts. The DCD strongly relies on the statistical properties of the input signal as they are reflected in the correlation matrix $\mathbf{R}(n)$. The WL-RLS-DCD variant is more attractive for hardware implementations, because it can function with an overall arithmetic workload proportional to $2L$.

However, regardless of the algorithm used for the SAEC, the WL-RLS-LSM is still susceptive to DT scenarios, when the noise component of $d(n)$ is much higher than the contribution of $y(n)$. A solution proposed in [7] is to alter the purpose of the initialization constant Φ used to avoid the singular nature of $\mathbf{R}(n)$ during the initial iterations of the algorithm. We can compute a regularization coefficient at every time index as

$$\Phi(n) = 2L\widetilde{\sigma}_x^2(n)(1+\sqrt{1+\widetilde{\mathrm{ENR}}})/(\widetilde{\mathrm{ENR}}), \qquad (1)$$

where $\widetilde{\sigma}_x^2(n)$ and $\widetilde{ENR}$ are estimates for the variance of the input signal, respectively the Echo-to-Noise Ratio (ENR). Consequently, we can compare the performances of the WL-RLS-LSM using the CG, respectively DCD, methods with their variable regularized counterparts in DT situations. The complexities of the namely VR-WL-RLS-LSM algorithms remain similar to the original versions.

III. SIMULATION RESULTS

In Figures 1 and 2, the WL-RLS-CG and WL-RLS-DCD methods were compared to the VR-WL-RLS-CG, respectively the VR-WL-RLS-DCD, by simulating a tracking scenario [2], followed by a DT occurence. The input signal is speech, and the ENR was set to 25 dB for Figure 1, respectively to 10 dB for Figure 2. We employed different number of N_u iterations for both algorithms [3], [4]. We can notice in Figure 1 that the VR versions of the algorithms perform better during the DT interval, with the compromise of slightly lower tracking speeds. Moreover, when the ENR is lower (Figure 2), the VR-WL-RLS-CG, respectively the VR-WL-RLS-DCD, clearly outperform their counterparts, including during the steady-state portions of the simulation.

IV. CONCLUSIONS

The VR approach for the WL-RLS-LSM improves the performance of the algorithm for smaller ENR values, respectively when DT situations occur. The increase of performances adds reasonable extra arithmetical effort, respectively slight losses in the tracking speeds. Considering the VR-WL-RLS-DCD has results similar to the VR-WL-RLS-CG, with less necessary arithmetic resources by an order of degree, the former is an attractive choice for practical applications. These regularized algorithms could be more suitable in echo cancellation and noise reduction scenarios, where the long length impulse responses and highly correlated input signals represent significant challenges for the LMS-based algorithms.

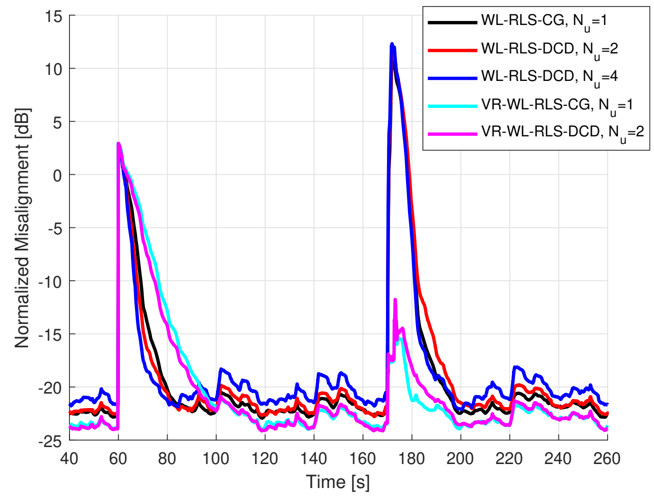

Figure 1. Normalized misalignment of the WL-RLS-CG, WL-RLS-DCD, VR-WL-RLS-CG, and VR-WL-RLS-DCD for different values of N_u. The four unknown echo paths have the length $L = 256$, and the ENR is experimentally set to 25 dB. The echo paths change at time index t_0 = 60 seconds, and a DT situation occurs in the time interval [170,174] seconds.

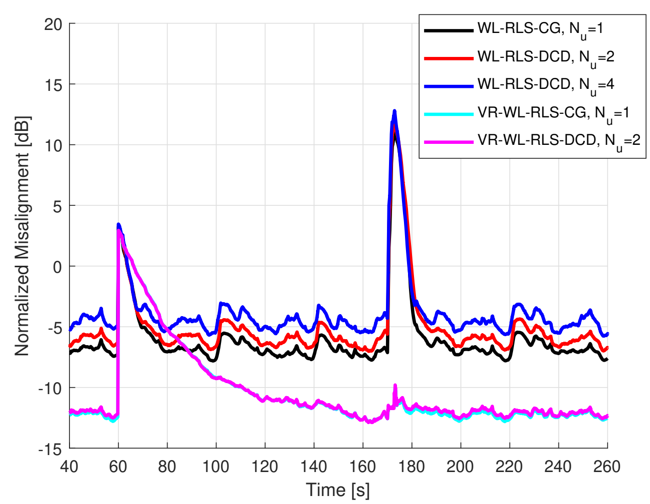

Figure 2. Normalized misalignment of the WL-RLS-CG, WL-RLS-DCD, VR-WL-RLS-CG, and VR-WL-RLS-DCD for different values of N_u. The four unknown echo paths have the length $L = 256$, and the ENR is experimentally set to 10 dB. The echo paths change at time index t_0 = 60 seconds, and a DT situation occurs in the time interval [170,174] seconds.

 ISBN: 978-1-68558-153-4

ACKNOWLEDGMENT

This work was supported by a grant of the Ministry of Research, Innovation and Digitization, CNCS–UEFISCDI, project PN-III-P4-PCE-2021-0438, within PNCDI III.

REFERENCES

[1] S. Haykin, *Adaptive Filter Theory – Fourth Edition*. Prentice Hall, 2002.

[2] J. Benesty, C. Paleologu, T. Gänsler, and S. Ciochină, *A Perspective on Stereophonic Acoustic Echo Cancellation*, vol. 4. Germany: Springer-Verlag, 2011.

[3] J. Liu, Y. V. Zakharov, and B. Weaver, "Architecture and FPGA Design of dichotomous coordinate descent algorithms," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 56, no. 11, pp. 2425–2438, 2009.

[4] C. Stanciu, J. Benesty, C. Paleologu, T. Gänsler, and S. Ciochină, "A widely linear model for stereophonic acoustic echo cancellation," *Signal Processing*, vol. 93, no. 2, pp. 511–516, 2013.

[5] Y. V. Zakharov, G. P. White, and J. Liu, "Low-complexity RLS algorithms using dichotomous coordinate descent iterations," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3150–3161, 2008.

[6] Y. V. Zakharov and V. H. Nascimento, "DCD-RLS adaptive filters with penalties for sparse identification," *IEEE Transactions on Signal Processing*, vol. 61, no. 12, pp. 3198–3213, 2013.

[7] C.-L. Stanciu, C. Anghel, and C. Elisei-Iliescu, "Regularized RLS adaptive algorithm with conjugate gradient method," in *International Conference on Speech Technology and Human-Computer Dialogue (SpeD)*, pp. 18–23, 2023.

 ISBN: 978-1-68558-153-4