Minimizing the Power by using Genetic Algorithms for Multi-User OFDM Systems

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***Abstract*—This paper considers the problem of minimizing power consumption of the base station in the downlink of the multi-user OFDM cellular network, characterized by frequency selective transmission channel. The channel gain is variable and specific for each user. In this case, the resources allocation must be determined so that the total power consumption is as low as possible. To solve this problem, we proposed an implementation ofsome genetic algorithms and we obtained a better solution than currently applied well-known methods.**

Keywords*—*OFDM; Genetic Algorithms; Multi-User; Frequency Selectivity; Resources Allocation

I. INTRODUCTION

The second and third generation of cellular network are characterized by single carrier transmission. For each user, one subcarrier is used during the communication that often causes lost of signal in the context of channel frequency selective. However, in 4G cellular systems [13], the multi-carrier transmission's technique OFDM (Orthogonal Frequency Division Multiplexing) [24] is used in order to fight against multipath phenomena and to provide a significant throughput to users. In the OFDM transmission technique, the total available bandwidth is split into many narrow bands subchannels at equidistant frequencies [21]. Each subchannel has approximately a constant channel gain and the bit loading can be adjusted according to the level of the channel gain.

The orthogonality of the subcarriers permits to reduce the width of the transmission subchannels; thereby, increases the spectral efficiency. The high number of the subcarriers makes possible to perform a more effective management, by adapting the subcarriers allocation according to the channel gain of the users [22].

In the context of frequency selective channel, where the channel gain is variable and specific for each user, the optimal resources allocation is an optimization problem. The adaptive resources allocation is a proposed method to improve the performance of cellular systems. The adaptive allocation can give 20dB of Signal-to-Noise-Ration in contrast to random allocation [2].

In multi-users OFDM cellular systems, the resources allocation consists in solving an optimization problem with constraint. It takes the channel estimated characteristics. In practice, the channel gain is given by receiver in logic channel CQIH (Channel Quality Indicator Channel) [8]. Note that, this optimization problem is highly nonlinear; therefore, it is unlikely that the algorithm with polynomial complexity be used [15].

Thus, several algorithms are developed for solving the resources allocation problem. The water-filling algorithm proposed by Tu et al. [23] proved to be optimal for solving the power-minimizing problem in the single-user case. Improvements have been introduced by Qi et al. [3], and Munz et al. [6], in order to reduce complexity of the water-filling algorithm. The Greedy algorithm proved to be optimal to solve the bit loading algorithm in single-user case [12]. However, the problem of resources allocation becomes very complex in multi-user OFDM systems. Wong et al. [4] proposed a suboptimal method to allocate subcarriers, but the method requires iterative process and the foreknown required number of subcarriers allocated to users [9]. Kim et al. [14] converted the non-linear optimization problem into a linear one with integer variables. The optimal solution can be achieved by integer programming (IP), but its computation is still very huge [9].

In this context, the evolutionary approach inspired by natural phenomena are introduced to improve the resources allocation [7]. Among this methods, we propose in this paper to solve the optimization problem with GAs (Genetic Algorithms). The main objective of this work is to propose an implementation of some GAs in order to have better results.

In Section 2, the related work about resources allocation is described. In Section 3, the modeling system and the formulation of the optimization problem are presented. In Section 4, the principle of genetic algorithms is given and in Section 5, the proposed algorithm is described. Finally, in Section 6, we compare the results of the proposed algorithm to others algorithms.

II. RELATED WORK

Genetic algorithms belong to the family of evolutionary algorithms inspired by biology. It is first introduced by H. Holand [11] and implemented by D. Goldberg [7] for solving optimization problems. Since the GAs is an effective search technique, it has been applied to wireless communications recently [1][5][20][17].

Most of the work based on the genetic algorithms use the same procedure. However, the difference lies in the implementation and the method of GAs's operators. In the work proposed by Zhu et al. [18], the natural selection's method is used, while it accelerates the algorithm but not ensure

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converging towards the global optimum [16]. In this paper, we used the method of selection by tournament, that keeps diversity of the population during the process and allows a better exploration of the search domain [10].

In many proposed implementation, the constraint of the optimization problem is considered in the evaluation of the fitness (objective function) of individual. This can deplete the population and limits the exploration of the search domain. In our proposed implementation, the fitness is only based on the power according to equation (4). To meet the constraints of the optimization problem, we defined the function named $adjust\_allocation$ so as to balance the throughput of users.

III. System modeling and Formulation of the problem

A. System modeling

We consider an OFDM multiple access system, in which the characteristics of transmission channel are assumed known by the transmitter and the receiver. The channel is assumed to be linear and stationary, with additive white gaussian noise of zero mean. It is decomposed into N flat subchannels and each of which is characterized by a constant gain, specific for the nth subcarrier and the kth user.

The transmission channel is frequency selective, and we assume that the users are in micro-mobility; therefore, the Doppler effect is ignored. We note that, there are no inter-symbol interferences, since subcarrier is allocated to a single user. We consider the mono cellular system. The number of users and subcarriers is constant during the GAs process.

B. Formulation of the problem

The channel of transmission is frequency selective. It can be considered as a filter, characterized by the transfer function varying with frequencies and of each user. The magnitude of the transfer function corresponds to the gain of the transmission channel.

Let $H_{k,n}$ the channel gain of the kth user on the nth subcarrier. The required power is given by Proakis [19]:

$$p_{k,n} = \frac{f(c_{k,n})}{H_{k,n}^2} \tag{1}$$

where

$$f(c_{k,n}) = \frac{N_0}{3}(2^{c_{k,n}} - 1)[Q^{-1}(\frac{BER_k}{4})]^2 \tag{2}$$

$c_{k,n}$ is the number of bits for the kth user on the nth subcarrier. N_0 is the power spectral density of the noise. BER_k is the bits error rate of the kth user. $Q(x) = erfc(x)$ is the complementary error function.

Note that, when the channel gain $H_{k,n}$ increases, the power required decreases. Thus, the nth subcarrier will be allocated in preference to the kth user having the smallest power to achieve the required throughput. As the number of subcarriers is high, it is not necessary that many users share the same subcarrier (Time Division Mutliplexing). In addition, the inter-symbol interferences can be avoided when one subcarrier is exclusively allocated to one user.

Let $\rho_{k,n}$ the allocation factor of the nth subcarrier to the kth user defined by:

$$\rho_{k,n} = \begin{cases} 1 & \text{if the } nth \text{ subcarrier is allocated to the } kth \text{ user} \\ 0 & \text{else} \end{cases} \tag{3}$$

The total power allocated to the kth user is given by:

$$P_k = \sum_{n=1}^{N} p_{k,n}.\rho_{k,n} = \sum_{n=1}^{N} \frac{f(c_{k,n})}{H_{k,n}^2}.\rho_{k,n} \tag{4}$$

Thus, the total power allocated to all users is given by:

$$P_T = \sum_{k=1}^{K} P_k = \sum_{k=1}^{K}\sum_{n=1}^{N} \frac{f(c_{k,n})}{H_{k,n}^2}.\rho_{k,n} \tag{5}$$

The total bits for the kth user is given by:

$$r_k = \sum_{n=1}^{N} c_{k,n}.\rho_{k,n} \tag{6}$$

The equation (1) shows that the required power for the kth user on the nth subcarrier is inversely proportional to the channel gain $H_{k,n}$. Therefore, it is more efficience to allocate the nth subcarrier to the kth user which presents the smallest channel gain $H_{k,n}$. Thereby, the power can be minimized while the constraints of QoS are satisfied.

Thus, the problem of resources allocation can be written:

$$min(P_T)$$

subject to

$$r_k > r_0$$

where r_0 is the minimal rate for the kth user.

IV. Genetic Algorithms

Genetic algorithms are inspired by Darwin's theory of evolution and by Mendel's works about recombination of species [16]. GAs are used to solve many problems of optimization.

Robustness is the main advantage of genetic algorithms relative to traditional resolution methods of optimization problems [7]. In other words, we can see the four major differences between the two methods:

1) GAs work with a coding of the set of parameters, while the classical methods use directly the parameters.
2) The solution given by GAs is a set of points (chromosomes) and the solution for a classical methods is a single point.
3) GAs use the objective function and the standards methods often use derivatives of function or other auxiliary knowledge.
4) Gas use probabilistic transition rules when the traditional methods use deterministic rules.

The principle of GAs is based on the evolution of an initial population under the effect of operators such as selection, mutation and crossover. At the end of the GAs's process, the best individual in the population will be the solution of the optimization problem.

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The different phases of the GAs are:

- Coding of chromosome
  A chromosome of the population represents a resource allocation scheme. The coding of the chromosome is to provide a structure corresponding to the resource allocation problem. In our implementation, chromosome is represented by an array of structure, containing a fixed number of subcarriers, numbered in increasing order, from 0 to $N-1$. In the nth cell, there are the index of the user to which the corresponding subcarrier is allocated. The required power and the quantity of bits are calculated from this allocation for every user. All chromosomes have the same fixed size which corresponds to the total number of subcarriers. The following table shows an example of the structure of the chromosomes .

TABLE I. STRUCTURE OF CHROMOSOME

Subcarrier	1	2	- - -	N
User	5	3	- - -	10

- Initialization of the population
  It consists to set the size of the population and to generate all chromosomes of the population. In this work, the chromosomes of the population are randomly generated and their size is the same and stays constant during the GAs process.
- Evaluation
  In this function, the total required power and the total throughput are calculated for each chromosome. The required power corresponds to a fitness (objective function) of the chromosome. The fitness or the objective function represents the criterion of the selection of chromosomes.
- Selection
  After evaluating the power of all chromosomes, the selection consists to retain the best chromosome by directly sorting (natural selection) or by organizing tournament between two chromosomes arbitrary selected (selection by tournament). The best chromosome of the population is the one that the requires power is the smallest while respecting the constraint of throughput.
- Mutation
  It consists to bring changes to the resources allocation scheme in order to have a better exploration of the search domain. In this work, the number of mutations and the index of the mutated subcarrier are randomly determined for each chromosome. Note that, all chromosomes will not be affected by the mutation.
- Crossover
  In this step, a new chromosome is created from two chromosomes in order to take advantages of their best characters. Two points of crossover are randomly chosen and two parts of the first chromosome are concatenated with one part of the second chromosome. So, a new chromosome is created from this concatenation.

V. DESCRIPTION OF THE PROPOSED ALGORITHM

input:
$N_{Carrier}$: number of subcarriers
N_{User}: number of users
$Size_{pop}$: size of the poplutation
$Ngen$:number of generation of the GAs process
Step 1: Coding of chromosome
$Structure\ subcarrier\{$
$user$
$power$
bit
$\}$
$chrom[N_{Carrier}]$ //vector of subcarriers
Step 2: Initialization population
$for\ t\ =\ 1\ to\ Size_{pop}$
$for\ n\ =\ 0\ to\ N_{Carrier}-1$
$\{$
$k \leftarrow rand(N_User)$
$chrom[t].user \leftarrow k$
$chrom[t].bit \leftarrow c[k][n]$
$chrom[t].power \leftarrow f(c[k][n], H[k][n])$ // According to equation 1.
$\}$ **Step 3: Evaluation chromosome**
$for\ i\ =\ 1\ to\ Size_{pop}$
$\{$
$P_{chrom}(i) \leftarrow 0$ // Initialization the power of the chromosome
$for\ n\ =\ 0\ to\ N_{Carrier}-1$
$\{$
$P_{chrom}(i) \leftarrow\ P_{chrom}(i)\ +\ chrom[n].power$
$\}$
$\}$
Step 4: Selection by tournament
$c_1 \leftarrow rand(Size_{pop})$
$c_2 \leftarrow rand(Size_{pop})$
$p_1 \leftarrow P_{chrom}(c_1)$
$p_2 \leftarrow P_{chrom}(c_2)$
$if\ p_1\ <\ p_2\ then$ // chrom c1 better than chrom c2
$population\_next\_generation \leftarrow chrom[c_1]$
$else$ // chrom c2 better than chrom c1
$population\_next\_generation \leftarrow chrom[c_2]$
Step 5: Mutation
$n_{chrom} \leftarrow rand(Size_{pop})$ // number of chromosome to mutate
$for\ i\ =\ 0\ to\ n_{chrom}$
$\{$
$c_1 \leftarrow rand(Size_{pop})$ // chromosome's index
$n_{mutation} \leftarrow rand(Size_{pop}/8)$ // number of mutation
$for\ j\ =\ 1\ to\ n_{mutation}$
$\{$
$n \leftarrow rand(N_{Carrier})$
$chrom[i].user \leftarrow rand(N_{User})$ // changes of user allocation
$\}$
Step 6: Crossover
$c_1 \leftarrow rand(Size_{pop})$
$c_2 \leftarrow rand(Size_{pop})$
$pc_1 \leftarrow rand(N_{Carrier})$ // first crossover point
$pc_2 \leftarrow rand(N_{Carrier})$ // second crossover point
$offspring \leftarrow concatenate(chrom[c_1], chrom[c_2], pc_1, pc_2)$
Step 4 to **6** are repeated until the number of generation Ngen is reached
Step 7: Search best chromosome

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$best\_chrom \leftarrow 1$
$for\ k\ =\ 2\ to\ Size_{pop}$
$\{$
$if\ P_{chrom}(k)\ >\ P_{chrom}(best\_chrom)$
$best\_chrom \leftarrow k$
$\}$

Step 8: Adjust allocation

In this step, the best allocation scheme resulting from the GAs process, is adjusted in order to achieve the throughput constraint. In this paper, the throughput constraint is not considered in the calculation of the objective function. Here, the proposed solution of the genetic algorihms is only based on the criterion of the power and requires to be eventually adjusted.

VI. SIMULATION

A. Parameters simulation

- BER (Bit Error Rate): 10^{-2} to 10^{-4}
- M-QAM: M in $\{4, 8, 16, 32, 64\}$ M is the parameter of the modulation QAM (Quadrature Amplitude Modulation). It corresponds to the number of symbols of the modulation. M equal to 2^n where n is the number of bits per symbol.
- Power spectral density of the noise: $N_0 = 0.01$
- Number of subcarriers: 128
- Size of the population: 200
- Symbols rate: $14kbauds$ (3GPP Standard)
- Minimal user's rate: $r_0 = 750kbits/s$. It is calculated according to the number of subcarriers and the number of users.

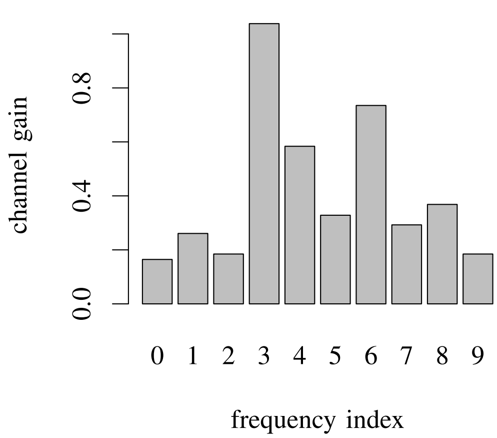

Figure 1. Variation of channel gain of one user

B. Results of simulation

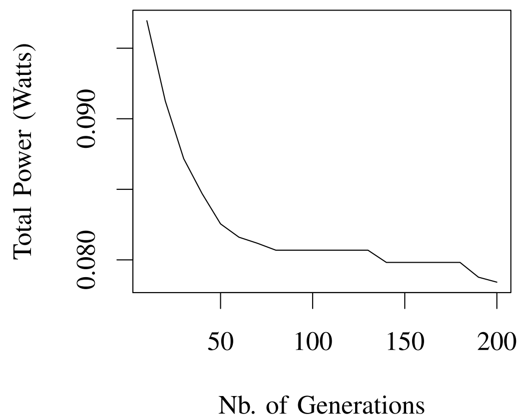

Figure 2. Convergence of our algorithm with 8 users, 128 subcarriers and $N_0 = 0.0001$.

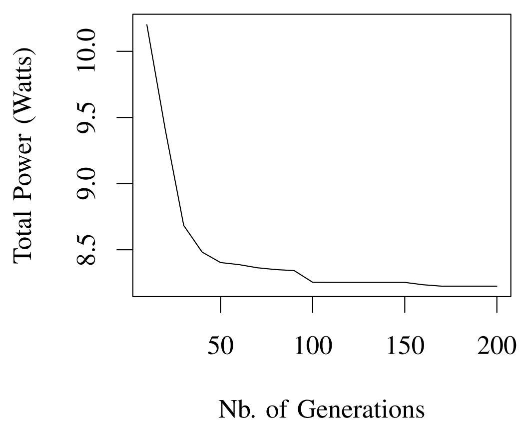

Figure 3. Convergence of our algorithm with 8 users, 128 subcarriers and $N_0 = 0.01$.

Figure 4. Variation of total power with different size of the population with 8 users, 128 subcarriers and $N_0 = 0.01$.

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TABLE II. COMPARISON OF OUR ALGORITHM AND THE ALGORITHM PROPOSED BY AHMADI ET AL. [1] WITH $N_0 = 0.0001$

	2 users	4 users	8 users
Our algorithm	0.01w	0.022w	0.061w
Ahmadi and Chew	0.03 w	0.05 w	0.12 w

TABLE III. COMPARISON OF OUR ALGORITHM AND THE ALGORITHM PROPOSED BY REDDY ET AL. [20] WITH $N_0 = 0.01$

	2 users	4 users	6 users	8 users
Our algorithm	0.98 w	2.09 w	3.77 w	5.68 w
GaReddy	5w	8 w	12w	15w

C. Analysis of results

The study of the evolution of the power, according to the number of generations (Figure 2), shows that the power remains relatively constant from the $80th$ generation and the corresponding resources allocation scheme is the solution of the optimization. Figure 4 shows some problems of convergence of the proposed algorithm; however, the total power decreases when the size of population increases. We also note that the result is better with large population size. The GAs that we implemented, gives better results than baseline algorithms met in the literature. The Tables II and III above, compare the consumption of the power for different numbers of users of the algorithm proposed in this work to the algorithms developed by Ahmadi et al. [1] and by Reddy et al. [20].

VII. CONCLUSION AND FUTURE WORK

In this paper, we have implemented the genetic algorithms to solve the problem of resources allocation in a frequency selective transmission channel. The implementation of the GAs that we proposed is characterized by the function named $adjusted\_allocation$ which permits to balance the resources allocation scheme of the obtained solution. The implementation of the proposed GAs in this paper gives better result than the algorithm proposed by Ahmadi et al. [1] and the algorithm proposed by Reddy et al. [20]. We have implemented the proposed GAs in this paper in C language, which gives us a good modeling. We hope that the execution of our model will be faster than others tools as Matlab or Scilab. In future works, we will try to improve the convergence of the proposed GAs.

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