A Tool for Generating Ambiguous Objects in Two Viewing Directions

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Abstract—**Ambiguous objects provide visual shape information that can be interpreted differently depending on the viewing direction. Generating effective ambiguity objects is difficult and therefore requires easy-to-use computational modelling. In this paper, we propose a Three-Dimensional (3D) modelling tool to generate an object that can be perceived differently from two different viewing directions. The tool uses solid models of cylindrical surfaces parallel to each of the viewing directions. These models are intersected at the central axis and rotated according to the viewing direction, using the intersection as the origin. Finally, by transforming each Two-Dimensional (2D) figure drawn by the user in each viewing direction into a cylindrical surface, a 3D ambiguous object can be generated. These 2D figures can be drawn using mouse click events. The generated ambiguous objects can be fabricated on a 3D printer to demonstrate the usability of the proposed tool. Our experiments show that ambiguous objects consisting of simple and complex shapes were successfully generated.**

Keywords-3D-Illusion; ambiguous object; ambiguous cylinder; 3D modelling; visual perception.

I. INTRODUCTION

An optical illusion is a phenomenon in which our perception of an object differs from its physical reality. This illusion is important in the study of human visual processing. The study of optical illusions began with Two-Dimensional (2D) images but has now been extended to Three-Dimensional (3D) objects. The former are ambiguous figures that represent a single figure but have multiple interpretable meanings, such as Edgar Rubin's “Face-Vase illusion” [1]. These figures are most often represented in binary images, where the white areas are foreground figures and the black areas are the intangible background, or vice versa. The boundaries shared by these areas play an important role in the figure assignment process. The latter are ambiguous objects, the shape of which can appear to be different depending on the direction in which they are viewed. Such objects can be generated in three ways: by making a discontinuous structure appear continuous from certain viewing directions [2], by the use of curved surfaces instead of planes [3], or by the use of angles other than 90 degrees to create a rectangular appearance [4]. Sugihara (2012) classified ambiguous objects into seven generations and showed that all objects are accompanied by illusions [5].

Computer-aided tools are available to assist in the generation of ambiguous figures and objects. The ambiguous figure generation tool finds partial matches by performing shape matching and deformation of the two figures and then stitching them together to produce the resulting image [6]. However, generating ambiguous objects is more complex. The tools used need to be built on a 3D modelling framework, with geometric modifications to the shapes also being performed in 3D [7]. Furthermore, unlike ambiguous figures, the generated ambiguous objects are viewing direction dependent, requiring the viewpoint to be determined prior to modelling.

In this study, we focused on the following two aspects when developing a tool to facilitate generating ambiguous objects. Firstly, the tool is implemented on top of a 3D modelling framework, but the shape of the individual objects is determined by drawing in 2D. Secondly, the shape of the generated ambiguous objects changes adaptively according to the viewing direction. This implementation allows users unfamiliar with 3D modelling to create ambiguous objects.

The rest of this paper is organized as follows. Section II describes related works on ambiguous object generation. Section III describes our proposed tool for generating the ambiguous objects in two viewing directions. Section IV summarizes our results. Finally, Section V concludes our study and discusses future work.

II. RELATED WORKS

The methods used to generate ambiguous figures and objects can be divided into three categories, as described below.

The first method generates ambiguous figures by manipulating the relationship between edges or faces. Shinohara et al. constructed a system that can portray impossible figures realistically using ray tracing [8]. Their system provides predefined basic parts, where the user can interactively manipulate the rendering depth of each surface with respect to the parts. Owada et al. proposed a system for generating ambiguous figures by taking a 3D object as input and interactively editing the edges of its 2D plane from a given viewing direction [9]. These operations can be performed using mouse events. Both systems facilitate the generation of ambiguous figures, but there is no continuity of edges and faces in the resulting 3D objects.

The second is Fukuda's method of generating ambiguous objects in two viewing directions [10]. These works were published in the book "One solid with two shapes” but are difficult to reproduce because the parameters and optimization methods required for their generation were not disclosed.

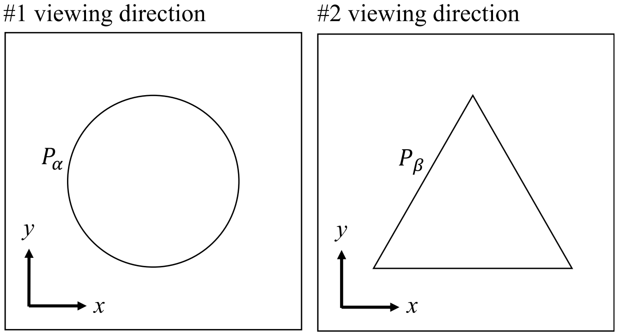


Figure 1. Two 2D figures created to be perceived from two viewing directions.

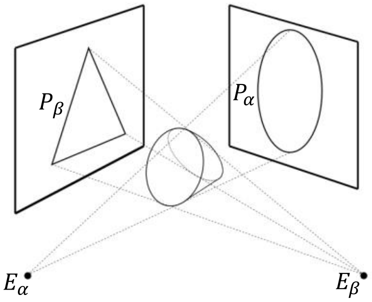


Figure 2. Two 2D figures integrated into a solid 3D object that can be perceived from two viewing directions.

The third is the method of generating ambiguous objects by solving linear equations. Sugihara experimented with generating an ambiguous object using 2D planar shapes such as flowers and stars as input. However, the results showed that the solution of the linear equations could not be obtained depending on the viewing direction and the given input geometry.

Although it does not fit into the above categories, a simple tool for generating ambiguous objects without the need to edit the object's shape has been proposed [11]. This tool generates ambiguous objects by adjusting the position and inclination of several square pillars placed in 3D space. However, due to the requirement of using square pillars, this tool cannot generate objects of arbitrary shape.

All of the above studies focused on modelling the shape of the ambiguous object in its generation but did not consider modelling the shape independent of the viewing direction. The automatic modification of the shape by changing the viewpoint would facilitate the generation of ambiguous objects.

III. OUR PROPOSED TOOL

The proposed tool uses an approach that allows the generation of 3D objects seen from each viewing direction, based on 2D figures. The generation of an ambiguous object involves the drawing of 2D figures and the integration of two solid 3D models.

A. *The drawing of 2D figures*

The tool provides two canvases for drawing each figure, where the user can draw a 2D figure by drawing a single stroke on each canvas. Note that the line segments composing each figure are not supposed to self-intersect. Figure 1 shows a circle P_α and a triangle P_β drawn on the canvases, respectively.

The figures on the canvas are stored as polygons. However, their shapes need to be optimized to make them equal in size and to remove unnecessary vertices caused by hand tremors during the drawing process. The proposed tool optimizes the figures drawn by the user through the following pre-processing.

a. *Normalization*

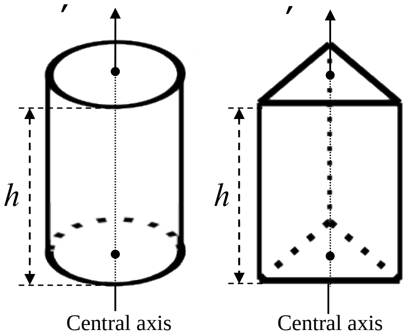


Figure 3. Two cylindrical surfaces for this study.

To make the two figures in an ambiguous object more visible, it is necessary to scale both figures equally. Therefore, the center coordinate

$$C(c_x, c_y) = \left(\frac{1}{n} \sum_{i=1}^{n} P_i^x , \frac{1}{n} \sum_{i=1}^{n} P_i^y \right) \tag{1}$$

of each figure is taken as the origin, and the vertices P_i that makes up the figure is then normalized by

$$P'_i = \frac{w}{\max(P^x, P^y) - \min(P^x, P^y)} \left(P_i^x, P_i^y \right), \tag{2}$$

where w is a user-defined scale of the figure, while P^x and P^y represent the coordinates of the vertices.

b. *Smoothing*

The following smoothing process is applied to each vertex to reduce the distortion of the figure caused by hand tremors during drawing.

$$P''_i = smooth(P'_i) = \frac{1}{3}(P'_{i-1} + P'_i + P'_{i+1}). \tag{3}$$

This process effectively reduces noise in the vertices caused by subtle hand tremors.

c. *Vertices pruning*

To make the density of the vertices that make up the figure uniform, the distance between vertices

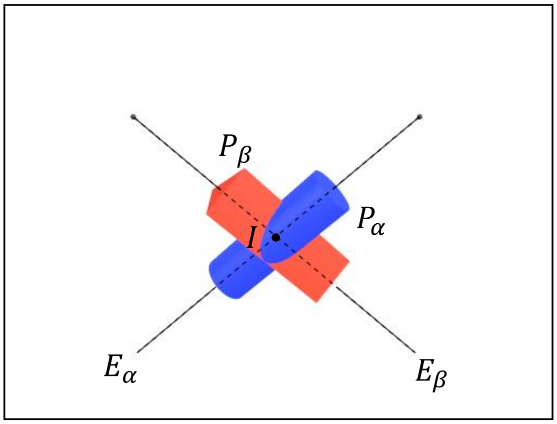


Figure 4. Two cylindrical surfaces A' and B' that intersect with respect to their viewing directions.

Shape	Figure #1	Figure #2
Simple		
Complex		

Figure 6. Figures for the construction of a simple and a complex ambiguous object.

$$D_i = \sqrt{\left({P''}_{i}^{x} - {P''}_{i+1}^{x}\right)^2 + \left({P''}_{i}^{y} - {P''}_{i+1}^{y}\right)^2} \quad (4)$$

that are less than a threshold is removed.

B. The integration of two solid 3D models

The integration of the two figures, as shown in Figure 1 into a single solid object must be done in a unique way. Figure 2 shows a possible integration in which the shape of P_α and P_β can be perceived from the viewpoint of E_α and E_β, respectively. This integration can be seen as the creation of an ambiguous 3D object representing the two figures. The shape of P_α is represented by an ellipsoid plane, while P_β is by a spherical triangle.

In this study, two cylindrical surfaces are used for the integration of the two figures. The integration takes the following steps.

Step 1: Generate a cylindrical surface for each figure. Figure 3 shows cylindrical surfaces A' and B' for P_α and P_β. The height of the cylindrical surface (h) should be high enough with respect to the viewing direction, as will be described later.

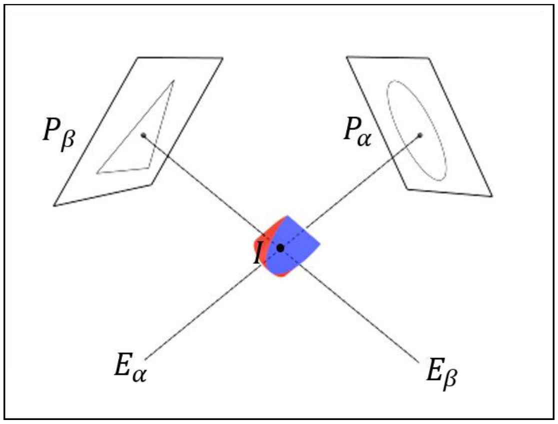
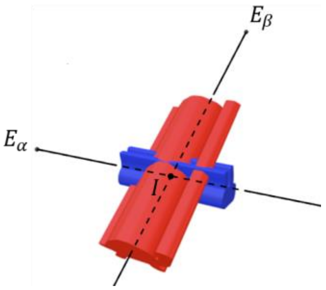
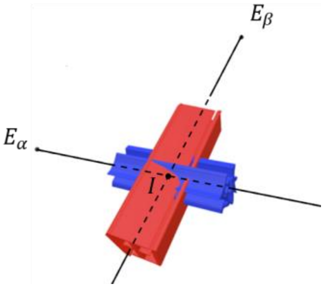


Figure 5. A solid object resulted from the Boolean intersection.



(a) Simple cylindrical surfaces



(b) Complex cylindrical surfaces

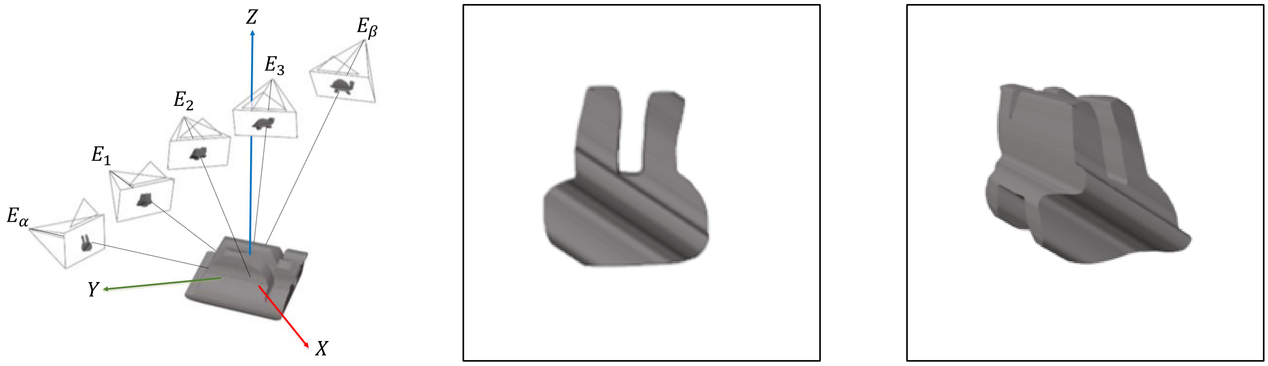
Figure 7. Cylindrical surfaces used to construct the ambiguous objects.

Step 2: Intersect the central axes (dashed lines) of the cylindrical surfaces A' and B', and rotate each cylindrical surface around the intersection point (I) to face the viewing points E_α and E_β, as shown in Figure 4.

Step 3: Perform a Boolean intersection to create a new solid from the intersection of the volumes of A' and B'. Figure 5 shows the resulting ambiguous object and its projected image plane as seen from E_α and E_β, respectively.

IV. EXPERIMENTAL RESULTS

Ambiguous objects consisting of simple and complex shapes were created using the proposed tool. Figure 6 shows

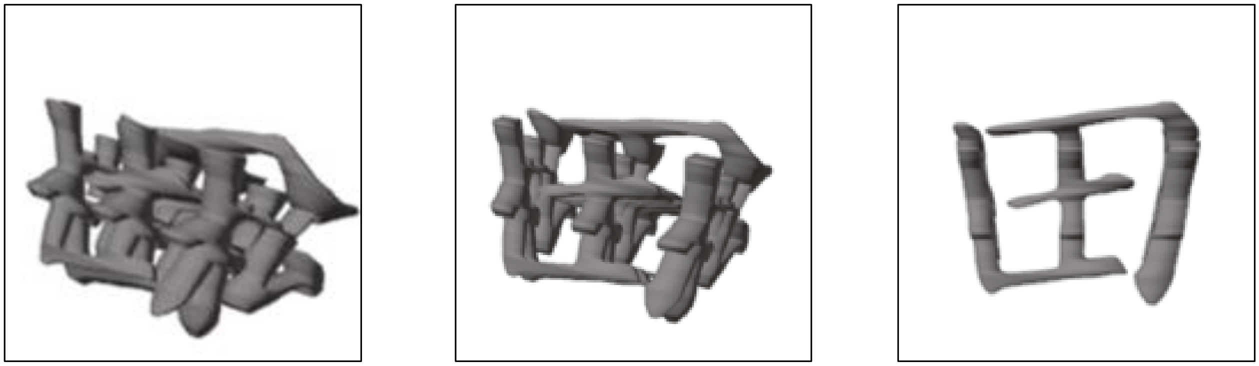


(a) Viewing directions. (b) The image plane viewed from E_α. (c) The image plane viewed from E_1.

(d) The image plane viewed from E_2. (e) The image plane viewed from E_3. (f) The image plane viewed from E_β.

Figure 8. The resulting ambiguous objects constructed by using simple figures.

(a) Viewing directions. (b) The image plane viewed from E_α. (c) The image plane viewed from E_1.



(d) The image plane viewed from E_2. (e) The image plane viewed from E_3. (f) The image plane viewed from E_β.

Figure 9. The resulting ambiguous objects constructed by using complex figures.

the 2D figures used to construct the ambiguous objects. The silhouettes of a turtle and a rabbit were used as simple figures. The figures are drawn almost entirely in outline, with no hollow areas. On the other hand, Japanese kanji characters were used as complex figures but were designed to be drawn with a single stroke.

Cylindrical surfaces associated to the simple and the complex figures were intersected to build ambiguous objects, as shown in Figure 7. Here, w=2.5 was used to rescale each figure to be 1m wide. The pre-processing shows that the cylindrical surfaces that intersect each other can be made to be of the same height. As both cylindrical surfaces are integrated by the Boolean interception, any change in viewing direction can generate a new ambiguous object. Unlike previous studies, the shape of the ambiguous object does not need to be edited each time the viewing direction is changed.

To show the difference between the correct and incorrect viewing directions of the generated ambiguous objects, the appearance of the objects in different viewing directions was captured, as shown in Figures 8 and 9. Let E_α and E_β be the correct viewing points and E_i the arbitrary viewing points (i=1,...,3), the original figures appear in its form when viewed from E_α and E_β.

V. CONCLUSION AND FUTURE WORK

In this paper, we propose a 3D modeling tool for generating ambiguous objects. These objects can be perceived differently from two different viewing directions. Users can easily generate ambiguous objects by simply drawing two 2D figures with this tool. Once the user defines the viewing direction of these figures, the ambiguous object is automatically generated. This approach differs significantly from those proposed in previous studies because the original 2D figures do not need to be modified by changes in viewing direction. Currently, there are several limitations in the proposed tool. First, if the angle between the two viewpoint directions is extremely small, the thickness of the generated ambiguous object becomes extremely thin, making it difficult to fabricate it with a 3D printer. Second, the appearance of the ambiguous object differs from the intended appearance even when it is viewed with only a slight viewpoint misalignment, because there is no tolerance process for viewpoint misalignment. Future work will include performing subjective evaluation experiments to evaluate the robustness of viewing the ambiguous objects generated by our tool.

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