

## Analysis of the Optimum Switching Points in an Adaptive Modulation System in a Nakagami- $m$ Fading Channel Considering Throughput and Delay Criteria

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**Abstract**— The adaptive modulation technique is a promising solution to resolve the problem of spectrum scarcity. A key issue that defines the performance of adaptive modulation systems is the ability to find the optimum switching points between neighboring modulations. In this paper, we analyze the influence of the fading channel model in the optimum switching points, assuming a Nakagami- $m$  fading model and both real and non-real-time traffic. Therefore, two criteria were considered to determine the optimum switching points: the maximum throughput criterion, for a real-time traffic scenario, and the delay criterion, for a non-real-time traffic scenario.

**Keywords**- Adaptive modulation; delay criterion; maximum throughput criterion; Nakagami- $m$  fading; optimum switching points.

### I. INTRODUCTION

Due to the exponential growing of the traffic in telecommunications networks in the last years, problems like the demand for higher transmission rates and the scarcity of spectrum have become extremely relevant. Several studies have been developed in order to improve the performance and ensure Quality of Service (QoS) for such networks. In this regard, the adaptive modulation technique has gained great attention as a promising solution to improve the performance of channels with time-varying conditions. For example, reference [1], published in ICN 2016, analyses the optimum switching point in an adaptive modulation system in a particular scenario.

Adaptive modulation technique consists of the dynamic adaptation of the modulation scheme as a function of the channel's state, according to a given performance criterion. The receiver makes an estimation of the channel state and sends this information back to the transmitter through a feedback channel. Based on this information, the transmitter modifies the modulation order, so that it better matches the conditions of the channel at that time [1] – [6].

The definition of the best points to switch between two modulations is a key issue in the adaptive modulation technique. In general, the change in modulation order occurs between neighboring modulations. Given a modulation with  $2^n$  points in its constellation, the neighboring modulation has  $2^{n-1}$  or  $2^{n+1}$  points in its constellation [1] [4] [5].

Several ways of determining the switching points between neighboring modulations have been mentioned in

the literature. The most common are to determine the switching points as a function of a target for the bit error rate (BER) [2] or a target for the packet error rate (PER) in the channel [6]. However, as proven in [4], these criteria do not optimize some QoS parameters, such as throughput and delay; thus, these authors proposed the calculation of the optimum switching points based on the maximum throughput criterion, considering real-time traffic, and the delay to transmit a correct PDU (Packet Data Unit) for non-real-time traffic. The analysis presented in [4] considers a memory-less channel, e.g., an AWGN (Additive White Gaussian Noise) channel in a wireless ATM (Asynchronous Transfer Mode) network. This approach, however, is not appropriate for several wireless channels, in which there is fading. Then, in [5], the authors extended the analysis presented in [4], taking into account Rayleigh fading channels. In [1], the authors extended the analysis presented in [4] and [5], considering a more general fading model: a Nakagami- $m$  channel. However, in [1], the authors analyzed the influence of the channel model in the optimum switching points, taking into account only the maximum throughput criterion.

In this paper, we extended the analysis presented in [1] [4] [5], examining in depth the influence of the channel model in the optimum switching points of an adaptive modulation scheme and considering a Nakagami- $m$  fading channel and two scenarios: real-time and non-real-time traffic. To compute the optimum switching points, we use the maximum throughput criterion for scenarios with real-time traffic and the delay criterion for non-real-time traffic, where a packet received with error can be retransmitted until it is correctly received.

The analysis in this paper considers transmissions in a wireless network with a Nakagami- $m$  block fading channel that uses adaptive  $M$ -ary Quadrature Amplitude Modulation ( $M$ -QAM).

The remainder of this paper is organized as follows: In Section II, we introduce the system and channel models; in Section III, we present the calculation of the exact PER in the channel, which is necessary to compute the throughput and the delay. The maximum throughput criterion is presented in Section IV, and the delay criterion is discussed in Section V; Section VI presents the numerical results, and finally, we present our conclusions in Section VII.

## II. SYSTEM AND CHANNEL MODELS

In this section, we define the characteristics of the system and the channel model considered in this paper.

### A. System Model

The system is composed of one base station, which manages all traffic, and several users that transmit over the wireless network, following the model presented in [4]. We assume a network using TDMA (Time Division Multiple Access), where time is divided into frames composed of the downlink and uplink periods.

In the downlink period, the base station communicates with the terminals through Time Division Multiplexing (TDM) and transmits the updated modulation information for users via broadcast. The modulation order is defined frame by frame based on the chosen performance criterion. In the uplink period, when users receive permission to transmit, they transmit data using TDMA. One TDMA frame is divided into  $X$  time slots, where each time slot allows the transmission of  $n_s$  bits.

In a communication system, data messages are usually transmitted in packets. In this paper, one data message containing  $n_d$  bits is fragmented into  $Z$  packets, each packet with  $n_s$  bits. Only one packet is transmitted in each time slot. In addition, each user has only one time slot per frame for their transmissions. Thus,  $Z$  frames are necessary to transmit a data message,  $Z = n_d/n_s$ . Figure 1 illustrates the frame structure and the transmission process in the uplink.

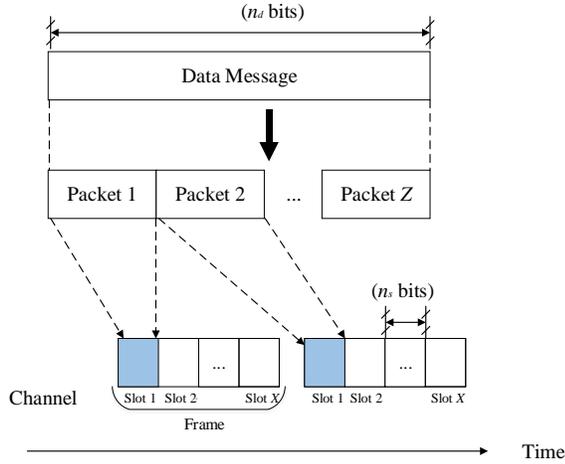


Figure 1. Frame structure and the transmission process in the uplink.

### B. Channel Model

We assume a slowly-varying Nakagami- $m$  block fading channel, whose complex gain values remains invariant over a single frame but may vary between adjacent frames. Thus, the choice of modulation order is made on a frame-by-frame basis [1] – [3] [6]. So, the probability density function (pdf) of the signal-to-noise ratio (SNR) is given by [2] [6]:

$$P_\gamma(\gamma) = \frac{m^m \bar{\gamma}^{m-1}}{(\bar{\gamma})^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right). \quad (1)$$

where  $\bar{\gamma}$  is the average received SNR,  $\Gamma(m)$  is the Gamma function, defined by  $\Gamma(m) = \int_0^\infty x^{m-1} e^{-x} dx$ , and  $m$  is the Nakagami fading parameter [2] [6].

The Nakagami- $m$  probability distribution is widely used in the literature to represent a wide range of well-known multipath fades [2]. The Nakagami- $m$  fading model is equivalent to a set of independent Rayleigh fading channels obtained by maximum ratio combining (MRC), where  $m$  represents the diversity order [7]. So, other distributions can be modeled with the variation of parameter  $m$ . For instance, when  $m = 0.5$ , the Nakagami- $m$  fading model represents the unilateral Gaussian distribution (which corresponds to the greatest amount of multipath fading scenarios); when  $m = 1$ , the Nakagami- $m$  distribution results in a Rayleigh distribution model, and when  $m > 1$ , there is a one-to-one mapping between the Nakagami fading parameter and the Rician factor, which allows the Nakagami distribution to approach the Rice distribution [2]. Moreover, reference [2] claims that the Nakagami- $m$  distribution often provides the best fit for urban and indoor multipath propagation. Thus, in this paper, we analyze the influence of the variation of the diversity order  $m$  in the optimum switching points in an adaptive modulation scheme.

## III. CALCULATION OF THE EXACT PER

To compute the throughput and the delay in the network, it is necessary to compute the PER. In this section, we summarize the approach used to compute the instantaneous and average PER.

### A. Instantaneous PER

In the scenario considered in this paper, the base station defines the best modulation in terms of the throughput or delay that the terminal should use to transmit data in the uplink frame. Six modulation schemes were chosen for our analysis:  $M$ -QAM with  $M = 8, 16, 32, 64, 128$  and  $256$ . Each  $M$ -ary modulation scheme has  $R_n$  bits per symbol, where  $n = 1, 2, \dots, 6$  and represents the modulation mode set at the moment.

In the current literature, the calculation of the PER is usually given as a function of the BER, and it is given as [6]:

$$PER = 1 - (1 - BER)^{n_s}, \quad (2)$$

where  $n_s$  represents the number of bit in a packet.

The expression (2) considers a system where the bits inside a packet have the same BER, with uncorrelated bit-errors. However, for large-size QAM constellations, the author of [6] claims that the PER calculation using (2) is not

accurate since the information bits of the same packet occur with different error probabilities for such constellations.

Thus, in this paper, we considered the approach proposed by [6] to compute the PER, which is based on the methodology proposed in [8] to compute the exact BER for an arbitrary rectangular M-QAM modulation.

Following [8], we assume that a rectangular M-QAM modulation can be modeled as two independent I-ary and J-ary pulse amplitude modulations (PAM), where  $M = I \times J$ . The exact BER calculation is obtained by observing regular patterns that occur due to the characteristics of Gray code bit mapping. The error probability of the  $k_{th}$  bit in-phase components of the I-ary PAM, where  $k \in \{1, 2, \dots, \log_2 I\}$ , is given by [8]:

$$P_I(k) = \frac{1}{I} \sum_{i=0}^{(1-2^{-k})I-1} \left\{ (-1)^{\lfloor \frac{i \cdot 2^{k-1}}{I} \rfloor} \left[ 2^{k-1} - \left\lfloor \frac{i \cdot 2^{k-1}}{I} + \frac{1}{2} \right\rfloor \right] \right. \\ \left. \times \operatorname{erfc} \left( (2i+1) \sqrt{\frac{3 \log_2(I \cdot J) \cdot \gamma_b}{I^2 + J^2 - 2}} \right) \right\} \quad (3)$$

where  $\gamma_b = E_b/N_0$  and denotes the average bit energy to noise density ratio and  $\lfloor x \rfloor$  denotes the largest integer to  $x$ .

For the quadrature components of the J-ary PAM, the error probability of the  $l_{th}$  bit, where  $l \in \{1, 2, \dots, \log_2 J\}$ , is obtained from [8]:

$$P_J(l) = \frac{1}{J} \sum_{j=0}^{(1-2^{-l})J-1} \left\{ (-1)^{\lfloor \frac{j \cdot 2^{l-1}}{J} \rfloor} \left[ 2^{l-1} - \left\lfloor \frac{j \cdot 2^{l-1}}{J} + \frac{1}{2} \right\rfloor \right] \right. \\ \left. \times \operatorname{erfc} \left( (2j+1) \sqrt{\frac{3 \log_2(I \cdot J) \cdot \gamma_b}{I^2 + J^2 - 2}} \right) \right\}. \quad (4)$$

With the results obtained by (3) and (4), the authors of [6] derive an exact closed-form to compute PER. So, the exact instantaneous PER for each modulation mode in a system with rectangular QAM symbols is calculated by [6]:

$$PER_n(\gamma) = 1 - \left\{ \prod_{k=1}^{\log_2 I} [1 - P_I(k)] \right\}^{(n_s / \log_2(I \cdot J))} \\ \times \left\{ \prod_{l=1}^{\log_2 J} [1 - P_J(l)] \right\}^{(n_s / \log_2(I \cdot J))}. \quad (5)$$

To compute  $P_I(k)$  and  $P_J(l)$ , we considered  $I = J = \sqrt{M}$  for square QAM modulations (256, 64 and 16-QAM),  $I = 8$  and  $J = 16$  for 128-QAM,  $I = 4$  and  $J = 8$  for 32-QAM, and finally  $I = 2$  and  $J = 4$  for 8-QAM.

## B. Average PER

In order to consider the influence of the channel fading in the system, we initially need to find the average PER value. For each modulation, the average PER can be determined by the integral of the instantaneous PER for the current modulation ( $n$ ) multiplied by the probability density function of the average symbol energy-to-noise density ratio ( $E_s/N_0$ ), which in this case is the pdf of a Nakagami- $m$  distribution [6] [9] [10]. So, the average PER is defined by:

$$\overline{PER}_n(\bar{\gamma}) = \int_0^{\infty} PER_n(\gamma) p_\gamma(\gamma) d\gamma. \quad (6)$$

Figure 2 shows the average PER as a function of the average symbol energy-to-noise density ratio for  $m = 1$ , representing a channel with Rayleigh fading.

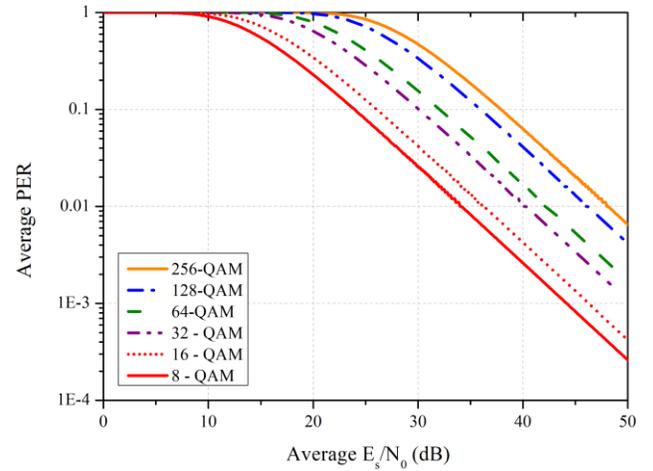


Figure 2. Average PER for a Rayleigh fading channel.

## IV. THE MAXIMUM THROUGHPUT CRITERION

The maximum throughput criterion has been considered in the real-time traffic scenario, where no error correction protocol was implemented. To compute the throughput, only successfully transmitted packets were considered (a parameter referred as *goodput* by some authors). We also assume an adaptive modulation scheme without error control coding, where all transmitted bits are information bits. Following [4], to compute the normalized throughput of the current modulations, we consider the following parameters:

- The ratio between the maximum number of transmitted bits and the maximum number of possible bits. In other words, the number of bits per symbol of the current modulation over the number of bits per symbol of a reference modulation;
- The percentage of packets correctly received.

So, the normalized throughput of the current modulation is given by:

$$\eta = \frac{\log_2 M_n}{\log_2 M_r} \cdot P_{cn} \quad (7)$$

where  $M_n$  is the number of points in the constellation of the current modulation,  $M_r$  is the number of points in the constellation of the reference modulation (in our case, 256-QAM) and  $P_{cn}$  is the probability of a data packet being successfully transmitted, given by:

$$P_{cn} = (1 - \overline{PER}_n(\bar{\gamma})). \quad (8)$$

In the numerical results, presented in Section VI, to compute  $P_{cn}$ , following [4], we considered that each packet has  $n_s = 424$  bits.

## V. DELAY CRITERION

In communication systems with non-real-time applications, in general, errors are corrected by retransmission using some retransmission protocol. In this context, the average time to transmit a correct data message is an important criterion to analyze the system performance [4] [5].

In our analysis, we consider that the ARQ (Automatic Repeat ReQuest) protocol is implemented at the data message level. Thus, if a single data packet is not successfully received, the entire data message is retransmitted. In this scenario, the average time to transmit a correct data message for a given modulation is given by [4]:

$$E(T)_n = \frac{T}{P_{dn}} + KT_Q \frac{(1 - P_{dn})}{P_{dn}} \quad (9)$$

where  $T$  is the time required to transmit a data message disregarding channel errors,  $K$  is the mean number of frames between the end of an incorrect transmission and the start of a retransmission of a data message,  $T_Q$  is the frame time, and  $P_{dn}$  is the probability of receiving a correct data message, given by:

$$P_{dn} = \left(1 - \overline{PER}_n(\bar{\gamma})\right)^Z \quad (10)$$

The time to transmit a data message is given by [4]:

$$T = \frac{n_s [(Z-1)X + 1]}{\beta_n B} \quad (11)$$

where  $X$  is the number of time slots in a frame,  $\beta_n$  is the bandwidth efficiency of the current modulation, given by  $\beta_n = \log_2 M_n$  [bps/Hz], and  $B$  is the bandwidth of the channel.

The frame time is calculated by [4]:

$$T_Q = \frac{n_s X}{\beta_n B} \quad (12)$$

The optimum switching points between neighboring modulations can be determined through a performance factor ( $\delta$ ), defined in [4] [5] by the ratio between the average time to transmit a data message for a  $2^n$ -QAM modulation and the average time to transmit a data message for a  $2^{n-1}$ -QAM modulation, with  $n = 8, 7, 6, 5$  and  $4$ . Thus, using (9), (11) and (12), the performance factor is given by [4] [5]:

$$\delta = \frac{(Z-1) \cdot X + 1 + KX(1 - P_{d(n-1)})}{(Z-1) \cdot X + 1 + KX(1 - P_{dn})} \cdot \frac{\beta_n}{\beta_{n-1}} \cdot \left( \frac{P_{dn}}{P_{d(n-1)}} \right). \quad (13)$$

where  $\beta_{n, (n-1)}$  is the bandwidth efficiency of the modulations  $n$  and  $n-1$ , respectively, and  $P_{dn, (n-1)}$  is the probability of a data message being correctly received for modulations  $n$  and  $n-1$ , respectively.

## VI. NUMERICAL RESULTS

In this section, we present numerical results for the optimum switching points using the maximum throughput criterion and delay criterion. We analyze the influence of the fading on the optimum switching points by varying the diversity order  $m$  of the Nakagami fading considering  $m = 0.5, 1, 2, 3$  and  $10$ , where the last value was used in order to consider an AWGN-like channel.

The calculation of the average PER is made by (3), (4), (5) and (6). Then, the PER is replaced in (8) to compute the probability that a packet was correctly received for throughput criterion and replaced in (10) to compute the probability that a data message was correctly received for delay criterion. All computations were performed using the Mathcad software.

When the system switches from one modulation to another one, the transmission power, or the average symbol energy, is kept constant. Thus, following [1] [4] [5], we analyze the system performance as a function of the parameter  $E_s/N_0$ , which represents the average symbol energy-to-noise density ratio.

### A. Throughput Criterion

To calculate the normalized throughput ( $\eta$ ), we replaced the result found by (8) in (7). We considered 256-QAM as the reference modulation and set the packet length, following [4], as  $n_s = 424$  bits.

The optimum switching point between two neighboring modulations is obtained by the crossover point of the corresponding curves of the throughput ( $\eta$ ).

Figures 3, 5, 7, 9 and 11 show the throughput curves as a function of the average symbol energy-to-noise density ratio, for 256-, 128- and 64-QAM modulations, considering  $m =$

0.5, 1, 2, 3 and 10, respectively. Figures 4, 6, 8, 10 and 12 show the throughput curves as a function of the average symbol energy-to-noise density ratio, for 64-, 32-, 16- and 8-QAM modulations, again considering  $m = 0.5, 1, 2, 3$  and 10, respectively.

Table I presents the optimum switching points and the throughput at these points. We can observe that the throughput at the switching points increases as  $m$  grows. In other words, if the channel fading becomes less severe, the throughput at these switching points increases.

In addition, we can see that the optimum points vary with the fading order (represented by the parameter  $m$  of the Nakagami model), and for a particular value of  $m$ , we can observe that the average symbol energy-to-noise density ratio in the switching points is not fixed, but varies with the neighboring modulations.

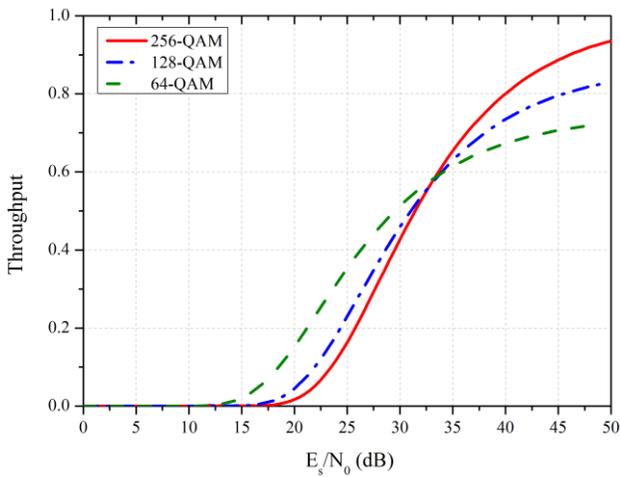


Figure 3. Throughput for  $m = 0.5$  and 256, 128, and 64-QAM.

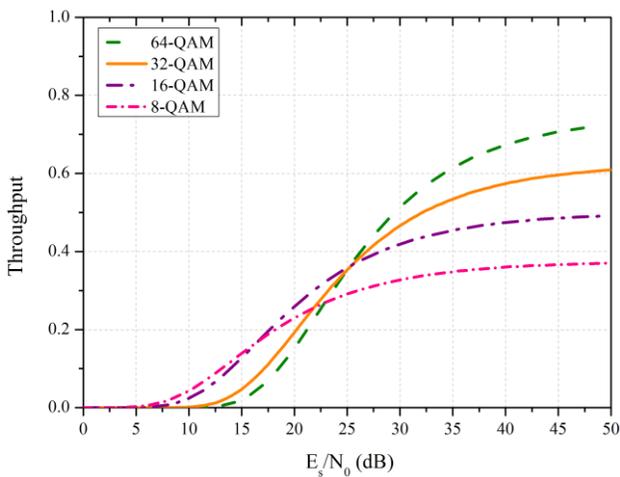


Figure 4. Throughput for  $m = 0.5$  and 64, 32, 16 and 8-QAM.

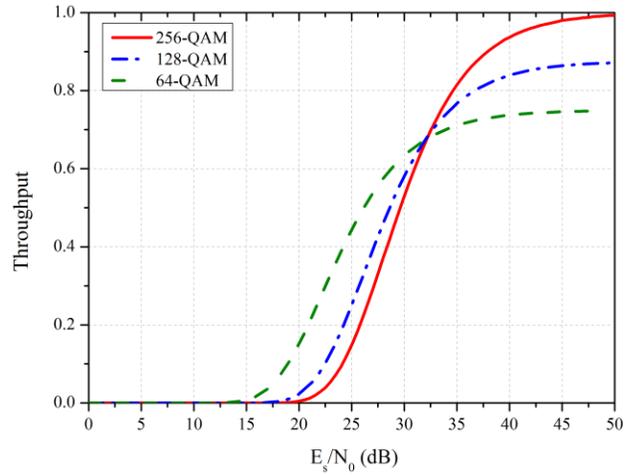


Figure 5. Throughput for  $m = 1$  and 256, 128 and 64-QAM.

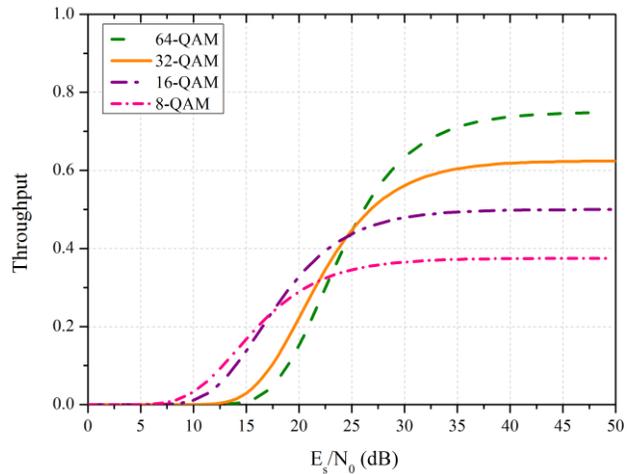


Figure 6. Throughput for  $m = 1$  and 64, 32, 16 and 8-QAM.

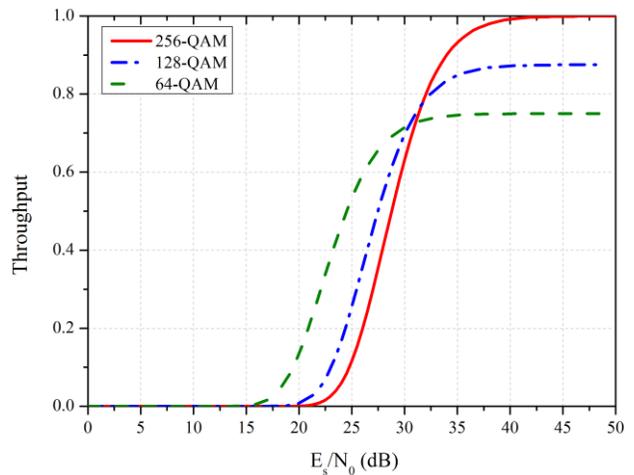


Figure 7. Throughput for  $m = 2$  and 256, 128 and 64-QAM

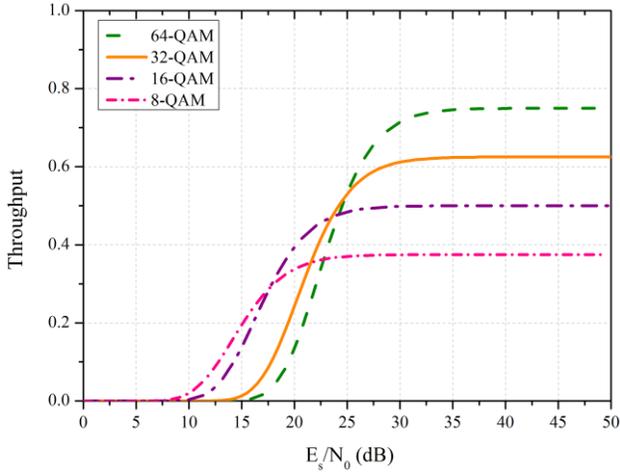


Figure 8. Throughput for  $m = 2$  and 64, 32, 16 and 8-QAM.

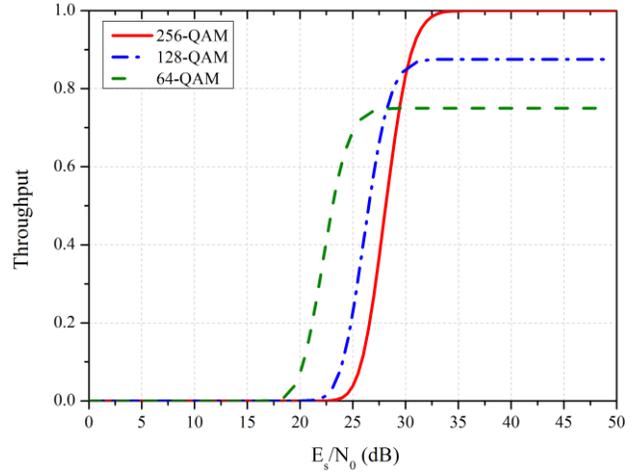


Figure 11. Throughput for  $m = 10$  and 256, 128 and 64-QAM

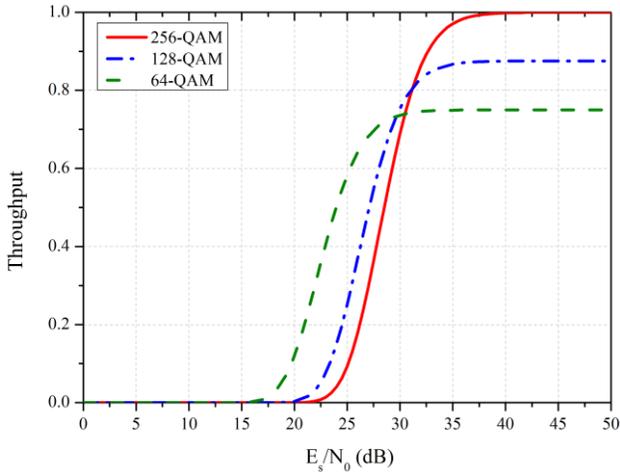


Figure 9. Throughput for  $m = 3$  and 256, 128 and 64-QAM

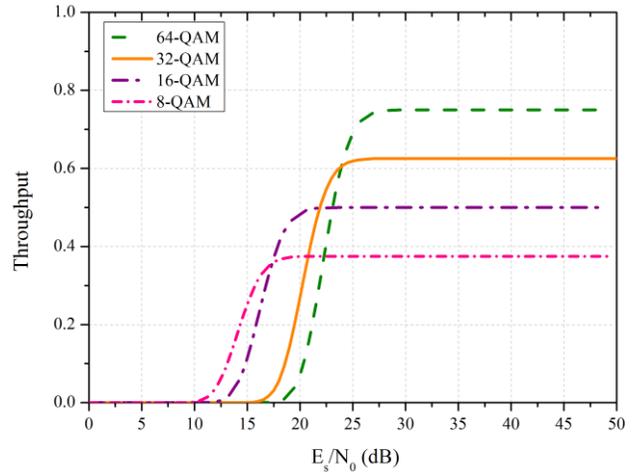


Figure 12. Throughput for  $m = 10$  and 64, 32, 16 and 8-QAM.

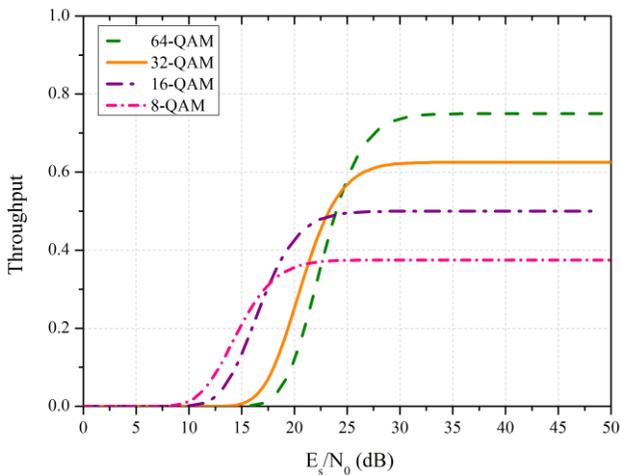


Figure 10. Throughput for  $m = 3$  and 64, 32, 16 and 8-QAM

Finally, through the analysis of Figures 3 to 12 and Table I, we can observe that some switching points between some neighboring modulations are close to each other. For example, the switching point between the neighboring modulations 256-QAM to 128-QAM is close to the switching point between the neighboring modulations 128-QAM to 64-QAM when  $m$  assumes the values  $m = 1, 2, 3$  and 10. The same behavior can be observed for the switching point between the neighboring modulations 64-QAM to 32-QAM and the switching point between the neighboring modulations 32-QAM to 16-QAM. Consequently, their throughput values are also very close to each other. Therefore, we can conclude that some modulations (like 128-QAM and 32-QAM) should not be considered for the implementation of an adaptive modulation scheme when the throughput criterion is considered.

Moreover, for the case of more severe fading, when  $m = 0.5$ , the switching point from 128 to 64-QAM occurs before the switching point from 256 to 128-QAM. The same occurs

for the switching point from 32 to 16-QAM, which occurs before the switching point from 64 to 32-QAM. So, we can conclude that 128-QAM and 32-QAM modulations should not be used in the adaptive modulation system in this case.

TABLE I. OPTIMUM SWITCHING POINTS AND NORMALIZED THROUGHPUT

Switch from	$m$	Switching points $E_s/N_0$ (dB)	Throughput ( $\eta$ )
256 to 128-QAM	0.5	32.7	0.559
	1	32.3	0.685
	2	31.6	0.77
	3	31.2	0.806
	10	30.1	0.846
128 to 64-QAM	0.5	33.7	0.592
	1	31.9	0.671
	2	30.4	0.718
	3	29.6	0.729
	10	28.2	0.746
64 to 32-QAM	0.5	25.1	0.353
	1	25.1	0.449
	2	24.9	0.527
	3	24.6	0.553
	10	23.9	0.603
32 to 16-QAM	0.5	25.4	0.364
	1	24.5	0.429
	2	23.5	0.469
	3	23	0.484
	10	21.8	0.492
16 to 8-QAM	0.5	16.5	0.168
	1	17.3	0.231
	2	17.6	0.289
	3	17.6	0.311
	10	17.3	0.353

### B. Delay Criterion

To compute the performance factor ( $\delta$ ), we employ the result obtained by (10) in (13) and, following [4], set the parameters  $X = 10$  and  $K = 1$ . Now, the optimum switching point between two neighboring modulations is obtained by the crossover point of the curves of the performance factor ( $\delta$ ) with the line  $\delta = 1$ .

#### 1) Analysis of the Influence of the Z:

Initially, we analyzed the influence of the length of the data message in the performance factor ( $\delta$ ). For this, we vary the parameter  $Z$  for a fixed slot size equal to  $n_s = 424$  bits. Figures 13 and 14 represent the performance factor curves between modulations 256- and 128-QAM and between modulations 128- and 64-QAM, respectively, considering  $Z = 1, 10, 100$  and  $1000$  and the diversity order  $m = 10$  (approaching an AWGN channel performance). We can observe that the optimum switching point depends on the data message length. As  $Z$  increases, so will the value of  $E_s/N_0$  at this optimum switching point.

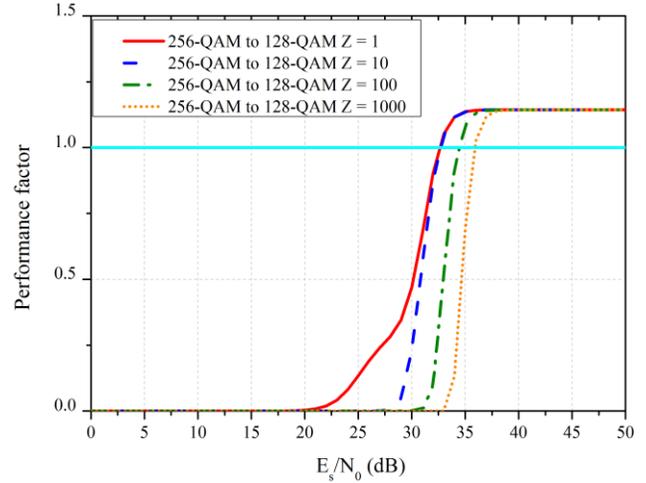


Figure 13. Performance Factor ( $\delta$ ) for  $Z = 1, 10, 100$  and  $1000$ ,  $m = 10$  and switching from 256- to 128-QAM.

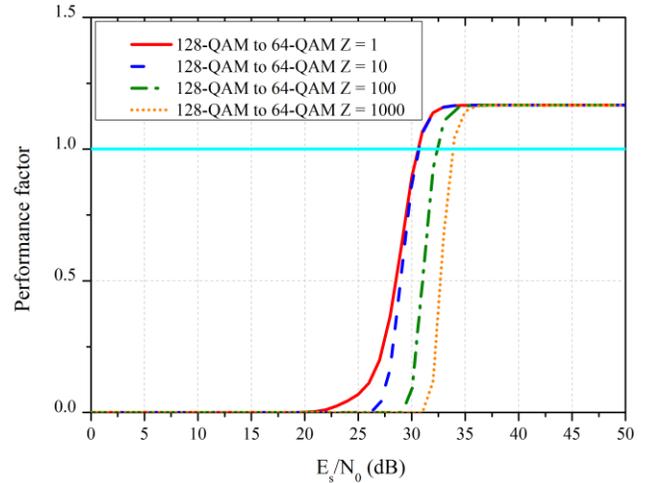


Figure 14. Performance Factor ( $\delta$ ) for  $Z = 1, 10, 100$  and  $1000$ ,  $m = 10$  and switching from 128- to 64-QAM.

#### 2) Comparing the results for throughput and delay criteria:

In this section, we compare the results obtained by the maximum throughput and the delay criteria. To be fair, we set  $Z = 1$ , and we compute the optimum switching points for both criteria. As  $Z = 1$ , the number of bits in a time slot,  $n_s$ , is equal to the length of a data message,  $n_d$ . Two packet lengths are considered in our analysis:  $n_s = 424$  and  $4240$  bits. Again, in order to consider the effect of the channel model in the optimum switching points, we consider the diversity order,  $m$ , equal to  $0.5, 1, 2, 3$  and  $10$ .

Figures 15, 16, 17, 18 and 19 show the performance factor curves for  $n_s = 424$  bits and  $m = 0.5, 1, 2, 3$  and  $10$ , respectively. Figures 20, 21, 22, 23 and 24 show the performance factor curves for  $m = 0.5, 1, 2, 3$  and  $10$ ,

respectively, but consider a larger packet size,  $n_s = 4240$  bits. The performance factor curves in these figures are between the following modulations: 256- and 128-QAM, 128- and 64-QAM, 64- and 32-QAM, 32- and 16-QAM, and 16- and 8-QAM.

Table II shows the optimum switching points between neighboring modulations for  $n_s = 424$  bits for both criteria. Table III shows the same but now considering  $n_s = 4240$  bits.

TABLE II. THE OPTIMUM SWITCHING POINTS FOR THROUGHPUT AND DELAY CRITERION  $n_s = 424$  bits

Switch from	$m$	Switching points $E_s/N_0$ (dB) for throughput criterion	Switching points $E_s/N_0$ (dB) for delay criterion
256 to 128-QAM	0.5	32.7	45.4
	1	32.3	41.5
	2	31.6	37.4
	3	31.2	35.6
128 to 64-QAM	0.5	33.7	49.6
	1	31.9	41.6
	2	30.4	36.1
	3	29.6	34
64 to 32-QAM	0.5	25.1	34.9
	1	25.1	33.7
	2	24.9	30.7
	3	24.6	29.1
32 to 16-QAM	0.5	25.4	39.4
	1	24.5	33.9
	2	23.5	29.3
	3	23	27.4
16 to 8-QAM	0.5	16.5	22.6
	1	17.3	24.8
	2	17.6	23.3
	3	17.6	22.2
16 to 8-QAM	10	17.3	19.8

Analyzing Tables II and III, and the Figures 15 to 24, we can observe that the optimum switching points for the delay criterion assume values of  $E_s/N_0$  larger than the optimum switching points for the throughput criterion. Thus, the optimum switching points depend on of the considered criterion. We can also see that the greater the length of the data message, the larger is the  $E_s/N_0$  in the optimum switching points.

We can observe again that the optimum switching points change as  $m$  varies. Thus, the optimum switching points depend on the channel model.

TABLE III. THE OPTIMUM SWITCHING POINTS FOR THROUGHPUT AND DELAY CRITERION  $n_s = 4240$  bits

Switch from	$m$	Switching points $E_s/N_0$ (dB) for throughput criterion	Switching points $E_s/N_0$ (dB) for delay criterion
256 to 128-QAM	0.5	34.8	47.5
	1	34.3	43.5
	2	33.6	39.3
	3	33.1	37.4
128 to 64-QAM	10	31.9	34
	0.5	35.7	51.6
	1	33.9	43.5
	2	32.2	37.9
64 to 32-QAM	3	31.5	35.7
	10	29.9	32
	0.5	27	37.1
	1	27.1	35.7
32 to 16-QAM	2	26.8	32.5
	3	26.5	30.9
	10	25.6	27.9
	0.5	27.3	41.4
16 to 8-QAM	1	26.4	35.7
	2	25.3	31
	3	24.8	29
	10	23.5	25.7
16 to 8-QAM	0.5	18.4	24.6
	1	19.2	26.7
	2	19.5	25.1
	3	19.5	23.9
16 to 8-QAM	10	19.1	21.3

It can also be seen that the average symbol energy-to-noise density in the optimum switching points using the delay criterion decreases as  $m$  increases.

Similarly to the throughput criterion, in the case of the delay criterion and  $m = 0.5$ , the switching point from 128- to 64-QAM occurs before the switching point from 256- to 128-QAM for both values of  $n_s$ . In addition, the switching point from 32- to 16-QAM occurs before the switching point from 64- to 32-QAM. Thus, again, for  $m = 0.5$ , the modulations 128-QAM and 32-QAM should not be used in the adaptive modulation system if the delay criterion is used.

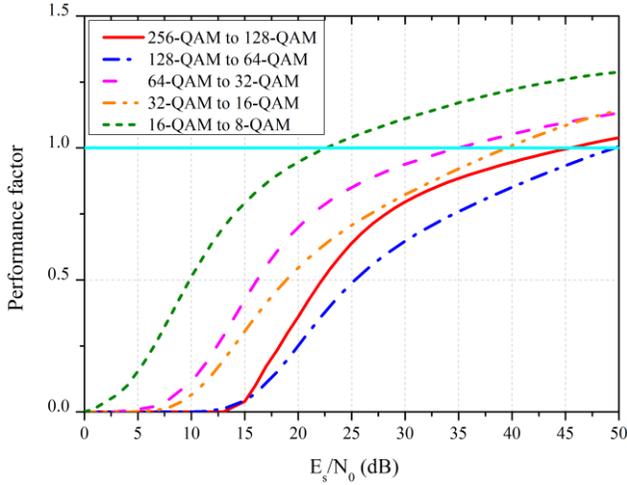


Figure 15. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 424$ ,  $m = 0.5$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

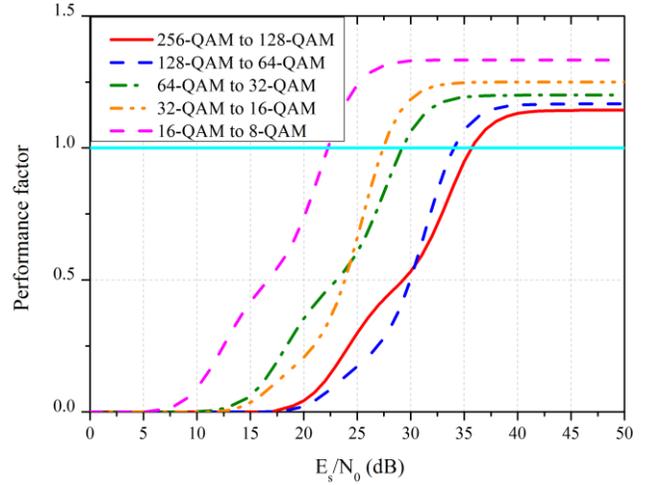


Figure 18. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 424$ ,  $m = 3$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

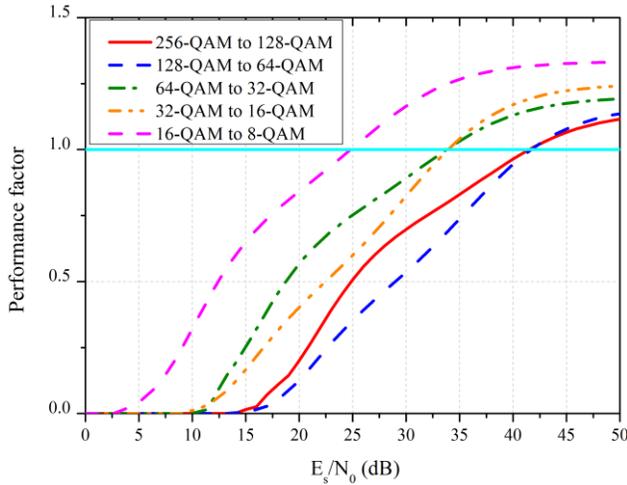


Figure 16. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 424$ ,  $m = 1$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

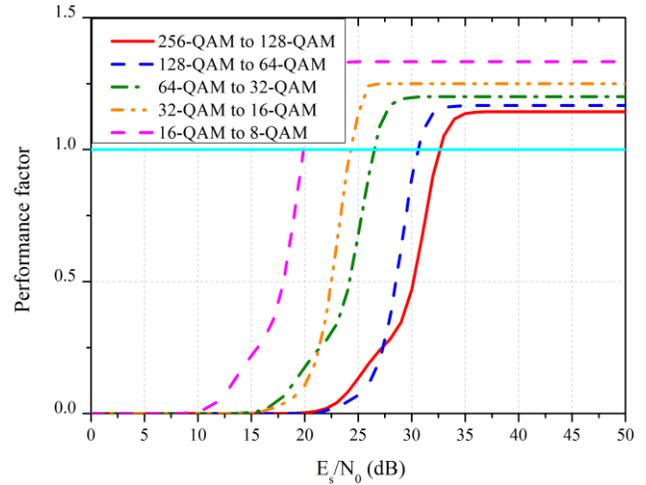


Figure 19. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 424$ ,  $m = 10$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

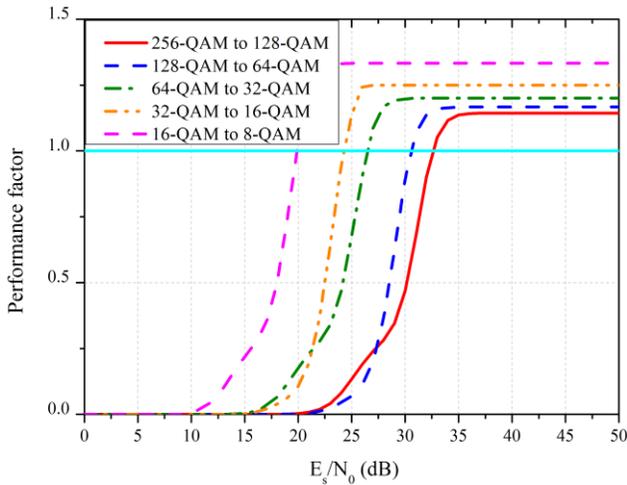


Figure 17. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 424$ ,  $m = 2$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

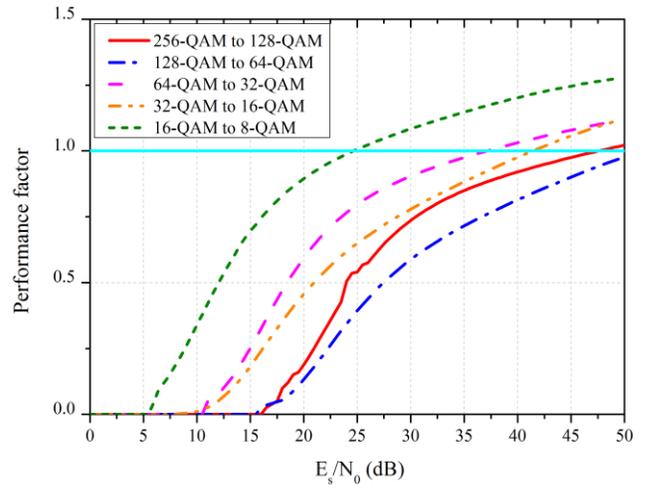


Figure 20. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 4240$ ,  $m = 0.5$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

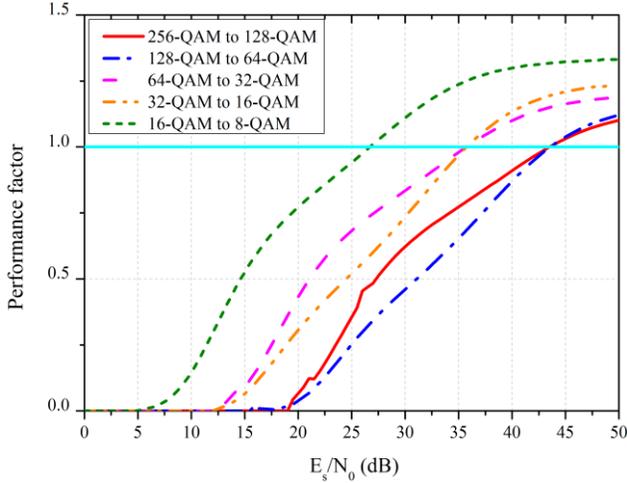


Figure 21. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 4240$ ,  $m = 1$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

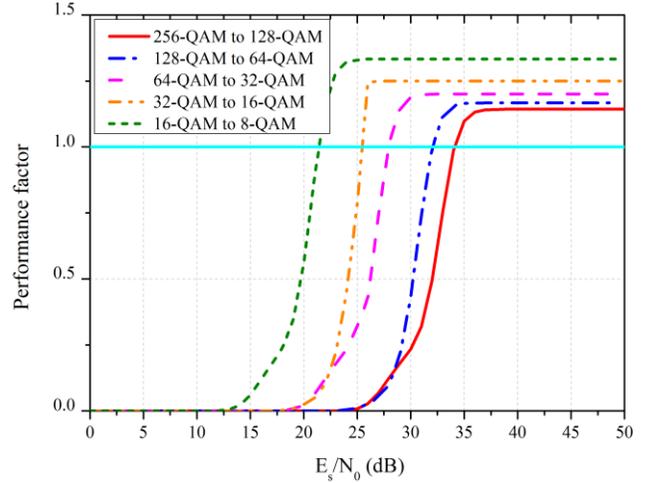


Figure 24. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 4240$ ,  $m = 10$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

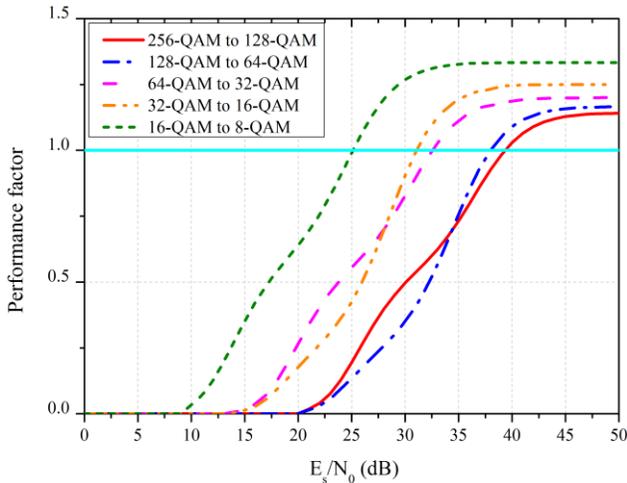


Figure 22. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 4240$ ,  $m = 2$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

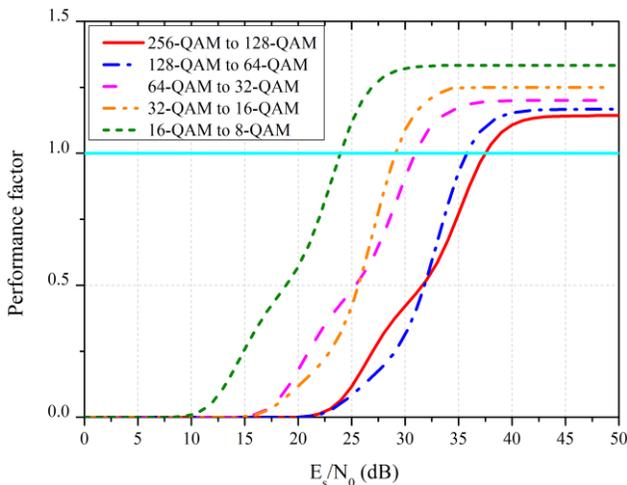


Figure 23. Performance Factor ( $\delta$ ) for  $Z = 1$ ,  $n_s = 4240$ ,  $m = 3$ , and (256,128), (128,64), (64,32), (32,16) and (16,8) QAM pairs.

## VII. CONCLUSION

In this paper, we considered a system with adaptive modulation and a Nakagami- $m$  fading channel. We analyzed the influence of the channel model in the optimum switching points between neighboring modulations considering two scenarios: real-time and non-real-time traffic. To compute these points, we employed the maximum throughput criterion for real-time traffic and the mean time to transmit a data message for non-real-time traffic. We considered  $M$ -QAM modulations, with  $M = 8, 16, 32, 64, 128$  and  $256$ , and we varied the diversity order of the channel setting, i.e.,  $m = 0.5, 1, 2, 3$  and  $10$ .

We concluded that the optimum switching points depend on the channel model and on the type of the traffic in the system (real-time and non-real-time traffic) that defines the best criterion to compute the switching points.

We also observed that for the throughput criterion, some modulations (like 128-QAM and 32-QAM) can be neglected in the implementation of a practical adaptive modulation scheme.

Furthermore, for systems where the fading channel is severe ( $m = 0.5$ ), the switching point from 128- to 64-QAM occurs before the switching point from 256- to 128-QAM, and the switching point from 32- to 16-QAM occurs before the switching point from 64- to 32-QAM, for both criteria (throughput and delay). Therefore, 128-QAM and 32-QAM should not be used in the adaptive modulation system in this case.

## ACKNOWLEDGMENT

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