

# The Analysis of the Specific Dictionaries for Compressive Sensing of EEG Signals

Monica Fira

Institute of Computer Science  
 Romanian Academy  
 Iasi, Romania  
[mfira@etti.tuiasi.ro](mailto:mfira@etti.tuiasi.ro)

Victor-Andrei Maiorescu<sup>1,2</sup>, Liviu Goras<sup>1,2</sup>

<sup>1</sup>“Gheorghe Asachi” Technical University of Iași,  
<sup>2</sup>Institute of Computer Science, Romanian Academy, Iasi  
[mandrei@etti.tuiasi.ro](mailto:mandrei@etti.tuiasi.ro)  
[lgoras@etti.tuiasi.ro](mailto:lgoras@etti.tuiasi.ro)

**Abstract**— In this paper the possibility of the electroencephalogram (EEG) compressed sensing based on specific dictionaries is presented. Three types of dictionaries (wavelet, temporal EEG signal specific, and channel specific) are analyzed and the results are expressed through quantitative measures (distortion) and by qualitative measures (stimulus classification rate of the Brain Computer Interface (BCI) paradigm - Spelling).

**Keywords**- EEG; Compressed sensing; BCI; P300.

## I. INTRODUCTION

In recent years, compressed sensing (CS) has attracted considerable attention in areas like applied mathematics, computer science, and electrical engineering by showing that, in certain conditions, it is possible to surpass the traditional limits of sampling theory. CS builds upon the fundamental fact that many signals can be represented using only a few non-zero coefficients in a suitable basis or dictionary. Nonlinear optimization can then be used to recover such signals from very few measurements [1]. The concept of compressed sensing is an example of practical use of new mathematical results. The difficulties for using in applications of such results are related to the way such concepts are understood, in a more or less intuitive manner, in order to facilitate the fusion between theory and applications.

The literature of recent years shows an impressive number of papers in the CS field, covering both 1D and 2D medical signals. Among the 1D signals the most frequently used in CS applications are the electrocardiogram (ECG) and electroencephalogram (EEG) since they are most used in the medical world as well. In the case of EEG signals, there is often a need of records for longer periods of time (i.e., during the night) or for a large number of channels.

In this paper, we propose a compression method for EEG signals based on CS using specific dictionaries for reconstruction in the sense that the atoms of the above dictionaries are actually segments of EEG signals and compare it with a wavelet representation.

To validate the proposed method EEG recordings of the competition for Spelling, BCI Competition III - Dataset II were used, and in order to evaluate the reconstructed signal, two types of evaluation were used, namely:

- **qualitative evaluation**, by P300 detection and prediction of the watched character in the case of the spelling paradigm for data taken from the “BCI III

competition Challenge 2005” on the reconstructed EEG signals and using the winner scripts (Alain Rakotomamonjy [9]).

- **quantitative evaluation**, using distortion measures such as PRDN (normalized percent of root-mean-square difference), NMSE (normalized Mean Square Error) and RMS (root mean square error) between the reconstructed and original signals.

## II. COMPRESSED SENSING

Shannon’s sampling theory represents, for many signal classes of interest in signal processing, a condition too strict for acquisition and representation of signals [1]. When signals are known to be compressible or sparse, only a much smaller fraction of samples may be needed to capture all the signal information, at the expense of having to use nonlinear reconstruction techniques. This area of research has evolved into the technique of “compressed sensing”, also known as compressive sensing, compressive sampling and sparse sampling), perfected in the past few years by prestigious researchers such as D. Donoho [3], E. Candès [4] and M. Elad [5], and drawing the attention of many others. It consists in capturing the information from sparse or compressible signals via a set of a few linear measurements, possibly random, followed by reconstructing a signal using optimization techniques for finding sparse solutions to underdetermined linear systems. The generality of the approach coupled with the prevalence of sparsity-related signal processing for big data is considered to have an enormous potential, with multiple implications and applications, in numerous fields of exact sciences.

As already mentioned, CS studies the possibility of reconstructing a signal  $x$  from a few linear projections, also called measurements, given the a priori information that the signal is sparse or compressible in some basis  $\Psi$ . The vectors on which  $x$  is projected onto are arranged as the rows of a  $n \times N$  projection matrix  $\Phi$ ,  $n < N$ , where  $N$  is the size of  $x$  and  $n$  is the number of measurements. Denoting the measurement vector as  $y$ , the acquisition process can be described as:

$$y = \Phi x = \Phi \Psi \gamma \quad (1)$$

$$\hat{\gamma} = \arg \min_{\gamma} \|\gamma\|_{l_0} \quad \text{subject to} \quad y = \Phi\Psi\gamma \quad (2)$$

$$\hat{x} = \Psi\hat{\gamma} \quad (3)$$

The system of equations (1) is obviously undetermined. Under certain assumptions on  $\Phi$  and  $\Psi$ , however, the original expansion vector  $\gamma$  can be reconstructed as the unique solution to the optimization problem (2); the signal is then reconstructed with (3). Note that (2) amounts to finding the sparsest decomposition of the measurement vector  $y$  in the dictionary  $\Phi\Psi$ . Unfortunately, (2) is combinatorial and unstable when considering noise or approximately sparse signals. Two directions have emerged to circumvent these problems: (i) pursuit and thresholding algorithms seek a sub-optimal solution of (2) and (ii) the Basis Pursuit algorithm relaxes the  $l_0$  minimization to  $l_1$ , solving the convex optimization problem (4) instead of the original.

$$\hat{\gamma} = \arg \min_{\gamma} \|\gamma\|_{l_1} \quad \text{subject to} \quad y = \Phi\Psi\gamma \quad (4)$$

In the past few years, techniques inspired from the mathematic fundamentals of CS have also been applied in the field of biomedical signals, both at the level of processing methods for electrocardiographic (ECG) and electroencephalographic (EEG) signals and of practical implementation in applications such as compression, transmission and reconstruction of the ECG signal using a personal device such as a smart-phone.

### III. BCI P300 SPELLER

The use of EEG signals as a vector of communication between man and machine is one of the new challenges in biomedical signal theory. The main element of this communication system known as "brain-computer interface" (BCI Brain-Computer Interface) is the proper interpretation of the EEG signals and the characteristic parameters of the brain electrical activity.

The P300 speller is based on the so-called oddball paradigm which states that rare expected stimuli produce a positive deflection in the EEG after about 300 ms.

A	B	C	D	E	F
G	H	I	J	K	L
M	N	O	P	Q	R
S	T	U	V	W	X
Y	Z	1	2	3	4
5	6	7	8	9	

Figure 1. Example of a 6 × 6 user display in P300 Speller

A P300 speller, based on this paradigm, has been introduced by Farwell and Donchin who developed a protocol whereby a subject is presented a 6 × 6 character matrix (see Figure 1) [11].

The dataset II of the BCI competition III 2005, from the competition webpage [10], has been recorded from two different subjects and 5 different spelling sessions. Each session is composed of runs, and for each run, a subject is asked to spell a word. For a given acquisition session, all EEG signals of a 64-channel scalp have been continuously collected. Before digitization at a sample rate of 240 Hz, signals have been bandpass-filtered from 0.1 to 60 Hz.

Each session is composed of runs, and for each run, a subject is asked to spell a word. Row/column intensifications were the block is randomized in blocks of 12. The sets of 12 intensifications were repeated 15 times for each character epoch (i.e., any specific row/column was intensified 15 times and thus there were 180 total intensifications for each character epoch). Each character epoch was followed by a 2.5sec period, and during this time the matrix was blank.

The training set contains 85 characters and the test set consists of 100 characters for each of the two subjects A and B. A more detailed description of the dataset can be found in the BCI competition paper [10].

The competition winners, Alain Rakotomamonjy and Vincent Guigue propose a method that copes with such variabilities through an ensemble of classifiers approach [9]. Each classifier is composed of a linear Support Vector Machine trained on a small part of the available data and for which a channel selection procedure has been performed. Their performances are a classification rate of 95.5% for the 15 sequences and 73.5% for 5 sequences [9].

### IV. METHOD

The key element in the success of signal compression based on compressed sensing is the right choice of the dictionary based on which the reconstruction will be done. Generally, the ECG and EEG biomedical signals have not a very high sparsity in standard wavelet dictionaries. Therefore, in most of cases, the authors propose specific dictionaries either specific to the signal or specific for the used database.

In the following an analysis of how the results are influenced by various dictionaries is made. Three types of dictionaries have been analyzed as follows.

#### A. Wavelet Dictionary

The first choice was the Daubechies 10 type dictionary so that for all channels a single dictionary will be used [6-8].

#### B. Temporal EEG signal specific dictionary

A second choice investigated as well in [7], was to build dictionaries from segments of certain predefined EEG channels, recorded at the same time with the target EEG compressed signal channel. These dictionaries are built with EEG signals from channels that are acquired in the classical

way; the dictionaries are the same for all compressed sensed channels.

To construct the necessary dictionaries for reconstruction, EEG signals from channels FPZ, F7, F8, C5, CZ, T8, PO7, PO8 and Oz were used. The rest of the channels, i.e. the 55 channels (Fig. 2) were segmented in windows with 1 second length (240 samples) and using a random matrix with size 240x24 we obtained EEG compressed signals (with size 24 and compression ratio CR = 10:1). In Figure 2 the red channels are used for dictionary construction and the signals in the white channels are compressed sensed.

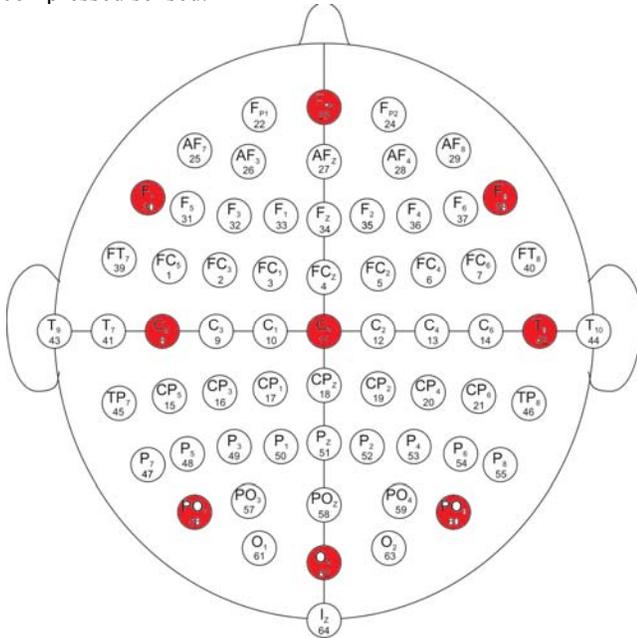


Figure 2. Electrode placement and channel name.

Thus, for each sample time we have one dictionary. Knowing the random matrix used for sensing and the dictionary with the EEG signals from the 9 channels acquired synchronously with the compressed sensed channels, the EEG signals for the CS channels could be reconstructed [7].

### C. Channel specific dictionary

Third, for each EEG channel a dictionary has been built. The atoms of dictionary are actually the EEG segments of the training set. In this case, for the data acquired on 64 channels, there are 64 dictionaries. Thus, each dictionary was composed from 2x85 atoms; for every epoch from the training set, 2 segments (from 240 samples) of EEG signal were randomly selected as atoms in the dictionary.

### D. Evaluation of the reconstructed signals

In order to evaluate and validate the methods, we used both quantitative measures (the reconstructed signal distortion) and signal quality measures (expressed by the classification rate of the characters tracked by a human

subject, which is exactly the problem of the BCI Competition III 2005 - dataset II – Spelling).

## V. EXPERIMENTAL RESULTS AND DISCUSSIONS

For the evaluation of the analyzed methods we used the dataset II of the BCI Competition III 2005 -P300 Spelling.

Thus, for evaluation of compression we used the compression rate (CR) (5) defined as the ratio between the number of bits needed to represent the original and the compressed signal.

$$CR = \frac{b_{orig}}{b_{comp}} \quad (5)$$

For qualitative evaluation of the method based on the classification rate in spelling paradigm, we used scripts from the winners, Alain Rakotomamonjy and Vincent Guigue [9]. The used scripts implement classification based on all 64 EEG channels.

To validate the compression we evaluated the distortion between the original and the reconstructed signals by means of the PRDN (the normalized percentage root-mean-square difference):

$$PRDN\% = 100 \sqrt{\frac{\sum_{n=1}^N (x(n) - \tilde{x}(n))^2}{\sum_{n=1}^N (x(n) - \bar{x})^2}} \quad (6)$$

where  $x(n)$  and  $\tilde{x}(n)$  are the samples of the original and the reconstructed signals respectively,  $\bar{x}$  is the mean value of the original signal, and N is the length of the window over which the PRDN is calculated.

For compression, the EEG signal was segmented into windows of length 1 sec, i.e. 240 samples and we used a random matrix with size 240x24 for CR = 10:1 and a random matrix with size 240x48 for CR = 5:1.

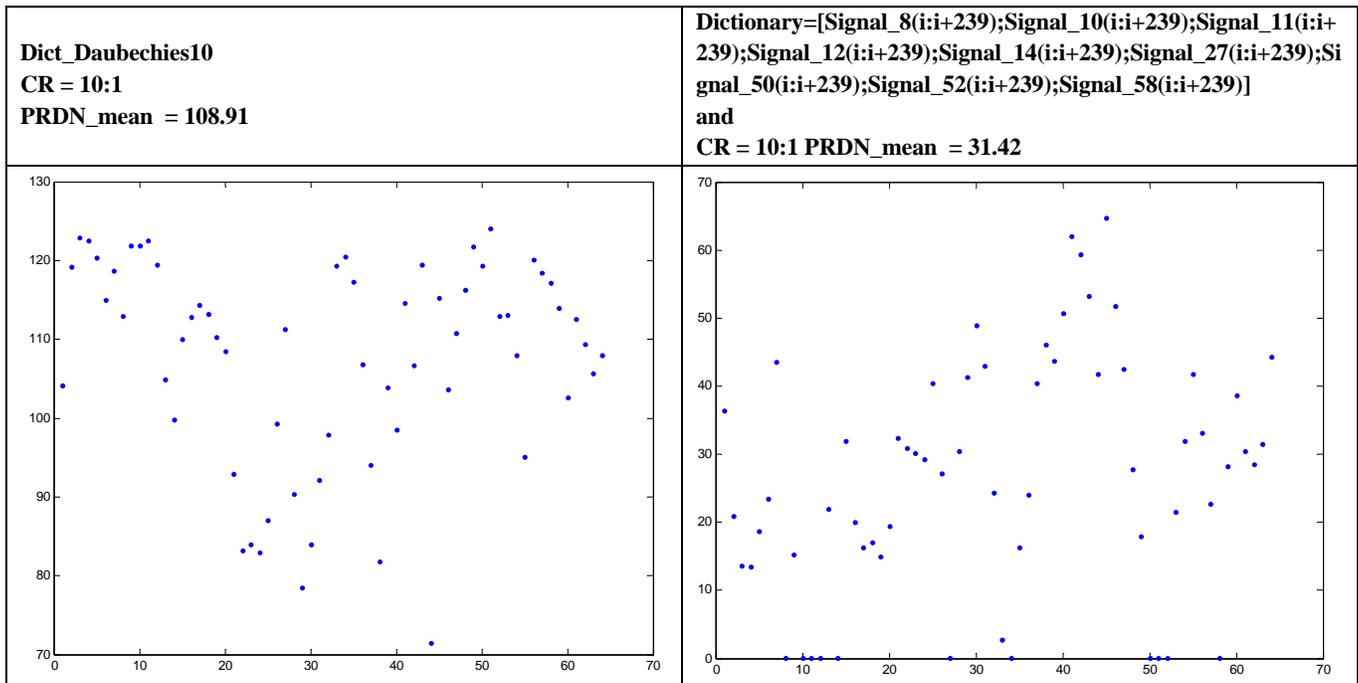
Table 1 presents the results of the three tested dictionaries. Note that in terms of reconstruction errors expressed via PDN, the smallest errors were obtained using specific in time EEG signal dictionaries, which consists of atoms regularly acquired at the same time point from the other channels. Considering the rate of classification for the spelling paradigm, the best results are obtained using channel-specific dictionaries.

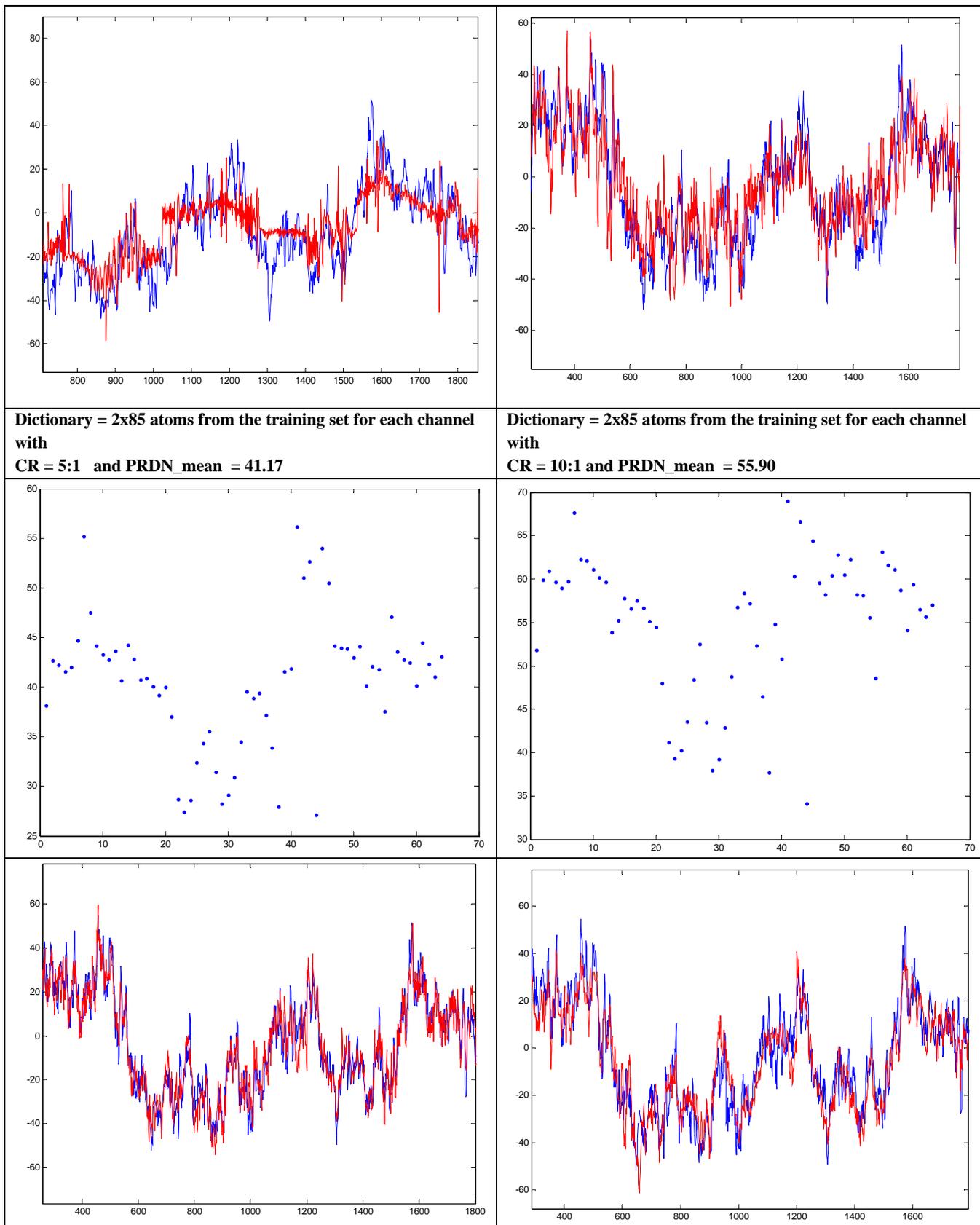
TABLE I. AVERAGE RESULTS: PRDN\_MEAN AND CLASSIFICATION PERFORMANCE IN % OF CORRECTLY RECOGNIZED CHARACTERS FOR THE B SUBJECTS AND FOR INCREASING NUMBER OF SEQUENCES.

P300 Spelling - classification performance for number of sequences (classification rate %)														PRDN mean	
1	2	3	4	5	6	7	8	9	10	11	12	13	14		15
<b>Original data – uncompressed and classified by Alain Rakotomamonjy and Vincent Guigue</b>															
-	38	60	70	69	80	84	86	89	92	93	95	96	96	96	0
<b>Dict_Daubechies10</b>															
10:1	7	17	16	18	27	29	37	40	38	41	48	45	51	48	108.91
<b>Dictionary=[Signal_8(i:i+239);Signal_10(i:i+239);Signal_11(i:i+239);Signal_12(i:i+239);Signal_14(i:i+239);Signal_27(i:i+239);Signal_50(i:i+239);Signal_52(i:i+239);Signal_58(i:i+239)];</b>															
5:1	22	31	42	50	54	61	68	70	72	77	82	85	79	79	35.38
10:1	23	35	42	44	54	57	60	63	68	70	76	78	76	78	<b>31.42</b>
<b>Dictionary = 2*85 atoms from the training set for each channel</b>															
5:1	<b>29</b>	<b>41</b>	<b>50</b>	<b>62</b>	<b>68</b>	<b>74</b>	<b>73</b>	<b>78</b>	<b>82</b>	<b>82</b>	<b>86</b>	<b>86</b>	<b>86</b>	<b>87</b>	41.17
10:1	19	30	43	52	50	60	60	65	73	71	78	82	84	82	55.90

In Table II PRDN vs. channel (top figure) are presented as well as examples of original and reconstructed EEG signal (figures below). The worst results are obtained using wavelet type dictionaries. Between the classification rate in spelling paradigm and error expressed as PRDN there is a discordance, namely not always the smallest PRDN errors lead to highest classification rate (see the results in table with bold). The explanation for the discrepancy between the classification rate and average PRDN is that each channel has a certain weight in the classification rate for the spelling paradigm. The obtained results lead to the conclusion that some channels that have a higher weight in the classification rate are rebuilt better than others which have lesser meaning. Thus it can be seen that for the channel specific dictionaries, in case of both compression rates of 5:1 and 10:1, the error for the channels 22-38 is much lower compared to the rest of the channels. In fact one can speak about a group of errors in three clusters, namely, a class for the channels 1-21, the second class for the 22-40 channels, and the third class for the channels 41-64. These three groups are closely interlinked to the placement of the cranial electrodes too (see Figure 1).

TABLE II PRDN\_MEAN VS. CHANNEL AND ORIGINAL VS. SIGNAL RECONSTRUCTED FOR THE THREE TESTED DICTIONARIES





## VI. CONCLUSIONS

In this paper a comparative analysis of the results obtained using three types of dictionaries for EEG signals compressed sensing is presented. The used dictionaries are: Daubechies 10 wavelet dictionary and two types of EEG signal specific dictionaries, namely, temporal EEG signal specific dictionary and channel specific dictionary. For the evaluation of the proposed method we used the dataset from the BCI Competition III 2005 - P300 Spelling. In order to evaluate the results of the EEG signal reconstruction the PRDN was used in parallel with the classification rate of the spelling paradigm assessed using the scripts from the winner of the competition (the version of classification using all 64 channels). Based on the analysis it is found that the worst results are obtained when standard wavelet dictionaries were used. The other two EEG signal specific dictionaries, lead to better results. Thus, for the channel specific dictionaries the best results in terms of classification at the spelling paradigm are obtained for CR = 5:1 and 10:1 when the achieved classification rate was 90%, respectively, 89% (for the original signals the classification rate was 95%). The temporal EEG signal specific dictionaries lead to the best results in terms of error expressed as PRDN, i.e. for a compression of 5:1 it was obtained a PRDN = 35.38 and for 10: 1 the obtained PRDN was 31.42.

The obtained results demonstrate that channel specific dictionaries and temporal EEG signal specific dictionaries provide much improved results compared to the standard wavelet dictionaries.

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