Theoretical Investigation of an Ultrasonic Array Transmitter with Anisotropic Directivity

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Abstract— A range sensor based on an ultrasonic array transmitter with anisotropic directivity is investigated theoretically. Although the sound power of an array transmitter with isotropic directivity decreases as the angle of the divergence increases, with anisotropic directivity, the decreases are limited and high power is obtained. The time delay required to control divergence is discussed.

Keywords—ultrasonic; array sensor; transmitter; range sensor

I. INTRODUCTION

Ultrasonic sound has a number of applications. In particular, it is widely used to measure the distance between two points and in range sensors, which can measure the three-dimensional positions and shapes of various objects [1–4]. Ultrasonic sensors have the advantage of being simple to construct, of being able to perform measurements in optically invisible conditions, and of being relatively non-intrusive when used to monitor daily activities. Since ultrasonic waves traveling through air are spread and absorbed, the range for measurements is limited to a few meters. Some investigations using spark discharge to make high-power ultrasonic transmitter in order to improve the measurable range [3]. However, the frequency is not controllable. We have constructed a high-power ultrasonic transmitter that consists of an array of transmitting elements [4] in order to improve the measurable range. We have constructed a high-power ultrasonic transmitter that consists of an array of transmitting elements and obtained the pressure 144 times higher than that of a single element [4]. The sound frequency is exactly 40 kHz. However, all elements of the array generate ultrasonic sounds that are in phase with each other, and the directivity is narrow. The measurement is thus limited to a small area. Therefore, we proposed an ultrasonic array transmitter in which the phase of the sound is controlled for each element [5], and which has isotropic directivity. As a result of these changes, we showed that the divergence of the directivity is successfully controlled by the phase control, and the directional measurement field covers a wide range. However, the sound pressure decreased significantly with the increase in the divergence. In many cases of measurement in air, a wide area is required in the horizontal direction but not in the vertical direction. Therefore, we consider an ultrasonic array transmitter with wide horizontal but narrow vertical divergence in order to retain a high level of sound pressure.

In the present paper, we discuss the theory of this system and investigate its behavior.

II. ULTRASONIC ARRAY TRANSMITTER

The present paper discusses the array transmitter that was developed in our laboratory. The array has (12 × 12) elements in the XY plane at a distance of 10 mm from each other. Each element has a radius of 4.3 mm, directivity of 100° (-6 dB), and operates at 40kHz.

![Figure 1. Schematic diagram of the system.](image)

The phase of the signal from each element is controlled by a signal-delay controller, as shown in Fig. 1. The phase delay has been controlled with the sampling rate of 1μs.

III. THEORY

A. Array Transmitter with Isotropic Directivity

There are many transmitting elements at positions $P(x_i, y_i, 0)$ in the $z = 0$ plane and directed toward the $z$-axis, as shown in Fig. 2 (ultrasonic array).

We now consider an array transmitter with isotropic directivity (the divergences are the same in the $x$ and $y$ directions). The coordinate system of the divergence is shown in Fig. 3. The sound pressure is obtained by the following equation [5]:

$$P_i = \frac{P_0}{d_i^2}$$

where $P_0$ is the sound pressure of the ultrasonic sound, and $d_i$ is the distance between the transmitter and the measurement point. The sound pressure decreases as the distance increases, and the directivity is narrow.
\[ P(x, y, z) = \frac{A}{r} \sum_{i} D(\theta_i) \exp \left\{ -2\pi i \left( x_2 \sin \theta_x + y_2 \sin \theta_y + \frac{x^2 + y^2}{2L} \right) \right\} \]  

where \( A \) is the amplitude of the sound pressure from one element, \( r \) is the distance between the origin (i.e., the center of the array) and the observation point \( P(x, y, z) \), and \( \theta_x \) and \( \theta_y \) are respectively the angles between the vector \( \overrightarrow{OP} \) and the \( yz \)- and \( xz \)-planes. \( D(\theta_i) \) is the directivity of the transmitting element \( L \) is the distance between the center of the divergence and the center of the array, as [5]

\[ L = \frac{n-1}{2} \frac{d}{\tan \phi}. \]  

Here, \( n \times n \) transmitter elements are located in a square with length of \( d \) in the \( x \) and \( y \) directions. The length of each side is obtained by \( (n-1) \times d \), and \( \phi \) is the angle of the divergence.

By modifying (1), the following equation is obtained for an array transmitter with anisotropic directivity:

\[ P(x, y, z) = \frac{A}{r} \sum_{i} D(\theta_i) \exp \left\{ -2\pi i \left( x_2 \sin \theta_x + y_2 \sin \theta_y + \frac{x^2}{2L_x} + \frac{y^2}{2L_y} \right) \right\} \]  

The time delay applied to each element is

\[ \Delta \tau = \frac{x^2}{L_x} + \frac{y^2}{L_y} \frac{1}{2v}. \]  

\[ \Delta \tau = \frac{L^2}{L_x} + \frac{L^2}{L_y} \frac{1}{2v}. \]  

B. Array Transmitter with Anisotropic Directivity

We next consider an array transmitter with anisotropic directivity (the divergences are different in the \( x \) and \( y \) directions). Figure 4 shows this coordinate system and the angle of divergence.

Here, \( \phi_x \) and \( \phi_y \) are the angles of the divergence in the \( x \) and \( y \) directions, respectively, and \( L_x \) and \( L_y \) are the distances between the center of the array and center of the divergence in the \( x \) and \( y \) directions, respectively. They are calculated as follows:

\[ L_x = \frac{n-1}{2} \frac{d}{\tan \phi_x}, \]  

\[ L_y = \frac{n-1}{2} \frac{d}{\tan \phi_y}. \]  

IV. RESULTS

Figure 5 shows the directivities of an array transmitter in which all the elements are in phase, as calculated using (1). The elements have a radius of 4.3 mm, and the transmitting elements are located in squares with lengths of 10 mm in the \( x \) and \( y \) directions. The sound pressure is normalized by \( A/r \).
Figure 5(a) is the 2D image, and Fig. 5(b) is the result along with the x-axis. The sound pressure is normalized by $A/r$ such that the normalized sound pressure of a transmitter with one element is 1. A sound pressure 144 times higher than that of a single element is obtained, and it has a directivity of $\pm 2$ deg. Note that this directivity is too narrow for some applications.

![2D image.](image1)

Figure 5. Sound pressure of the array transmitter in phase. The sound pressure is normalized by $A/r$.

Figure 6 shows the directivities of an array transmitter in which the divergence of the directivity is controlled, as in (1). The angle of the divergence used in the equation is 20 deg in the $x$ and $y$ directions. The sound pressure is normalized by $A/r$.

The length $L$ is calculated to be 155 mm. The time delay is calculated by (3), and it is 0 to 1.45 ms. Although the angle of divergence is about $\pm 20$ deg, the sound pressure is only 1/20th that of a single element, and it is 7 times lower than that of an array transmitter that is in phase.

![In the x direction.](image2)

(b) In the $x$ direction.

Figure 6. Sound pressure of the array transmitter with isotropic phase. The sound pressure is normalized by $A/r$.

Figure 7 shows the directivities of an array transmitter for which the anisotropic directivity is calculated by (6). The angle of divergence used in the calculation is 20 deg in $5$ deg in the $x$ and $y$ directions, respectively. The sound pressure is normalized by $A/r$.

The angle of divergence is about 20 deg and 5 deg in the $x$ and $y$ directions, respectively. The peak pressure is about 48 times higher than that of a single element transmitter and 2.5 times higher than that of an array transmitter with isotropic directivity. The measuring area at the distance $r=30$ m is $\pm 10$ m horizontally and $\pm 2.6$ m vertically.

Figure 8 shows the dependence of the sound pressure on the angle of the divergence in the $y$ direction. The angle of the divergence in the $x$ direction is changed. The sound pressure decreases with an increase in the angle of divergence for the isotropic array transmitter ($\phi_x = \phi_y$).
However, by using an anisotropic array transmitter, the decrease of the peak power can be controlled. The sound pressure is about 50 times higher than that of a single element when the vertical angle of divergence is kept within 5 deg for an array transmitter with a wide horizontal divergence.

V. CONCLUSIONS

An ultrasonic array transmitter with anisotropic directivity has been proposed and its characteristics have been investigated theoretically. The signal phase of each of the elements was changed, and the divergences were successfully controlled. By making the directivities anisotropic, a high sound pressure with a wide range of measurement was obtained. The detailed investigations comparing the simulation results with experimental results of actual device is now in progress.

REFERENCES