Handling Positioning Errors in Location-based Services

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Abstract—Location-based services (LBS) shall typically only be provided within an authorized zone. This is enforced by location-based access control (LBAC) and affected by occurring positioning errors. Recent research has brought up different approaches for LBAC strategies. However, up to now it is unclear which strategy should be chosen for a given LBS and positioning system under realistic boundary conditions. In detail, the false authorization decision may cause severe additional costs when operating the underlying LBS. Hence, this paper presents a methodology to analyze the expected costs of LBAC strategies under the occurrence of positioning errors. The correlation to the practical costs occurring under realistic conditions is evaluated in an extensive case study. It is shown that in certain situations risk-based authorization is easily mislead by imprecise position error estimates and thus games away its theoretical superiority. In such situations, ignoring estimated errors may even yield lower expected costs of operating the LBS. The presented methodology contributes to finding the most suitable authorization strategy when deploying a LBS. This finally helps to reduce costs occurring from false authorization decisions when operating the LBS.

Keywords—Location-based Access Control; Risk-aware Authorization; Positioning Errors

I. INTRODUCTION

Mobile devices with their integrated positioning capabilities enabled LBS, which nowadays are of substantial importance for most users and service providers [1]. A subclass of LBS are zone-based. Here, a user is granted the authorization to use a LBS if he resides within a predefined authorized zone. For example, imagine a museum that provides an audio guide for mobile devices. The guide’s explanations for an exhibition room shall only be audible if the user payed the room’s entrance fee and is inside. In order to enforce such authorization semantics, LBAC systems have been developed. Those systems employ the user’s current location measurement to decide about the permission to use a given LBS. Such LBS require precise location measurements, which are unfortunately inherently affected by a varying degree of uncertainty, for example due to changing environmental influences or imprecise sensors. Thus, the most precise formalization of the real user location is done by adhering an according probability density function (pdf) as the position estimate, which is finally derived from measurements.

An appropriate LBAC strategy is crucial, as false authorization decisions typically cause costs for false negatives and false positives. Early approaches to LBAC focused on extending the expressiveness of existing access control policies with spatial information. In those approaches, possibly occurring errors are ignored and only the most likely geographical point is used as location estimate. Here, authorization decisions are derived by checking whether the punctual location estimate is contained within the polygon of the authorized zone. Those approaches ignore possibly occurring positioning uncertainty and costs and are thus called risk-ignoring for the rest of this paper. The second category comprises threshold-based approaches to LBAC. Here, the position estimate pdf is employed to derive the probability that the user resides within the authorized zone. A threshold is predefined by the policy designer as the minimum required probability in order to be authorized for using the LBS. Here, costs are not considered and often, the derivation of an appropriate threshold is left unspecified. Even risk-based LBAC strategies were developed. Here, authorization decisions are finally derived based on cost functions and the probability that the user resides within the authorized zone. In detail, such LBAC systems only grant the authorization if the expected costs of a false positive undershoot the expected cost of a false negative. This method is theoretically optimal [2].

However, up to now it is unclear which LBAC strategy should be chosen in practice. Furthermore, it has not been studied how the superiority of the risk-based approach depends on the cost functions for false authorization decisions and the accuracy of the positioning system. In detail, there is urgent need to clarify the effect of statistically imperfect position estimates on the superiority of the risk-based approach. A methodology for choosing that LBAC strategy with the lowest expected cost of false authorization decisions when operating the LBS is non-existent. Nevertheless, such a methodology is urgently needed in order to avoid costly wrong decisions when choosing the authorization strategy for a LBS.

In order to solve this problem, this paper presents a novel approach to analyze the expected costs of false authorization decisions in the forefront of a LBS’s deployment. In detail, a methodology for computing the expected costs when operating the risk-ignoring, threshold-based and risk-based LBAC strategy is proposed. Given the error characteristics of the underlying positioning system, this facilitates the computation of expected savings when operating the theoretically optimal risk-based strategy compared to the risk-ignoring strategy. This allows to illustrate the LBS’s sensitivity to statistically imprecise position estimates of the positioning system. The need for such analysis is demonstrated for an indoor–LBS in a typical office environment with WiFi fingerprinting as location provider. The theoretical optimality of the risk-based approach is shown to be highly dependent on the LBS’s parameters and the quality of the reported position estimates. The rest of the paper is structured as follows: Section II gives a detailed view on related work. Next, Section III presents the theoretical approach to analyze the superiority of the risk-based LBAC strategy. Section IV presents a case study to illustrate the urgent necessity of a detailed analysis before choosing a LBAC strategy. Finally, Section V concludes the paper.
II. RELATED WORK

Location information has been widely used for spatial authorization systems and provisioning of LBS in particular. Often, these methods are called LBAC or spatial access control. One important subset of LBAC systems employs the estimated location in order to determine if the user resides within a prescribed authorized zone. If true, the access right is granted. Important approaches provide sophisticated spatial extensions to role–based access control (RBAC) [3]–[5]. However, even recently published work, for example from Abdunabi et al. uses the reported user location without any consideration of measurement uncertainty [6]. Unfortunately, these approaches do not show the effectiveness of this strategy when applied to realistic and error–prone location providers. It is left unclear, if this strategy is suitable for a given authorized zone and a concrete location provider. Ardagna et al. proposed an approach which employs a confidence value for the probability that the user resides within a predefined authorized zone [7]. If this value overshoots a predefined threshold, the access right is granted. Thresholds are derived empirically based on estimates about the positioning system’s sensitivity to changing weather and environmental conditions. Also, the number of sensors is mentioned as an important factor when defining a threshold. However, no concrete methodology to provide any justification of derived thresholds is given. Shin et al. define an authorization policy, which also grants access if the user resides within an authorized zone with a confidence value larger than a predefined threshold [8]. Here, the uncertainty of a position fix is modeled as a probability distribution. The confidence value is derived by integrating the probability distribution over the authorized zone. The thresholds are derived for each access rule individually depending on whether the authorized zone demands high security or is an area of low sensitivity. Again, only very abstract and vague statements about deriving a suitable threshold are mentioned. Krustanov et al. consider costs when making authorization decisions based on the values of discrete attributes with uncertain values [2]. A threshold–based authorization strategy is shown to be cost–optimal in their scenario for a certain threshold based on cost functions. However, the approach does not show the influence of the uncertainty estimates’ quality on the cost–effectiveness of their strategy. Marcus et al. proposed a risk–aware approach for trajectory–based authorization using probabilistic trajectories derived from an adapted particle filter in combination with WiFi fingerprinting [9]. Here, expected costs and the corresponding risk are minimized by adhering assigned cost functions of false authorization decisions and the probability that the user’s trajectory satisfies the authorization condition.

Error estimators of positioning systems are needed to operate threshold–based and risk–based LBAC strategies. Basically, given an estimated location \( \mu \), an error estimate is a probability distribution \( P(\mu | x) \) describing the likelihood to observe an estimated location of \( \mu \) when standing at position \( x \) in the real–world. Hightower et al. [10] use a commercial infrared system and an ultrasound time–of–flight badge system. The infrared error estimates are a static bivariate Gaussian with a covariance matrix \( \Sigma = \begin{pmatrix} 2.1 & 0.0 \\ 0.0 & 2.3 \end{pmatrix} \). The values are derived from the vendor specification of the system’s range. The error estimates of the ultrasound system are retrieved from a lookup table built from previously recorded measurement errors within the test lab. Zandbergen et al. observed GPS errors with a root mean square error of 9–11 m for modern Smartphones, which highly increase in urban areas [11]. Zandbergen et al. also found that the positioning errors of GPS are not perfectly approximated by Gaussian distributions and hence, outliers need to be expected [12]. The error distribution of GPS is found to be approximate to a Rayleigh distribution. Marcus et al. proposed an error estimator for SMARTPOS, an indoor positioning system based on WiFi fingerprinting [13]. Here, the errors were shown to be approximately normally distributed with a mean of 1.2 m.

III. LOCATION–BASED AUTHORIZATION UNDER POSITIONING ERRORS

This section first discusses the characteristics of positioning systems and subsequently describes the methodology to theoretically derive the expected costs for each LBAC strategy. The expected savings of uncertainty–aware strategies are compared to the risk–ignoring strategy.

A. Positioning Systems and Error Estimators

The key technology for LBS are positioning systems, which determine the user’s location either terminal– or infrastructure–based. In outdoor scenarios, GPS emerged as the most important positioning technique, while in indoor scenarios WiFi fingerprinting showed very promising results [12] [13]. In the following, the returned position measurements are called position fixes and noted as \( \mu \) with \( \mu \in \mathbb{R}^2 \). In all cases, position fixes are subject to physical perturbations due to interference, reflections, multipath propagation, humidity, imprecise sensors, and so on [1]. Consequently, all position fixes are inherently affected by an error varying degree as discussed in Section II. The user’s ground truth position around the returned position fix \( \mu \) can be modeled as a probability density function (pdf), which is derived by error estimators.

In all cases, such error estimates are derived from singularities of the underlying measurement by an according error estimator. For example, in case of WiFi fingerprinting, the distribution of the k nearest neighbors around \( \mu \) was shown to be a good indicator for the occurring error [13]. Given a position measurement with an estimated position of \( \mu \), the error estimator finally derives a scale parameter \( \Sigma \) which defines a pdf around \( \mu \) in order to describe the ground truth location. In case of WiFi Fingerprinting, the scale parameter represents the covariance matrix of the underlying bivariate normal pdf. For the rest of this paper, position fixes \( \mu \) are reported with a scale parameter \( \Sigma \) of an appropriate error estimate pdf and finally written as \( (\mu, \Sigma) \). The larger the estimated scale parameter \( \Sigma \), the more uncertainty exists with the position fix \( (\mu, \Sigma) \). In the following, the accuracy of positioning systems is described by a distribution \( F_{err} \) of reported scale parameters \( \Sigma \). Practical experiments have shown that inverse Gaussian distributions give a very good fit for \( F_{err} \) in case of WiFi fingerprinting. However, the distribution of \( F_{err} \) is finally needed to analyze the suitability of single LBAC strategies for a concrete scenario and needs to be known for the employed positioning system.

B. Risk–ignoring, threshold– and risk–based LBAC strategies

The task of a LBAC strategy is to derive an authorization decision \( auth \in \{\text{true}, \text{false}\} \) based on a position fix...
The most basic LBAC strategy is the risk-ignoring authorization strategy as employed in [3]–[6]. Here, only the estimated position \( \mu \) is considered, when deriving the authorization decision. In detail, this strategy performs a simple point in polygon test to determine, if the estimated position \( \mu \) is contained within the authorized zone \( Z \):

\[
\text{auth} \Leftrightarrow \mu \in Z
\]  

(1)

The main advantage of such systems is the low computational overhead and the efficiency of point in polygon tests. In detail, no error estimate needs to be derived and no complex numerical operations need to be performed.

In order to consider the occurring uncertainty of a position fix \((\mu, \Sigma)\), the threshold-based LBAC strategy derives its authorization decision based on the probability \( p_Z \) that the user resides within the authorized zone \( Z \). \([2, 7, 8]\). This probability is derived from the estimated position \( \mu \) and the error estimate \( \Sigma \) and needs to overshoot the threshold:

\[
\text{auth} \Leftrightarrow p_Z(\mu, \Sigma) > \text{threshold} \tag{2}
\]

A drawback of this strategy is the dependence on reliable error estimators and the computational overhead of computing \( p_Z(\mu, \Sigma) \). Furthermore, it is not clear, which threshold makes sense for a given LBS.

A more sophisticated strategy is the risk-based strategy \([2, 9]\). Here, the expected costs of granting or denying the authorization request are compared. In particular, the authorization request is only granted, if the expected cost \( c_{fp} \) of a false positive undershoot the expected cost \( c_{fn} \) of a false negative. The expected cost can also be interpreted as the risk of each outcome:

\[
\text{auth} \Leftrightarrow (1 - p_Z(\mu, \Sigma)) \cdot c_{fp} < p_Z(\mu, \Sigma) \cdot c_{fn} \tag{3}
\]

This has the same computational complexity as the threshold-based strategy. Its main advantage is its decision-theoretical optimality given statistically perfect error estimates \( \Sigma \), which will be discussed in detail later in Section IV.

The risk-based strategy is a real generalization of the threshold-based strategy. Obviously, according to (3), the risk-based strategy depends on the static costs \( c_{fp} \) and \( c_{fn} \) and the ratio \( \frac{c_{fp}}{c_{fn}} \) in detail. For each ratio \( \frac{c_{fp}}{c_{fn}} \), there exists exactly one value of threshold such that a threshold-based strategy with threshold behaves exactly like a risk-based strategy with \( \frac{c_{fp}}{c_{fn}} \). This easily follows from resolving (3) to \( p_Z \), which finally represents the corresponding value of threshold:

\[
\text{threshold} = \frac{c_{fp}}{1 + \frac{c_{fp}}{c_{fn}}} \tag{4}
\]

This correspondence is depicted in Figure 1. Clearly, the higher the cost of a false positive compared to the cost of a false negative, the higher the corresponding value of threshold, which converges to 1. The knowledge of this correspondence has two positive effects. On the one hand, existing LBAC policies based on the threshold-based strategy can be assigned comprehensible thresholds given the cost functions of the underlying service or resource to be granted. This correspondence finally allows to derive such values of threshold that the threshold-based strategy also yields risk-optimal decisions given the specific cost functions for the LBS to be deployed.

On the other hand, analysis of a risk-based strategy deployed for a LBS also allows to assess a threshold-based strategy with a corresponding threshold. However, in real situations, both strategies are subject to the quality of the underlying error estimator. Statistically imperfect error estimators may under- or overestimate the real error, which is a severe weak point compared to the risk-ignoring strategy. In order to finally derive the most suitable LBAC strategy for a given LBS, first, the theoretically expected costs for each LBAC strategy are derived in the next section.

C. The Expected Costs of LBAC strategies

In this section, a methodology is derived to compare the LBAC strategies w.r.t. the expected costs of false authorization decisions. These expected costs are highly dependent on \( c_{fp} \) and \( c_{fn} \) and the uncertainty of underlying position fixes. In order to decide about the authorization of a requesting user to use a given LBS, the underlying LBAC strategy is provided a position fix \((\mu, \Sigma)\) to check the authorization conditions according to the aforementioned methods. Given this position fix, the statistical distribution of the user’s ground truth position \( x \) around \( \mu \) is denoted as the probability density function \( F_{\mu, \Sigma}(x) \). Note, the distribution of error estimation scale parameters \( \Sigma \) is denoted as \( F_{\Sigma} \), in the following.

In the next step, a methodology is presented, which allows to assess the expected costs \( E(\text{costs}_i) \) for each LBAC strategy \( i \in \{\text{risk-ignoring, risk-based, threshold-based}\} \). This is achieved by employing a function \( \text{costs}_i(\mu, \Sigma) \) denoting the expected costs arising from a possibly false decision when authorizing a user with position fix \((\mu, \Sigma)\) with LBAC strategy \( i \). Therefore, for each LBAC strategy \( i \), the expectation of occurring costs is derived w.r.t. the set \( \mathcal{R} \subseteq \mathbb{R}^2 \) of possible estimated locations \( \mu \in \mathcal{R} \) and the distribution of occurring error estimates \( F_{\Sigma} \):

\[
E(\text{costs}_i) = \frac{1}{|\mathcal{R}|} \int_{\mathcal{R}} \int_0^\infty \text{costs}_i(\mu, \Sigma) \cdot F_{\Sigma}(\Sigma) \, d\Sigma \, d\mu \tag{5}
\]

The rest of this section focuses on deriving the function \( \text{costs}_i(\mu, \Sigma) \) for each LBAC strategy \( i \) in order to finally derive its expected overall costs \( E(\text{costs}_i) \) according to (5).

In order to derive \( E(\text{costs}_i) \) for each LBAC strategy \( i \), the function \( \text{costs}_i(\mu, \Sigma) \) needs to be specified first. Here, a prerequisite is the computation of the probability \( p_Z(\mu, \Sigma) \) that a user with position fix \((\mu, \Sigma)\) resides within the authorized zone \( Z \):

\[
p_Z(\mu, \Sigma) = \int_{x \in Z} F_{\mu, \Sigma}(x) \, dx \tag{6}
\]
In the following, \( p \) is used as an abbreviation for \( p_z (\mu, \Sigma) \) if no ambiguities exist. Finally, this allows to derive the expected costs for each LBAC strategy given a position fix \((\mu, \Sigma)\).

1) Deriving Expected Costs of Risk-ignoring Approaches: Given a position fix \((\mu, \Sigma)\), the risk-ignoring approach simply checks if \( \mu \in Z \). Nevertheless, the user’s ground truth position \( x \) is distributed according to the position fix’ pdf \( F_{\mu, \Sigma} (x) \) and consequently lies outside of \( Z \) with probability of \((1 - p)\). Thus, if the authorization request with the estimated location \( \mu \) is denied, the probability of a false negative is \( p \). Contrary, if the authorization is granted, the decision is a false positive with probability \((1 - p)\). Assume both cases to cause costs \( c_{fp} \) and \( c_{fn} \), respectively. This finally allows to derive the expected costs:

\[
\text{costs}_{\text{risk-ignoring}} (\mu, \Sigma) = \begin{cases} 
(1 - p) \cdot c_{fp}, & \text{iff } \mu \in Z \\
p \cdot c_{fn}, & \text{else}
\end{cases}
\] (7)

Note, that given a position fix \( \mu \), the risk-ignoring strategy unfortunately may even choose that authorization decision with higher expected cost.

2) Expected Costs of Threshold-based Approaches: Given a position fix \((\mu, \Sigma)\), the threshold-based strategy derives its authorization decision based on checking whether probability \( p \) that the user resides within \( Z \) exceeds a predefined threshold. The probability is derived according to (6). Finally, this allows to compute the expected costs for the threshold-based strategy for each possible position fix \((\mu, \Sigma)\):

\[
\text{costs}_{\text{threshold-based}} (\mu, \Sigma) = \begin{cases} 
(1 - p) \cdot c_{fp}, & \text{iff } p \geq \text{threshold} \\
p \cdot c_{fn}, & \text{else}
\end{cases}
\] (8)

Again, despite the expected cost of an authorization decision, only the satisfaction of the threshold is considered.

3) Expected Costs of Risk-based Approaches: The risk-based LBAC strategy first computes the expected costs of either granting or denying an issued authorization request. Clearly, this results from multiplying the cost of \( c_{fp} \) and \( c_{fn} \) with their individual probability of occurrence \((1 - p)\) and \( p \). Finally, that authorization decision is taken, which promises lower expected costs. Formally, the expected costs for granting or denying the authorization request compute as:

\[
\text{costs}_{\text{risk-based}} (\mu, \Sigma) = \min \left((1 - p) \cdot c_{fp}, p \cdot c_{fn}\right)
\] (9)

Clearly, the risk-based approach always derives that authorization decision with the minimal expected costs. Thus, whenever the risk-ignoring or threshold-based strategy choose that authorization decision with lower expected costs by chance, the risk-based strategy will consequently also choose that decision. This implies, that the expected costs of the risk-based strategy theoretically are a lower bound for the expected cost of any other LBAC strategy.

D. Analyzing the Expected Costs of LBAC strategies

As seen above, the expected costs of the risk-based LBAC strategy are a theoretical lower bound for the expected costs of the risk-ignoring and threshold-based strategy. The percentaged savings \( E(S) \) w.r.t. any LBAC strategy \( j \in \{\text{risk-ignoring}, \text{threshold-based}\} \) compute as:

\[
E(S) = \frac{E(\text{costs}_j) - E(\text{costs}_{\text{risk-based}})}{E(\text{costs}_j)}
\] (10)

The theoretically expected savings \( E(S) \) from fewer false authorization decisions are laying the foundation for deciding about the overall most cost-effective authorization strategy. Given the boundary conditions \( Z, F_{err} \), and \( R \) of a LBS, the expected savings \( E(S) \) strongly depend on the ratio \( \frac{c_{fn}}{c_{fp}} \) of the LBS’s costs for false authorization decisions. Hence, the theoretically expected savings \( E(S) \) for a ratio \( \frac{c_{fn}}{c_{fp}} \) will finally play a major role when choosing the practically most cost-effective LBAC strategy. This will be explained later in Section IV in detail.

The dependence of \( E(S) \) on \( \frac{c_{fn}}{c_{fp}} \) is exemplary depicted in Figure 2 for five theoretical examples with increasingly more inaccurate positioning systems and an authorized zone of \( 5 \times 5 \) m. Note, \( \mu \) is the mean of \( F_{err} \) in this figure. The curves are derived using (5). Clearly, the expected savings \( E(S) \) show a minimum for each distribution of errors \( F_{err} \). The valleys around the minima of the graph \( E(S) \) spread with increasing accuracy of the positioning system. This coincides with the intuition, that the expected savings from operating the risk-based strategy are higher if the positioning system is more inaccurate.

The dependence of the expected savings on the ratio of costs \( \frac{c_{fn}}{c_{fp}} \) is a direct consequence of the dependence of \( \text{costs}_{\text{risk-based}} (\mu, \Sigma) \) on \( \frac{c_{fn}}{c_{fp}} \). Given a fixed value of \( \Sigma \), savings can only arise for such estimated locations \( \mu \) where the risk-based approach takes a different authorization decision than the risk-ignoring approach. Clearly, the larger the set of such estimated locations \( \mu \), the larger the expected savings. This correlation is depicted in Figure 3 in 1D for a authorized zone of 5 m, and Gaussian error estimates with a fixed value of \( \Sigma = 3.5 \) m. The difference of cost functions for the risk-based and risk-ignoring LBAC strategy is marked green. Here, cost functions \( c_{fp} = c_{fn} = 1 \) were chosen, i.e., the risk-based LBAC strategy only authorizes a request with position fix \((\mu, \Sigma)\) if \( p > 0.5 \) according to (4). As the risk-ignoring strategy authorizes all \( \mu \in Z \), the risk-based approach only derives a more cost-effective decision for such \( \mu \) with \( p(\mu, 3.5 \text{ m}) < 0.5 \).

Fig. 2. Theoretically computed expected savings \( E(S) \) for five different distributions of inverse Gaussian distributions of \( F_{err} \).
by the ratio of costs finally overshoots the maximum of \( p \) within \( Z \), all authorization requests will be denied by the risk–based strategy. Equally, if the ratio is decreased, the threshold required by the risk–based strategy decreases according to (4). If the required threshold undershoots the value of \( p \) on the boundary of \( Z \), the set of location estimates authorized by the risk–based strategy also includes \( \mu \notin Z \). If the ratio of costs converges to 0, the set of authorized \( \mu \) and the expected savings converge to infinity. Both authorization strategies yield the same expected costs if the threshold implied by the ratio of costs according to (4) corresponds to the value of \( p \) on the boundaries of \( Z \). In that case, the risk–based and the risk–ignoring approach show identical behavior. If the distribution of \( F_{err} \) shows a high probability for such \( \Sigma \) which cause an identical or nearly identical behavior of the risk–based and risk–ignoring strategy, the expected savings \( E(S) \) will finally have a lower minimum. Intuitively, the lower the value of this minimum and the wider the valley around it, the more sensitive is the risk–based strategy to statistically imprecise error estimates \( \Sigma \). In detail, the risk–based strategy might be misleadingly identified as being the optimal choice and game away the theoretically small benefit. Thus, the next section evaluates the weakness of the risk–based strategy when deployed with realistic error estimators.

IV. USE CASE: DEPLOYING A ZONE–BASED LBS IN AN OFFICE ENVIRONMENT

The expected savings of the risk–based strategy are now exemplary evaluated in a use case in a typical office environment. Here, WiFi fingerprinting is used as the underlying positioning system. A radiomap of 206 fingerprints was recorded within an area of 1400 m² as depicted in Figure 4. An overall set of 1500 test fingerprints was recorded, each labeled with the room where it was recorded. All areas outside the labeled rooms shown in Figure 4 were assigned the label outside. Positioning is performed according to prior work, where a kNN approach with \( k = 4 \) is used [13]. The position estimate is derived as the weighted mean of the nearest neighbors. Two error estimators, a Laplace and Gaussian error estimator, were used in order to compare the impact of the statistical quality of returned error estimates on the expected savings \( E(S) \). The Gaussian error estimator returns bivariate normal distributions and is defined according to prior work [13]. In detail, the aforementioned scale parameter \( \Sigma \) corresponds to the covariance matrix of the returned Gaussian and is defined as \( \Sigma = ( \sigma \ 0 \ 0 \ \sigma ) \). Here, \( \sigma \) is derived as the weighted average of the kNNs’ distances to the position fix. In contrast, the Laplace error estimator returns rotational symmetric bivariate Laplace distributions with a mean of \( \mu \) and a diversity \( b \) of \( \Sigma = b \). Again, the scale parameter is derived as the weighted average of the kNNs’ distances to the position fix \( \mu \).

The distribution of estimated scale parameters \( \Sigma \) is shown in Figure 5 and mainly follows an inverse Gaussian with parameters \( \mu = 0.8 \) and \( \lambda = 9.5 \). In the evaluation, the scale parameter is estimated for a derived position estimate \( \mu \) and used twice as the variance for a Gaussian and accordingly as the diversity for the Laplace distribution. A set of authorized zones was defined as the labeled rooms shown in Figure 4. In order to compare the effects of the error estimators, the recorded testset was applied to each of the authorized zones, once using the Laplace error estimator and once using the Gaussian error estimator. In order to identify the impact of the authorized zones’ size, a second run was performed, where the authorized zones consisted of all possible unions of labeled neighboring rooms from Figure 4. The results are depicted in Figure 6. All runs approximate the theoretically derived shape with a single minimum. For both runs, with single or aggregated rooms, the percentaged expected savings \( E(S) \) obtained by employing the Laplace error estimator clearly overshoot the value of \( E(S) \) obtained when applying a Gaussian error estimator. However, the theoretical optimality of the risk–based strategy is not given in all cases here. All runs except for the one with a Laplace error estimator and non–aggregated
rooms have a negative minimum. For both, the Laplace and the Gaussian error estimator, the expected savings for the run with aggregated rooms are slightly lower than for the single, non-aggregated rooms. This stems from the intuitive fact, that the application of the risk-based strategy is more promising if the rooms are small compared to the estimated errors. Consequently, for aggregated rooms, the minimum for the theoretically expected savings is lower than for non-aggregated rooms and hence, the small superiority is gained away more easily by imprecise error estimates. Finally, the evaluation results show several important implications. First, the optimality of the risk–based strategy strongly depends on the statistical correctness of the underlying error estimator. The extent of its statistical error compared to the authorized zone also shows a negative effect on the expected savings of the risk–based LBAC strategy. Hence, the cost–optimal LBAC strategy can only be determined by recording a set of test data around the authorized zone of the LBS in the forefront of its deployment. This test data needs to be evaluated with a suitable error estimator for the underlying positioning system. If the expected savings \( E(S) \) are negative or near 0 for the LBS’s ratio of costs \( \frac{c_{fp}}{c_{fn}} \), the application of the risk–based strategy is most likely inferior to the risk–ignoring strategy. However, when such evaluations are performed in order to decide about the most suitable LBAC strategy, a large number of test data is necessary in order to obtain statistically sound results.

V. CONCLUSION AND FUTURE WORK

This paper examined the problem of choosing an appropriate location–based authorization strategy, for example needed for LBS, under the occurrence of positioning errors. First, expected costs of operation were theoretically derived for three distinct authorization strategies. The risk–ignoring, threshold–based and risk–aware strategy. It was shown that the superiority of the risk–aware to the risk–ignoring strategy strongly depends on the ratio of costs of false positive and false negative decisions and is minimal for a specific ratio of these costs. In a practical evaluation, the risk–aware policy was shown to be easily mislead to suboptimal decisions when operated with statistically imperfect error estimators. This clearly shows that in practice the widely accepted theoretical superiority of risk–based authorization strongly depends on the ratio of costs and the quality of the error estimator. Furthermore, it is shown that the superiority of the risk–based approach needs to be empirically asserted if the LBS’s ratio of costs is near the theoretical minimum of expected savings. Clearly, when deploying a LBS, choosing the right authorization strategy is crucial in order to minimize the expected costs arising from false authorization decisions. Regardless of the importance of the correct choice, this question has not been studied under realistic assumptions up to now. Thus, the results presented in this paper show that the theoretically optimal strategy is not the most effective strategy in all cases under realistic boundary conditions. A methodology to assess the theoretically expected savings of the risk–based and threshold–based strategy is presented. This finally allows to analyze if the risk–aware strategy shows only little improvement given a specific LBS and finally indicates if its application needs to be empirically justified. Finally, the presented approach helps to deploy LBS more cost–efficiently and thus supports their acceptance and efficiency. Future work is seen in developing quality of service metrics for LBS based on the expected costs of their operation. In detail, the effects of imprecise position estimates on LBS require further research.

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