Nonlinear Error Modeling of Reduced GPS/INS Vehicular Tracking Systems Using Fast Orthogonal Search

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Abstract—Land Vehicle Tracking systems depend mainly on Global Navigation Satellite Systems (GNSS), such as Global Positioning System (GPS). However, GNSS suffer from signal blockage and degradation in urban areas. At the same time, most land vehicles, nowadays, come with low-cost low-power Inertial Measurement Units (IMU). Although these IMU can be used an accurate short-term tracking system by integrating a reduced IMU system with GPS to enhance the tracking accuracy of land vehicles in downtown and urban areas. Commonly, GPS/INS integration is based on Kalman Filter (KF), where a linearized dynamic models for INS errors is utilized. If Low-Cost MEMS-based inertial sensors with complex stochastic error nonlinearity are used, performance degrades significantly during short periods of GPS-outages. This paper presents a nonlinear INS-errors modelling using a fast nonlinear identification technique called fast orthogonal search (FOS). During reliable GPS coverage, the corrected vehicle state and sensors measurements are input to FOS and the FOS models outputs are trained to predict the INS deviations from GPS. During GPS-outages in urban areas, the trained FOS models along with the most recent vehicle state are used to predict INS deviations from GPS. The predicted INS deviations are then feedback to the system Kalman Filter, as updates to estimate all INS errors. The experimental setup of this work used a very low-cost IMU from Crossbow Inc. (USA based), the vehicle odometer measurements along with a GPS receiver from Novatel, Inc. (Canada based). Experiments were performed in Kingston, Ontario, Canada. Initial results show promising improvement of tracking accuracy in challenging GNSS-denied areas.

Keywords—Land Vehicles Tracking; Reduced IMU; GPS; INS/INS integration.

I. INTRODUCTION

Inertial Navigation Systems (INS) utilize inertial sensors to provide navigation information continuously with time [1][12][24]. In a Strapdown 3D INS with full Inertial Measurements Unit (IMU) [24][25], three acceleration sensors (Accelerometers) and three angular rate sensors (Gyrosopes) are utilized. The accelerometers measure the acceleration of the moving body in three orthogonal directions. Gyroscope measures the rotation rate around these three basic orthogonal axes. The essential functions in INS are defined as follows: 1) Determination of the angular motion of a vehicle using gyroscopic sensors, from which its attitude relative to a reference frame may be derived. 2) Measure the acceleration using accelerometers. 3) Resolve the acceleration measurements into the reference frame using the knowledge of attitude. 4) Account for the gravity component. 5) Integrate the resolved accelerations to estimate the velocity and position of the vehicle. Although INS systems have good short term accuracy, there are two main problems in using such a scheme. The first problem is the sensor imperfections and drifts [2][8]. The second problem is that the measurements of such sensors must be mathematically integrated to provide velocity, position, and attitude information. Integration causes errors to accumulate [2][8] resulting in huge drifts over time that growth without bounds.

On the other side, GPS systems provide consistent long term accuracy giving position and velocity updates using GPS satellites signals processing [1][12]. A major problem of GPS is signal blockage and multi-path in urban canyons, under buildings, and tunnels. In these environments, signal may be difficult to acquire or number of satellites available may be not sufficient to provide position information [25].

Based on the complementary error characteristics of INS and GPS, an integrated solution using both systems is often used. Although there are many approaches to fuse data from both systems, KF is most widely used [1][12][19]. KF utilizes an error dynamic model of the INS system errors to implement two main steps: Prediction step and Update step. Prediction step is done as long as no GPS update is available. In this step, the system uses the error dynamic model to estimate the INS errors. In the update step, GPS velocity and position measurements are used to get optimal estimate of INS errors. Thus, by subtracting INS errors from the INS output, accurate navigation information is obtained. This integration scheme is called loosely coupled which is utilized here in this work. This scheme is shown in Fig. 1. The challenge with INS/GPS systems is that during GPS
outages, the system depends only on the INS error dynamic model which is, in most of the cases, an approximate linearized model. This leads to poor errors estimates during GPS outages. Thus, the performance degrades significantly during GPS outages. This paper presents an enhanced GPS/Reduced INS integrated navigation system that is based on nonlinear systems identification technique called Fast Orthogonal Search (FOS). The novelty aspects of this work lies in the utilization of fast nonlinear modeling of INS errors using FOS. Compared to existing linear estimation, such as KF [1], and existing nonlinear filtering techniques such as Particle Filter [28], the utilization of FOS is significantly faster and more reliable.

Moreover, these methods use models with sophisticated parameters that need to be estimated during good GPS availability, which add complexity and computational load to the navigation system.

The primary objectives of this research is as follows:
- To propose a KF algorithm with new GPS outage bridging scheme to mitigate large drifts during GPS outages.
- The bridging technique should not add much complexity to the integrated INS/GPS solution to be suitable for real-time realization.
- To provide this INS/GPS vehicular navigation system at low cost using a reduced IMU consists of single vertical gyro and two level accelerometer aided by vehicle speed measurements.

III. METHODOLOGY

A. GPS/Reduced IMU System

The proposed GPS outages bridging technique is realized on low-cost 3D land-vehicles tracking system using Reduced IMU integrated with GPS, based on Kalman Filtering. A low-cost 3D Reduced IMU platform consists of one MEMS grade vertically aligned gyroscope, two horizontal accelerometers, and vehicle odometer. This platform is shown in Fig. 2.

The state of the vehicle is determined by the vector:
\[
\{A_k, r_k, p_k, \phi_k, \lambda_k, h_k, Ve_k, Vn_k, Vu_k\}
\]
where \(A_k\) is the latitude of the vehicle, \(\lambda_k\) is its longitude, and \(h_k\) is its altitude, \(Ve_k, Vn_k, Vu_k\) are the East, North, and up velocity, respectively, \(p_k\) is the pitch angle (inclination), \(r_k\) is the roll angle, and \(A_k\) is the azimuth angle (heading from north). The INS error state vector \(x_k = [\delta A_k, \delta r_k, \delta p_k, \delta \phi_k, \delta \lambda_k, \delta h_k, \delta Ve_k, \delta Vn_k, \delta Vu_k, \delta a_{od}, \delta f_x, \delta f_y, \delta w_z]\) where \(\delta a_{od}, \delta f_x, \delta f_y, \delta w_z\) are errors of the odometer-derived acceleration, transversal
accelerometer, forward accelerometer, and the gyroscope, respectively.

The nonlinear vehicle state dynamic model is generally given by

$$ x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) $$  \hspace{1cm} (1)

where $u_k$ are the sensors reading, and $w_k$ is the noise contaminating sensors measurements (process noise). The detailed mathematical equations of this dynamic model can be found in [28]. The measurement model involves GPS velocity, position, and azimuth updates and it is given generally as

$$ z_k = h(x_k, v_k) $$  \hspace{1cm} (2)

where $z_k$ are sub-set of Reduced INS system error state vector whose elements can be directly observed from the difference between Reduced INS output and GPS measurements which are velocity, position, and azimuth.

The $v_k$ is the GPS measurement noise.

In KF reduced INS/GPS integrated system, both Reduced INS errors and GPS measurement dynamic models in (1) (2) are linearized using Taylor series expansion [1] to apply Kalman Filtering. After linearization, systems models are given by

$$ x_k = \Phi x_{k-1} + G u_{k-1} + w_{k-1} $$ \hspace{1cm} (3)

$$ z_k = H x_k + v_k $$ \hspace{1cm} (4)

When GPS observations are available, deviations of Reduced INS output (position, velocity, and azimuth) from GPS measurements ($z_k^{GPS}$) is used as observations to KF which use the difference between the actual system output ($z_k$) and the observation ($z_k^{GPS}$) to derive the system to correct its state which is Reduced INS error state vector. Having the observations ($\Delta z_k = z_k - z_k^{GPS}$), Reduced INS error state $x_k$ is now partially known from $\Delta z_k$. Hence, KF performs the update step to estimate the complete Reduced INS error state vector $x_k$ as follows:

$$ x_k = x_k + K \Delta y_k $$ \hspace{1cm} (5)

where $K$ is the Kalman gain. Hence, Reduced INS navigation output is corrected by subtracting errors state from it. For more details about KF equations and Kalman gain derivations, we refer the reader to [1][12][19].

B. Fast Orthogonal Search (FOS)

Orthogonal Search [26][27] is a general purpose nonlinear systems identification tool that can model any general system as seen in Fig. 3, and as explained in the following figure, using the following general model:

$$ Y[n] = \sum_{m=0}^{C-1} a_{jm} P_m[n] + e[n] $$ \hspace{1cm} (6)

where $P_m[n]$ is a set of arbitrary candidates, $a_{jm}$ are coefficients and $e[n]$ is the residual errors. The purpose of FOS is to select the best set of candidates $P_m[n]$ and the coefficients $a_{jm}$ that minimizes $e[n]$. The candidates $P_m[n]$ can be any arbitrary function of system inputs and outputs. For example, in an autoregressive model, the candidates $P_m[n]$ would be the system input delayed with specific number of samples $(x[n-l], l = 1, 2, \ldots, L)$. In Orthogonal Search techniques, a Gram–Schmidt procedure [26][27] is used to replace the functions $P_m[n]$ by a set of orthogonal basis functions $W_m[n]$ where the model for a specific $j$ is represented by the following corresponding model:

$$ Y[n] = \sum_{m=0}^{C-1} g_m W_m[n] + e[n] $$ \hspace{1cm} (7)

In orthogonal basis function space, the coefficients $g_m$ that minimize the mean square error over the observations is given by

$$ g_m = \frac{\bar{Y}[n] W_m[n]}{W_m^2[n]} $$ \hspace{1cm} (8)

where the over-bar in denotes the time average. The mean square error is given by:

$$ \bar{e}^2 = \left[ Y[n] - \sum_{m=0}^{C-1} g_m W_m[n] \right]^2 = Y[n] - \sum_{m=0}^{C-1} Q_m $$ \hspace{1cm} (9)

Where

$$ Q_m = \frac{\bar{Y}[n] W_m^2[n]}{W_m^2[n]} $$ \hspace{1cm} (10)

The reduction in mean square error resulting from adding a term $a_m P_m[n]$ is $Q_m$. The fast orthogonal search procedure makes use of the fact that it is not necessary to create the orthogonal functions $W_m[n]$ explicitly. Only their correlations with $P_m[n]$, the data $Y[n]$, and with themselves are required. By eliminating the generation of the orthogonal functions $W_m[n]$ explicitly, the FOS performance is much faster than existing traditional modeling techniques. This enables the FOS to work well in real-time applications that require superior performance,
such as video streaming, for high speed networks, such as ATM and Internet.

![Diagram](image)

**Figure 3. FOS Nonlinear Systems Identification Technique**

**C. Bridging GPS outages using FOS**

During GPS reliable availability, we have valuable information that should not be ignored. We have the corrected vehicle state which is the most accurate state estimation according to all available knowledge till the moment. In addition, we have the current sensors readings and the Reduced INS output deviations from GPS measurements. This valuable information represents the error characteristics of INS or Reduced INS solution by giving us data points that map the current vehicle state with current sensors readings (as input) and the INS or Reduced INS errors in position, velocity, and attitude (as output). If enough number of data points is collected in a data set in the format shown in Table 1, a FOS model can learn this data set mapping [18] and provide predictions of data points that are not seen before in the data set we already collected.

<table>
<thead>
<tr>
<th>INPUT</th>
<th>OUTPUT</th>
</tr>
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<tbody>
<tr>
<td>Corrected Vehicle State</td>
<td>Sensors Readings</td>
</tr>
<tr>
<td>(velocity and Attitude)</td>
<td>Reduced INS output deviations</td>
</tr>
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**Table 1. Input/Output Data for FOS Modeling**

During GPS outages, FOS equations (3) and (4) are used to predict the Reduced INS output deviations (part of error state vector $x$), which then are fed to KF as a virtual GPS updates to estimate all Reduced INS output errors. The mechanism is shown in Fig. 4 and Fig. 5.

![Diagram](image)

**Figure 4. FOS-Aided Reduced INS/GPS Mechanism in training**

**IV. EXPERIMENTAL WORK AND RESULTS**

The developed INS/GPS loosely coupled KF algorithm was tested on physical road data records collected over two different trajectories. The set of equipment used in experiments are as follows: Honeywell HG1700 AG11 tactical grade Inertial Measurement Unit (IMU), Novatel GPS receiver, CarChip E/X (8225) data logger [17] of a General Motors Passenger Van, and Laptop computer to control the equipments and log recorded data. Novatel CDU interface software was used to record GPS and IMU data which provide USB ports interface. G2 Pro-Pack Span unit developed by Novatel provides a tightly coupled INS/GPS navigation solution, which was used as a reference to evaluate proposed technique. Fig. 6 shows the testing trajectory as it appears in GPS Visualizer tool.

![Map](image)

**Figure 6. Testing Trajectory**

The Root Mean Square error (RMSE) [8] of the horizontal position of the vehicle during GPS outage was used as a performance measure. RMSE during 20 minutes of GPS-outage is shown in Fig. 7, which is compared with and without the FOS-bridging technique. The FOS was trained for only 6 minutes of good GPS availability period before the GPS-outage starts.
Fig. 8 shows the relationship between the GPS-outage period and the improvements in RMSE obtained by applying the proposed FOS-based bridging technique. Obviously the FOS performs better with longer GPS-outages.

V. CONCLUSION AND FUTURE WORK

This work presented an enhanced multi-sensors INS/GPS tracking systems for land vehicles using Fast Orthogonal Search (FOS) as a nonlinear identification technique, the proposed bridging scheme of Kalman Filter INS/GPS tracking systems successfully prevents the large drifts that occur during long GPS outages periods. The bridging scheme utilized FOS-based measurements prediction to enable Kalman Filter to perform update step on virtual aiding measurements. Experimental results show great RMSE improvement in longer GPS-outages. The proposed bridging scheme can be used with any AI-based modeling method or non-linear systems identification technique. Future work includes applying the same mechanism on full 3D IMU/GPS configuration [2] instead of the reduced IMU configuration. In this case, it is expected that more FOS candidates may be required [26].

REFERENCES


