Fairness index in single and double star Networks
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Abstract
In wireless network, the communication works in half duplex mode and nodes can interfere together. In this context, fairness is not obvious. This paper will focus on fairness in the received packets by each node. Fairness is evaluated for static networks topologies called Single Star Network or Double Star Network. The fairness is quantified by its index. In this work, the evaluation of fairness index for double star network is given. Some value of this index are not possible for double star network topology. For example the index of one can only be possible if the double star network is similar to star network. Then the star networks are studied and some simulations are used to illustrate the way to get fairness in the network by controlling the flow rates.

1 Introduction
The performance of wireless 802.11 MAC protocol is generally evaluated with two parameters: collision probability and fairness [11],[13]. The fairness algorithm was widely studied by different research groups. Jain, Chiu and Hawe give us a mathematical definition in [6]. Their paper introduces the fairness index for any kind of resource sharing. This definition will be applied to the packet rates received by nodes.

TCP fairness was studied in [4], where the authors propose a distributed algorithm on neighbors to improve TCP fairness. In another paper [11] the authors propose a scheduling algorithm to get fairness in a multi-hop wireless network. Some papers are also based on the study of a distributed algorithm to maximize throughput and fairness in Ad Hoc networks [2],[3].

This paper is focused in fairness on node reception rates in an Ad-Hoc wireless network. A general introduction for fairness and fairness index is given. After that, a description of the network characteristic is done. For a theoretical analysis of the problem, the fairness index is evaluated for a basic network called Double Star Network and Star network. The existence of some value of fairness index is proved. We recall form [15] that exact fairness, such that fairness index is one, is not possible until Double Star Network degenerates in a Single Star Network. The fairness of Star network is studied and an algorithm is recalled from [14]. The simulation is done with NS2-2.33, will show that fairness can be accomplished by limiting the transmission rate of some nodes.

2 Network model and fairness
This work is done in the area of Ad-Hoc wireless network. An Ad-Hoc network is made of wireless nodes which establish wireless communications between themselves. In this context, there is no central infrastructure. This means that the nodes are equivalent. A node can be in two states at a given time, transmission or reception. The limitation of radio communications implies that the communications of a channel are limited in distance and they act in half duplex mode. These characteristics are described as follow:

2.1 Network model
We consider an Ad-Hoc network such as:
- The network is packet-switched
- The nodes are in half-duplex mode
- Only nodes in some distance can communicate
- The time is divided in time slots
- Packets are sent in time slots
2.2 fairness index

In an Ad-Hoc network, the nodes have an equivalent role. Because the communications are in half-duplex mode, the position of the node in the network topology is dramatically related to its transmission rate. Some nodes with high degrees of connectivity in transmission will interfere with a high number of neighbors. This implies that the transmission and reception rates of the nodes are different.

In this paper, we try to give a fair access to each node. Fairness can be expressed as a mathematical formula given in [6]. The formula is based on a resource independent model and can be used to express fairness for any shared resource. It is also independent of network scalability. The resource will be applied to the reception rate of each node.

We recall the fairness index definition form [6]:

Definition 2.1. The fairness index of a shared resource \( x_i \) is given by

\[
 f(x_i) = \left( \frac{\sum_{i=1}^{n} x_i}{n \sum_{i=1}^{n} x_i^2} \right)^2
\]

We will apply this approach to the reception rate of a node. Here are some of the notations:

Notation. The following notations will be used in this work:

- Let \( x_i \) be the reception rate of node \( X_i \)
- Let \( r_{j,i} \) be the reception rate of node \( X_i \) of the packets send by \( X_j \)
- Let \( D_i \) be the degree of node \( X_i \)
- Let \( S_{j}^{i} \) be the transmission rate of node \( X_j \) which is a neighbor of node \( X_i \)

Remark 1. Using the previous notations, we have a reception rate \( x_i \) of node \( X_i \):

\[
x_i = \sum_{j=1}^{D_i} r_{j,i}
\]

Node \( X_j \) has transmission rate \( S_j \). Therefore we have \( r_{j,i} = S_{j}^{i} \). This gives us:

\[
x_i = \sum_{j=1}^{D_i} S_{j}^{i}
\]

3 Double Star Network

Compute a fairness condition on reception rates is not easy for some Ad-Hoc networks. From a theoretical approach, the star double network and star network are introduced. This network is simple enough to compute a relation on transmission rates to get fairness. It can represent a sub-graph of an Ad-Hoc network composed of a central nodes and its neighbors.

3.1 Definition

Definition 3.1 (Double Star network). The double star network \( SN_{n,m} \) is composed of \( n + m + 1 \) nodes \( \{X_0, X_1, \ldots, X_n, X_n + 1, \ldots, X_{n+m} \} \) where:

- \( \{X_i, i \leq n \} \) are neighbors of the node \( X_0 \), \( \{X_0, X_i, n + 1 \leq i \leq n + m \} \) are neighbors of the node \( X_n \)
- and there is no connection between \( X_i, X_j \) for \( i, j > 0 \) only \( X_0 \) is the neighbor of \( X_n \).

This is an example:

Figure 1. The \( SN_{8,3} \) double star network

When the central nodes \( X_0 \) or \( X_n \) sends a packets, it will be received by their neighbors and this is not fair. Each set of packets respectively sent by \( X_i \) where \( 0 < i < n + 1 \) and or by \( X_j \) where \( n < j < m + n + 1 \) will be seen respectively only once by \( X_0 \) or \( X_n \).

3.2 Fairness index of the double star network

We can compute the fairness index for a double star network:

Lemma 3.1. For a double star network \( SN_{n,m} \), the fairness index \( \alpha \) for the reception rate is given by the equation:

\[
2(2 - \alpha(n + m + 1)) X^2 - 2\alpha(n + m + 1) Y^2 + Q(S_0, S_n) = 0
\]
where:

\[ X = \sum_{i=1}^{n-1} S_i + \left(\frac{(2n - \alpha(n + m + 1))S_0}{2(2 - \alpha(n + m + 1))}\right) + \left(\frac{(2(m + 1) - \alpha(n + m + 1))S_n}{2(2 - \alpha(n + m + 1))}\right) + \left(\frac{(2(m + 1) - \alpha(n + m + 1))S_n^2}{2(2 - \alpha(n + m + 1))}\right) \]

\[ Y = \sum_{i=n+1}^{n+m} S_i - \frac{\alpha(n + m + 1)S_0 - \alpha(n + m + 1)S_n}{2\alpha(n + m + 1)} \]

\[ Q(S_0, S_n) = AS_0^2 + 2BS_0S_n + CS_n^2 \]

is a quadratique form where

\[ A = -\frac{(n - 1)(n + m + 1)\alpha(n + m + 1)\alpha - n - 1}{(n + m + 1)\alpha - 2} \]

\[ B = \frac{m(n - 1)(n + m + 1)\alpha}{\alpha(n + m + 1) - 2} \]

\[ C = -\frac{(n + m + 1)^2\alpha^2(2m - 3) - 2(n + m + 1)\alpha(m^2 - 2)}{2(\alpha(n + m + 1) - 2)} \]

Proof. Let \( \alpha = f(x) \)

some basic computation gives the equation (3).

The equation (3) might have no solutions. Let’s see which value \( \alpha \) will give no trivial solutions. Equation (10) is a quadratique relation in \( X \) and \( Y \) it has a constant term \( Q(S_0, S_n) \).

3.2.1 The coefficients of \( X^2 \) and \( Y^2 \)

The coefficient of \( Y^2 \) is \(-2\alpha(n + m + 1)\) it is negative because \( \alpha \) is a fairness index and therefore \( \alpha \) is positive.

The coefficient of \( X^2 \) is \(2(2 - \alpha(n + m + 1))\). Its sign depends on \( 2 - \alpha(n + m + 1) \).

The following lemma gives some results about the existence of solution of (3):

**Lemma 3.2.** For a double star network, the existence of fairness index \( \alpha \) is submit to the following rules:

- If \( m^2 - 2 > 2m + 3 \) then there exits no trivial solutions for equation (3).
- If \( m^2 - 2 > 2m + 3 \) then there exists no trivial solutions for equation (3) if \( Q(S_0, S_n) > 0 \).

Proof. If \( m^2 - 2 > 2m + 3 \) then the coefficient of \( X^2 \) and \( Y^2 \) have opposite sign. This implies the existence of solutions. If \( m^2 - 2 > 2m + 3 \) then the coefficient of \( X^2 \) and \( Y^2 \) have same sign. To have no trivial solution, the quadratique form \( Q(S_0, S_n) \) has to be positive.

The lemma 3.2 shows the importance of the sign of the quadratique form \( Q(S_0, S_n) \) if \( \alpha > \frac{m^2 - 2}{2m + 3} \). In the next part, the sign of quadratique form will be studied.

3.2.2 The sign of the quadratique form \( Q(S_0, S_n) \)

Recall that:

\[ Q(S_0, S_n) = AS_0^2 + 2BS_0S_n + CS_n^2 \]

In the lemma 3.2, the sign of the quadratique form \( Q(S_0, S_n) \) is important for \( \alpha > \frac{2}{m^2 + m + 1} \). Let’s suppose that \( (n + m + 1)\alpha - 2 > 0 \), then the denominator of \( A, B \) and \( C \) are positive. Let’s study their numerator:

- The sign of \( A \) is the sign of \( n + 1 - (n + m + 1)\alpha \).
- The sign of \( B \) is positive because \( n > 1 \).
- The sign of \( C \) is the sign of \( 2(m^2 - 2) - (n + m + 1)\alpha(2m - 3) \).

The next lemma will give conditions on \( n \) and \( m \) to have \( A, B \) and \( C \) positive.

**Lemma 3.3.** If \( \frac{m^2 - 2}{m + m + 1} < \alpha \) and \( m \geq 1 \) then

\[ \frac{m^2 - 2}{2m - 3} \geq 1 \]

and \( C \) is positive if

\[ \frac{2}{m + m + 1} < \alpha \leq \frac{2(m^2 - 2)}{(n + m + 1)(2m - 3)} \]

Proof. If \( \alpha > \frac{2}{m + m + 1} \) then \( C \) is negative else \( C \) is positive. The sign of \( C \) is given by

\[ 2(m^2 - 2) - (n + m + 1)\alpha(2m - 3) \]

Let’s prove that:

\[ 2(m^2 - 2) - (n + m + 1)\alpha(2m - 3) < 0 \]

This implies that:

\[ \frac{2}{m + m + 1} < \alpha \]

**Lemma 3.4.** \( A \) is positive if:

\[ \frac{2}{n + m + 1} < \alpha \leq \frac{n + 1}{n + m + 1} \]

Proof. The sign of \( A \) is given by:

\[ n + 1 - (n + m + 1)\alpha \]

The next theorem will give a condition for the quadratique form \( Q(S_0, S_n) \) to be positive.
3.2.3 Maximal fairness index for double star network

The corollaries 3.1 and 3.2 give a upper bound for the fairness index of a double star network. This is shown in the next lemma:

**Lemma 3.5.** Let \( \alpha \) be a fairness index of a double star network \( SN_{n,m} \) which has no zero data rate reception.

- If \( 1 < n \leq m \) then \( \alpha \leq \frac{n+1}{n+m+1} \) exists
- If \( 1 < m < n \) then \( \alpha \leq \frac{n+2}{n+m+1} \) exists

To validate the theoretical analysis, the next section will present some simulations.

3.3 Double star network simulations

NS2-2.33 is used for the next simulations. CBR over UDP traffic is used. The nodes \( X_i, 1 \leq i \leq n - 1 \) and \( X_{j}, n + 1 \leq j \leq n + m \) have a CBR rate of 0.5Mb. The nodes \( X_0 \) and \( X_n \) have a CBR rate going for 0.1Mbps to 1.0Mbps with a step of 0.5Mbps. The Ad-Hoc routing protocol DSDV is active. The CBR traffic goes from \( X_0 \) to \( X_i, 1 \leq i \leq n \), for \( X_n \) to \( X_j, n + 1 \leq j \leq n + m \) and backwards.

3.3.1 The \( SN_{3,8} \) double star network.

In this case, \( n = 3, m = 8 \) and \( n < m \). The lemma 3.5 shows that there can exist fairness index \( \alpha \) such that

\[
\alpha \leq \frac{1}{3}
\]

We do simulation for 1000 time slots, and we get the following results show in this figure 3.3.1:

![Figure 2. \( SN_{3,8} \) fairness index](image)
We notice that the maximal fairness index is given for $S_0 = 1.0M$bps and $S_n = 0.1M$bs. Its value is 0.2008 which is in the range given by lemma 3.5.

### 3.3.2 The $SN_{8,3}$ double star network.

In this case, $n = 8$, $m = 3$ and $n > m$. The lemma 3.5 shows that there can exist fairness index $\alpha$ such that

$$\alpha \leq \frac{5}{12}$$

We do simulation for 1000 time slots, and we get the following results show in this figure 3.3.2:

![Figure 3. SN_{8,3} fairness index](image)

We notice that the maximal fairness index is given for $S_0 = 0.5M$bps and $S_n = 0.5M$bs. Its value is 0.1501 which is lower than $\frac{1}{6} = \frac{2}{n+m+1}$.

If $\alpha$ will be greater than $\frac{1}{6}$, then the equation 3 will be the equation of an ellipse with variable $X$ and $Y$. It will be a very lucky to have $X, Y$ on this ellipse. If the fairness index is lower than $\frac{1}{6}$ then equation 3 is easier to solve.

We have given results of existence of fairness index if $\alpha \leq \frac{n+2}{n+m+1}$ or if $\alpha \leq \frac{n+12}{n+m+1}$, but what happens if the fairness index is greater as this values, say $\alpha = 1$. The next section will give an answer in this case.

### 3.4 Fairness for double star network.

A network is fair if the fairness index is 1. This was studied in the conference [15], and we recall the main results in this section.

To be fair in a double star network, the fairness index of the network must be 1. This implies the following lemma:

**Lemma 3.6.** A double star network $SN_{n,m}$ is fair if and only if:

$$n(a - X - Y)^2 + m(a + Y)^2 + mn(n - 1)a^2 = 0 \quad (5)$$

where

- $a = S_0 - S_n$
- $X = \sum_{i=1}^{n-1} S_i$
- $Y = \sum_{i=n+1}^{n+m} S_i$

**Proof.** If the network is fair, the fairness index is $f(x) = 1$

this gives:

$$\left( \sum_{i=1}^{n} S_i + nS_0 + \sum_{i=n+1}^{n+m} S_i + S_0 + mS_n \right)^2 = (n + m + 1) \times$$

$$\left( \sum_{i=1}^{n} S_i \right)^2 + (n + 1)S_0^2 + \left( \sum_{i=n+1}^{n+m} S_i \right)^2 + mS_n^2$$

By direct computation, we get the condition (5).

**Remark 2.** In equation (5), we can observe that all terms are positive or null. This implies that each term has to be zero for the relation to be validate. We have to discuss about the value of $n$ and $m$.

### 3.4.1 Case $n \neq 0, m = 0$

If $m = 0$ and $n \neq 0$, the relation (5) becomes:

$$n(a - X)^2 = 0 \quad (6)$$

This implies that $a = X$. This is the result given for a star network in the paper [14].

### 3.4.2 Case $n = 0, m \neq 0$

If $n = 0$ and $m \neq 0$ the relation (5) becomes:

$$(a + Y)^2 - a^2 = 0 \quad (7)$$

This implies that $Y = 0$ or $Y = -2a$. Because $n = 0$ we have $a = 0$ and then $Y = 0$. In this configuration, no node is transmitting.
3.4.3 Case \( n = 1, m \neq 0 \)

If \( n = 1 \), then \( X = 0 \), and the relation (5) becomes:
\[
(a - Y)^2 + m (a + Y)^2 = 0 \tag{8}
\]
Because \( m \neq 0 \), this implies that:
\[
\begin{cases}
  a - Y = 0 \\
  a + Y = 0
\end{cases}
\]
In this case, we get \( Y = 0 \) and \( a = 0 \). Then only \( X_0 \) and \( X_1 \) are transmitting with the same rate \( S_0 \).

3.4.4 Case \( n = 1, m = 0 \)

If \( n = 1 \), then \( X = 0 \), and the relation (5) becomes:
\[
(a - Y)^2 = 0 \tag{9}
\]
This implies that \( Y = a \), and then we get:
\[
S_0 = \sum_{i=1}^{m+1} S_i
\]
This is the result for the star network \( SN_{m+1} \).

3.4.5 Case \( n \neq 0, n \neq 1, m \neq 0 \)

In this case, the equation (5) has no coefficient equal to zero. Then every term should be equal to zero:
\[
\begin{cases}
  a - X - Y = 0 \\
  a + Y = 0 \\
  a = 0
\end{cases}
\]
The only solution is \( a = 0, X = 0, Y = 0 \) there is no packet transmitted.

The next theorem was proved:

**Theorem 3.2.** The fairness of packet transmitted in a \( SN_{n,m} \) double star network is given by:

- \( S_0 = \sum_{i=1}^{n} S_i \), if \( m = 0 \) and \( n > 0 \) this is a \( SN_n \) star network,
- \( S_1 = \sum_{i=2}^{m+1} S_i \) if \( n = 1 \) and \( m > 0 \), this is a \( SN_m \) star network,

If \( m > 0 \) and \( n > 1 \), then transmitting a packet broke the fairness condition.

This theorem shows that exact fairness can’t exist in a non degenerate double-star network. In the next section, we recall the results for [14] which gives the results in the degenerate case called star network.

4 Star Network

**Definition 4.1** (Star network). The star network \( SN_n \) is composed of \( n + 1 \) nodes \( \{X_0, X_1, \ldots, X_n\} \) where \( \{X_1, \ldots, X_n\} \) are neighbors of the node \( X_0 \), and there is no connexion between \( X_i, X_j \) for \( i, j > 0 \).

The next graph shows a star network:

![Figure 4. A star network](image)

In this network, the central node \( X_0 \) has a degree of \( n \), and each of its neighbors has a degree of 1. This is a very unfair communication channel repartition. When the node \( X_0 \) sends a packet, it will be received by the \( n \) neighbors. This packet will be seen \( n \) times in the network. On the other hand, each packet send by \( X_i, i \geq 1 \) will be seen only once by \( X_0 \). Intuitively, we see that to get fairness, the transmission rate of \( X_0 \) must balance the transmission rates of all other nodes. This will be shown in the next section.

4.1 Fairness of the star network

We can compute the fairness conditions for a star network:

**Lemma 4.1.** For a star network \( SN_n \), the fairness for the reception rates hold only and only if:
\[
S_0 = \sum_{i=1}^{n} s_i = 0 \tag{10}
\]

**Proof.** Using the expression (2) of \( x_i \), the reception rate of node \( X_i \), we have:
\[
f(x) = \frac{\left( \sum_{i=0}^{n} \sum_{j=1}^{D_j} s_j \right)^2}{(n+1) \sum_{i=0}^{n} \left( \sum_{j=1}^{D_j} s_j \right)^2}
\]
If the network is fair, we must have \( f(x) = 1 \). By a direct computation, we get:
\[
S_0 = \sum_{i=1}^{n} s_i = 0
\]
The relation gives a direct condition on transmission rates to achieve fairness. It is much simpler to compare transmission rates than to evaluate the fairness index. The fairness index is based on a continuous function for transmission rates. This implies that if the difference (10) is close to zero, then the fairness index is close to 1. To get a fair star network, the condition (10) must be made closer.

Remark 3. The star network is fair only and only if the transmission rate $S_0$ is the sum of the transmission rates of all the neighbors of $X_0$.

4.2 Fairness algorithm

An Ad-Hoc network is seen from a node $Y_i$ as a star network $SN_d$ where $d$ is the degree of the node $Y_i$. Following (3), we can imagine that the node $Y_i$ adjusts its transmission rate such that it corresponds to the sum of the reception rates. It gets from its neighbors. This gives us the following algorithm running on each node $Y_i$ and using a parameter $s$ given by the administrator of the network:

1. Computes the sum of the neighbors transmission rates $A_i$.
2. Compares the sum $A_i$ to the node $Y_i$ transmission rate $S_i$.
3. If $S_i - A_i > s$ then reduce $S_i$, if $S_j$ becomes negative, then set it to 0.
4. If $S_i - A_i < s$ then increase $S_i$ if it is possible.
5. If $S_j = 0$ for a long time, then increase it.
6. Go to step 1

This algorithm acts only on the transmission rates. It tries to adjust the difference $S_i - A_i$ to be close to zero. To do this, it needs to have some control over $S_i$. For a star network, the theoretical approach shows that minimizing the difference $S_i - A_i$ will increase fairness. But it can also be used on any Ad-Hoc networks.

The parameter $s$ controls the sensibility of the algorithm to the standard access algorithm. If $s$ is null, the algorithm will try to always get an exact fairness. This is not realistic, and this can decrease the performance of the network. If $s$ is too high, then the algorithm will have no influence on fairness.

4.3 Simulations

We use the network simulator ns2 to do the simulations. The DSDV routing protocol is used. First, we will do the simulation with the original ns2. After that, we modify ns2 to simulate our algorithm. The transmission rate will be computed on TCP packets sent by each nodes. We use FTP agent to simulate traffic.

4.3.1 The star network $SN_6$

We will now use $SN_6$ star network for simulation where two FTP connections (up and down) are established between $X_0$ and $X_i$, $i > 0$. The following graph shows the fairness index in a function of time:

![Figure 6. Fairness of a $SN_6$ star network](diff.png)

We can also compute the difference (10) in function of time slots:

![Figure 7. Rate difference to fairness](diff.png)
We notice that the difference (10) increases in time, which is coherent with the fact that the fairness index is decreasing. The aim of our algorithm is to keep the difference (10) close to zero.

We apply our algorithm to this network with $s = 500$. To reduce $S_j$, the algorithm changes the rate from 1Mb to 0.5Mb. When the delay becomes too long, the algorithm reset the node in the standard rate.

This gives us the following graphs:

![Figure 8. Fairness of a $SN_6$ star network with modified access algorithm](image)

Remark that the fairness index goes to 0.43 which is better than the simulation done by the default algorithm of ns2.

We can also compute the difference (10) in function of time slots. This is shown in the next graph, see how the algorithm acts on to minimize the difference.

![Figure 9. Rate difference to fairness](image)

4.3.2 The star network $SN_8$

We will now use the $SN_8$ star network for simulation where two FTP connections (up and down) are established between $X_0$ and $X_i, i > 0$. The following graph shows the fairness index in a function of time:

![Figure 10. Fairness of a $SN_8$ star network](image)

We can also compute the difference (10) in function of time slots:

![Figure 11. Rate difference to fairness](image)

We can notice that the fairness index is around 0.432 at time slot 500 and the difference (10) is around -14. This confirms that for the star network $SN_8$, the behavior is not fair. Now we will try to see what is happening with our algorithm. The fairness index is shown in the following graph:

![Figure 12. Fairness of a $SN_8$ star network](image)

This shows that the fairness index is better than the ns2 standard case. At 500, the fairness index is higher than 0.49.
and is still increasing. Therefore the algorithm gives better results. Let’s take a look at the difference of rate to fairness given by (10): 

\[ \text{Difference rate to fairness} \]

\[ \begin{array}{c|c|c|c|c} \hline \text{Time Slots} & 0 & 50 & 100 & 150 & 200 \\ \hline \text{Value} & 0 & 0.5 & 1 & 1.5 & 2 \\ \hline \end{array} \]

Figure 13. Difference of rate to fairness of a \(SN_8\) star network

This graph shows that the algorithm is working well. The difference is less that -0.6 at time slot 500. The algorithm seems to be efficient.

4.4 A no star network

In this example, the algorithm is applied to no star network. The topology of the network is given by the graph:

\[ X_0 \rightarrow X_1, X_2, X_3, X_4, X_5, X_6 \]

Figure 14. A no star network

There is an FTP traffic simulated for each node to its neighbors. When we applied the standard ns2 simulator, this gave for the fairness index the following result:

\[ \text{Fairness index} \]

\[ \begin{array}{c|c|c|c|c} \hline \text{Time Slots} & 0 & 50 & 100 & 150 & 200 \\ \hline \text{Value} & 0.66 & 0.68 & 0.7 & 0.72 & 0.74 \\ \hline \end{array} \]

Figure 15. Fairness of the no star network

We can see that the node \(X_0\) is connected to every other node in the network. This node can play the same role as the central node for a star network. The rate difference (10) can be evaluated for this node. This gives the next graph:

\[ \text{Difference rate of the no star network} \]

Figure 16. Difference rate of the no star network

The difference is increasing according to the fairness index of the network. This let us supposes that the fairness index is related to the rate difference of \(X_0\) to its neighbors. The algorithm for star networks can be apply to reduce the rate difference (10). When the algorithm is used, the rate difference (10) reacts as following graph:

\[ \text{Difference rate of the no star network with our algorithm} \]

Figure 17. Difference rate of the no star network with our algorithm

The rate difference goes down from 65 to less than 40. The fairness index is shown in the next graph:

\[ \text{Fairness index of the no star network with algorithm running} \]

Figure 18. Fairness index of the no star network with algorithm running

Thus the fairness index tends to be closer to 0.8.

In this example, we applied our algorithm to a no star network. The algorithm improves the fairness index.
5 Conclusion

In this work, the study is focused on the fairness of the reception rate. After some generalities, a double star network and a single star network are introduced. This network enables us to compute the fairness index. For double star network, we give some upper bound on fairness index to guaranty their existence. We prove that fairness can only exist if double star network degenerates in star network. Fairness is studied in star networks. Then we elaborate an algorithm to get fairness. The algorithm needs only to know the reception rate and the transmission rate of the node and it can be used on every Ad-Hoc network. But in this case the influence on the fairness index is not developed. Never less and example are shown where the algorithm improves fairness. The simulation shows that the fairness index is improved for a star network if we apply our algorithm. But the fairness index doesn’t seem to react very efficiently. We can expect better results if the simulation time goes to infinity.

In some further work, we propose to apply our algorithm to more complex networks to approach a general Ad-Hoc network. A first step is to compute the maximal fairness index for some topology and then we expect to modify our algorithm to reach this maximal fairness index for a more general Ad-Hoc network.

References


