

Localization for Electromagnetic Radio Underwater Sensor Networks

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Abstract—Electromagnetic (EM) radios have been recently proposed for underwater sensor networks. Due to the short transmission range of EM radio transmissions underwater, new localization techniques are required for underwater sensor networks using these radios. We propose and evaluate three localization approaches for EM-based underwater sensor networks: (1) multilateration with the aid of autonomous underwater vehicles (AUVs), (2) multi-dimensional scaling (MDS) without the AUVs, and (3) a hybrid approach combining MDS and multilateration. We compare performance of the three approaches with simulations. The main disadvantage of AUV-aided localization with multilateration is that it does not always provide complete localization coverage of the network. On the other hand, MDS provides complete localization coverage, but its performance decreases in sparse networks. The hybrid approach provides complete network coverage and improves the performance of MDS in sparse networks.

Keywords-Underwater Localization; Multi-dimensional Scaling; Multilateration; AUV-aided Localization.

I. INTRODUCTION

The importance of underwater sensor networks (UWSNs) for environmental monitoring is becoming more important as demonstrated by man-made disasters such as the recent oil spill in the Gulf of Mexico, natural disasters such as Tsunamis, as well as ongoing concerns about climate change. The monitoring capabilities of UWSNs are essential to limit the impact of disasters and to predict the future of Ocean resources. An important service required for effective environmental monitoring is localization of environmental sensors, which ensures that the environmental information can be mapped. Due to the unavailability of the Global Positioning System (GPS) underwater, UWSN localization is a very challenging problem. In this work, we tackle the localization problem for UWSNs using electromagnetic (EM) based underwater radios.

The EM underwater radio is a recent technology, which promises a cost-effective and reliable underwater communications [1]. Current underwater acoustic communication technologies suffer from serious drawbacks: they are unreliable due to their susceptibility to environmental noise and require expensive signal processing to deal with the multi-path underwater acoustic channel. Underwater, EM-based signals do not suffer from multi-path arrivals and are not susceptible to environmental noise, so they do not require expensive processing for signal decoding.

The major drawback of using the EM-based radio underwater is its limited communication range due to the rapid attenuation of EM waves in the water. This short communication range requires a large number of nodes to provide connectivity across large areas. However, since the cost of EM-based radios is significantly lower than that of acoustic based radios we do not expect the cost of these networks to be large. In fact, a network with hundreds of EM-based nodes would be less expensive than a network with only a few nodes using acoustic modems.

Localization of nodes in a network is a highly desired capability for sensor network applications. Location stamps are used to tag sensor measurements, ensuring that observations can be mapped to the location where they were taken. Even though UWSNs do not have access to the GPS, localization can be achieved in acoustic-based UWSNs [2]. Due to their short range, localization of EM-based UWSNs needs to be handled differently than localization in acoustic-based UWSNs.

We propose and evaluate three different localization algorithms for UWSNs using EM-based radios:

- (i) Multilateration using an autonomous underwater vehicle (AUV) to act as a mobile beacon. The AUV may not reach all nodes, so this localization approach does not in general cover the entire network.
- (ii) Multi-dimensional scaling (MDS) using neighbourhood distance information. MDS requires all inter-node distances, which may not be available due to the limited transmission range of the nodes. Missing distances are estimated with a shortest path algorithm, which introduces localization errors.
- (iii) A hybrid approach, which uses MDS, but where instead of finding the missing inter-node distances with shortest-path algorithms, the missing distances are calculated from the positions estimated with multilateration. The hybrid approach improves the performance of MDS in sparse networks.

An exhaustive review of localization techniques for acoustic UWSNs is available in [2]. Here we briefly point out why many of these techniques cannot be used for EM-based UWSNs, due to their shorter communication range.

Current localization approaches in underwater acoustic networks rely heavily on the use of anchors [2]. Surface anchors have a known location through the GPS [3]–[5],

while dive'n'rise (DNR) anchors get their position on the surface from the GPS and then dive underwater for localization [6], [7]. Underwater anchors get their position from the surface anchors or from the DNR anchors. Given locations of the anchors and distances to the anchors measured by directly communicating with them, UWSN nodes can localize themselves [4]–[7]. For large networks, the anchors may not be able to communicate directly with all nodes. A solution is to use already localized nodes as anchors to localize other nodes in the network, which are then used as anchors, until the entire network is localized [3]. Another approach is to use mobile anchors [8], which can move and reach every node in the network, so that all nodes are covered and localized.

The approaches using direct communication with anchors would not work in EM-based UWSNs since they would require an enormous number of anchors to reach all nodes in a network. In the case of surface anchors, the approach may not work at all if the UWSN is located at a high depth. The incremental approach may also require a large number of anchors to provide desired performance, since the localization errors propagate as new nodes become anchors. Mobile anchors [8] can get closer to the nodes in the UWSN, which is the reason why we propose their use in this work. Moreover, the MDS approach used in this paper is especially good at eliminating the propagation of localization errors in large sensor networks.

II. SYSTEM ARCHITECTURE

The underwater network considered in this paper is used for environmental monitoring similar to the floats used in the ARGO project [9]. The nodes are dropped off as a swarm to monitor and collect environmental information. Normally, the nodes dive below the surface of the ocean to collect data. Periodically, they rise to the surface and report their data to the sink over a satellite link or are picked up from the water at the end of the monitoring missions and their information is physically retrieved. In either case, the collected information is used for off-line analysis of a given environmental phenomena.

The network may include a roaming AUV for the purpose of augmenting the localization process. The AUV may also be used for other purposes, such as facilitating delay tolerant networking (DTN) if satellite links are not available [10]. In DTN, the data is picked when the AUV is near a sensor and transmitted to the satellite at a later time by the AUV.

All nodes are assumed to be equipped with EM-based radios for peer-to-peer underwater communications [1]. Since EM waves suffer extremely high attenuation in the underwater medium, the radio range of the nodes is several orders of magnitude shorter than that of an acoustic-based networks. We provide more details on node range in Section IV.

The nodes are required to stamp their data with the location of where the data is collected. As the nodes may be

moving due to currents, the location where the information is retrieved (by satellite or by ship) may be very different from the location at the time the environmental data was collected. However, since the collected information is analyzed off-line, the sensor nodes do not actually require the knowledge of their location at the time that observations are made – the location estimates are only needed during the analysis of the observed data. Hence, location estimation can be done off-line, in tandem with the analysis of the observed data. This means that, the localization related information does not need to be sent to a central processor on the fly, even for centralized localization algorithms, significantly reducing the communication cost of sensor localization.

As we show later, geometrical information about the network (distances between nodes and between nodes and the AUV), is sufficient for localizing the nodes. In our system, the sensor nodes tag the observations with the geometrical data instead of estimating their own location and tagging the observations with the estimated location. The observation data and the geometrical data are later transferred to the sink for off-line processing. During off-line processing, the location estimates are obtained from the geometrical data and the observed data is then mapped with the location estimates.

While it may seem that this amount of geometrical data is large (in the order of the number of nodes in the network), in practical situations each node is only connected to a few neighbours at any given time. In our simulations, a typical number of neighbours is only 20 even for networks with several hundred nodes. With the current advancement in computer hardware, this storage requirement can be achieved with off-the-shelf hardware.

III. LOCALIZATION ALGORITHMS

We now discuss three localization approaches tailored for EM-based underwater networks: multilateration, MDS and a hybrid approach. All three approaches use distance information to localize the nodes. For notational purposes, we assume that there are n nodes in the network and that they are distributed on a plane, i.e., two-dimensional localization is considered. The extension to three dimensions is trivial.

Multilateration uses distance measurements from the AUV to a node i , which are the m distance measurements $d_1^{(i)}, \dots, d_m^{(i)}$ to the AUV and the corresponding set of m AUV positions at which the measurements were taken $(x_1^{(i)}, y_1^{(i)}), \dots, (x_m^{(i)}, y_m^{(i)})$. MDS uses the set of distances from each node to each of its neighbours. This information is represented by an $n \times n$ Euclidian distance matrix D in which an entry D_{ij} is the distance between nodes i and j . The hybrid approach uses both sets of geographical information: the distance measurements to the AUV and the positions of the AUV, and the Euclidean distance matrix D .

A. A Two-way Ranging Protocol

Distances between pairs of nodes, or a node and the AUV, are obtained with a two-way ranging protocol. If node i sends a ranging packet at time t_1 , which is received by node j at time t_2 , and at time t_3 node j responds with a ranging packet received by node i at time t_4 , the propagation time can be found with

$$\begin{aligned} t_{ij}^{(prop)} &= \frac{1}{2} [(t_4 - t_3) + (t_2 - t_1)] \\ &= \frac{1}{2} [(t_4 - t_1) + (t_3 - t_2)] \end{aligned}$$

and the distance between i and j can be computed with

$$d_{ij} = t_{ij}^{(prop)} v_{EM}$$

where v_{EM} is the propagation speed of electromagnetic waves in the water. Note that node j can be the mobile AUV.

The two-way ranging protocol time-stamps the ranging packets and also sends the clock differences required to find the propagation delays. For example, node j sends the difference $t_2 - t_1$ in its ranging packets, while time-stamping the packet with t_3 . The time-stamp and the clock difference is sufficient information for node i to determine the distance to node j . In a two-way ranging exchange with the AUV, the AUV also sends its current location in addition to the time difference.

The exchange of ranging packets is performed periodically in order to provide the current distance information in the presence of node movement. We note that this type of packet exchange is found in networks for synchronization purposes [11]. If the network supports synchronized medium access control, the ranging process comes for "free" and does not require extra communications between nodes.

Since the ranging packets are time-stamped with the computer time and not the actual time, the distance estimate contains errors due to inaccurate clock reading and clock skew. However, since the propagation speed of EM waves in the water is relatively low (10^5 m/s) and the actual propagation time in EM-based networks is in the order of milliseconds, the clock errors contribute a very small amount of perturbation to the distance estimate (in the order of centimeters).

B. Multilateration from AUV Measurements

The multilateration localizes each individual sensor node using the distance measurements from the node to the AUV. The set of distance measurements to the AUV and the positions of the AUV associated with those measurements are used to form a system of m non-linear equations

$$\left(x_j^{(i)} - x_i \right)^2 + \left(y_j^{(i)} - y_i \right)^2 = \left(d_j^{(i)} \right)^2, 1 \leq j \leq m$$

where (x_i, y_i) is the unknown position of the node i . In order to localize a node on a plane, $m \geq 3$ distance

measurements from non-collinear AUV locations for each system of equations.

The unknown square terms can be eliminated by subtracting the last equation from the first $m - 1$ equations to arrive at a linear set of equations for each node

$$\mathbf{A}_i \begin{bmatrix} x_i \\ y_i \end{bmatrix} = \frac{1}{2} \mathbf{b}_i, \quad (1)$$

where

$$\mathbf{A}_i = \begin{bmatrix} (x_1^{(i)} - x_m) & (y_1^{(i)} - y_m) \\ \vdots & \vdots \\ (x_{m-1}^{(i)} - x_m) & (y_{m-1}^{(i)} - y_m) \end{bmatrix}$$

and

$$\mathbf{b}_i = \begin{bmatrix} \left(d_m^{(i)} \right)^2 + \left(x_1^{(i)} \right)^2 + \left(y_1^{(i)} \right)^2 \\ \vdots \\ \left(d_m^{(i)} \right)^2 + \left(x_{m-1}^{(i)} \right)^2 + \left(y_{m-1}^{(i)} \right)^2 \\ - \begin{bmatrix} \left(d_1^{(i)} \right)^2 + \left(x_m \right)^2 + \left(y_m \right)^2 \\ \vdots \\ \left(d_{m-1}^{(i)} \right)^2 + \left(x_m \right)^2 + \left(y_m \right)^2 \end{bmatrix} \end{bmatrix}.$$

An estimate of the node position is a least squares solution to (1), which minimizes the Euclidian norm

$$\left\| \frac{1}{2} \mathbf{b}_i - \mathbf{A}_i \begin{bmatrix} x_i \\ y_i \end{bmatrix} \right\|.$$

This solution is given by

$$\begin{bmatrix} \hat{x}_i \\ \hat{y}_i \end{bmatrix} = \frac{1}{2} (\mathbf{A}_i^T \mathbf{A}_i)^{-1} \mathbf{A}_i^T \mathbf{b}_i. \quad (2)$$

Geographical information collected through the ranging process with the AUV is sufficient for each node to use multilateration to estimate its location without any exchange of information with its neighbours. However, there are several problems with the AUV approach, which may prevent its effective use. First, it is unlikely that the AUV can reach each node in the network from multiple non-collinear locations, meaning that some nodes may not have sufficient number of measurements to localize themselves. Second, since the AUV is underwater, it does not have access to the GPS satellites and must use dead-reckoning techniques to estimate its location. Dead-reckoning only provides a very rough estimate of the AUV's location and has the disadvantage of error propagation (errors grow with time), which ultimately affects the localization performance. Third, the AUV takes time to visit every node in the network, by which time some nodes may have moved away. So, the location estimates obtained with the AUV may become outdated quickly.

C. Multi-dimensional Scaling

The MDS localization algorithm [12] simultaneously finds the position of all of the nodes in the network. Unlike the multilateration approach with the use of AUV, this approach provides complete localization coverage of the network. In addition, MDS is known to be relatively resilient to distance measurement errors due to the over-determined nature of the solution. The output of the algorithm is the estimated relative positions of the nodes

$$\tilde{\mathbf{P}} = \begin{bmatrix} \tilde{x}_1 & \tilde{y}_1 \\ \vdots & \vdots \\ \tilde{x}_m & \tilde{y}_m \end{bmatrix}$$

The algorithm works on the full Euclidian distance matrix \mathbf{D} , which contains the distances between all pairs of nodes. First, the MDS algorithm calculates the square distance matrix $\Delta^{(2)}$ in which each entry corresponds to a square entry in the distance matrix

$$\Delta_{ij}^{(2)} = (D_{ij})^2$$

Then, the MDS algorithm calculates an estimate of the Gram matrix $\tilde{\mathbf{B}} = \tilde{\mathbf{P}}\tilde{\mathbf{P}}^T$ by applying double centering to the square distance matrix

$$\tilde{\mathbf{B}} = -\frac{1}{2}\mathbf{J}\Delta^{(2)}\mathbf{J}, \quad (3)$$

where $\mathbf{J} = \mathbf{I} - 1/n\mathbf{1}\mathbf{1}^T$, \mathbf{I} is an $n \times n$ identity matrix, and $\mathbf{1}$ is an $n \times 1$ column vector of 1s.

Finally, the position matrix is recovered from the Gram matrix with the use of eigenvalue decomposition. Given the eigendecomposition of $\tilde{\mathbf{B}}$

$$\tilde{\mathbf{B}} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^T, \quad (4)$$

where \mathbf{Q} is the matrix of eigenvectors and $\mathbf{\Lambda}$ is the diagonal matrix of eigenvalues, the estimate of the position matrix is given by

$$\tilde{\mathbf{P}} = \mathbf{Q}_+\mathbf{\Lambda}_+^{1/2}, \quad (5)$$

where \mathbf{Q}_+ is an $n \times 2$ matrix obtained from \mathbf{Q} by retaining the two eigenvectors corresponding to the two largest eigenvalues and $\mathbf{\Lambda}_+^{1/2}$ is an $n \times n$ matrix obtained by retaining the columns of $\mathbf{\Lambda}$ corresponding to the two largest eigenvalues and taking their square root and making all other entries in the matrix 0.

The last step in the MDS algorithm minimizes the "strain" error between the position matrix and its Gram matrix [12]

$$\left\| \tilde{\mathbf{P}}\tilde{\mathbf{P}}^T - \tilde{\mathbf{B}} \right\|.$$

The MDS approach assumes that the distance matrix \mathbf{D} is fully populated. However, this is only the case if all nodes can communicate directly with each other. In general, the distance matrix is sparse and missing distances should be approximated or estimated. To fill in the missing entries,

we use the standard method where a shortest path algorithm [13] estimates the missing distances from available distance measurements. In our simulations we use the all-pair Ford-Fulkerson algorithm to fill in the missing entries in the distance matrix.

D. Hybrid MDS-Multilateration Localization

Even though the MDS algorithm estimates the positions of all nodes in the network, its major drawback is that it relies on estimates of inter-node distances obtained by the shortest path algorithm. When a network is dense and has a regular shape, the shortest path distance corresponds well to its Euclidean distance. However, if a network is sparse or has an irregular shape, a shortest path distance will not match its Euclidean distance, resulting in localization errors.

To improve the performance of MDS localization in sparse UWSNs, we propose a hybrid approach, which combines the multilateration estimates and the MDS algorithm. In the hybrid approach, the position estimates from multilateration are used to calculate the missing inter-node distances in the distance matrix. These estimates have the potential to improve the shortest path estimates as long as the error from multilateration is small.

IV. SIMULATION RESULTS

To analyze and compare the performance of the proposed localization algorithms, we perform a set of Monte-Carlo simulations using Matlab. In each Monte-Carlo run, sensor nodes are uniformly distributed over a disk with a 1000 m radius. The transmission range of the nodes is assumed to be 100 m, corresponding to the range of EM radio signals in water [1]. The AUV path and the locations of beacon transmissions are the same for every Monte-Carlo run. For each run, we calculate the inter-node distances and the distances from each node to the AUV. We pass the distance smaller than the maximum range to the localization algorithms.

The AUV moves in a spiral pattern with a trajectory given by the coordinates in time $x_{uuv}(t + t_0) = At \cos(t + t_0)$ and $y_{uuv}(t + t_0) = At \sin(t + t_0)$, where $A = 10$ and $-19\pi \leq t \leq 19\pi$ are chosen to ensure that the AUV can visit every node in the network and t_0 is the uniform random variable chosen from the interval $[0, 2\pi]$. The AUV sends out a beacon every 10 s, nodes in the 100 m radius of the AUV at the time of the beacon transmission can perform two-way ranging with the AUV.

Since the AUV moves underwater and only occasionally updates its coordinates using the GPS, most of the time it uses dead-reckoning to determine its coordinates. We model the error due to dead reckoning by nudging the AUV away from its nominal path with a random perturbation. The AUVs coordinates are randomly sampled from a uniform disk with a given radius centered at the point on the AUVs

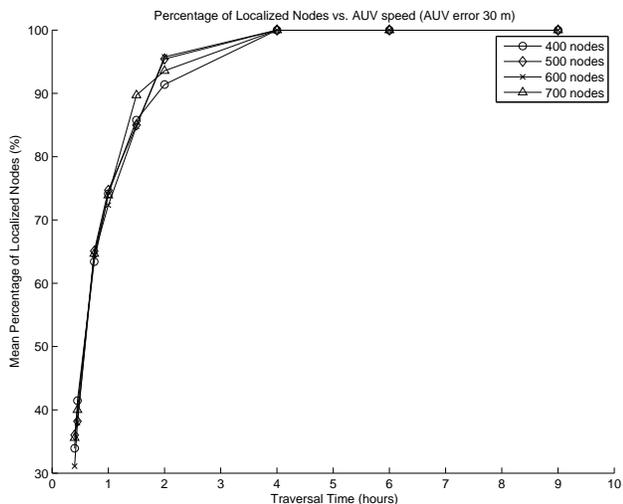


Figure 1. Mobile Beacon Coverage.

nominal path. This radius of this circle is indicated as "AUV error" in our plots.

We vary the speed of the AUV to achieve different amount of node coverage. We use speeds from about 10 m/s to about 0.5 m/s, corresponding to the network traversal time from about 30 minutes to about 9 hours, respectively. At a high speed the AUV can only send a few beacons before it traverses its entire pre-programmed path. The consequence of high speed then is that many nodes may not receive a sufficient number of beacons to localize themselves. At a lower speed the AUV sends out more beacons while traversing its path, thus increasing the number of nodes that received more than 3 non-collinear beacons.

Figure 1 shows the localization coverage of the mobile beacon approach as the total time to traverse the network (AUV speed) changes. We see that at high speeds the coverage is very low (about 30 %). The coverage can be 100 % at lower speeds, albeit at the cost of longer time to cover the entire network. These results are consistent with previously published results [8] for acoustic based networks. The figure also shows that the coverage does not improve at higher node densities.

Figure 2 and Figure 3 compare the performance of the three localization algorithms. Figure 2 shows the performance for a relatively sparse network (400 nodes), while Figure 3 shows the performance results for a relatively dense network (600 nodes). In both cases, the AUV error is 30 m. The estimated position error is the average across all node errors for all runs.

For the sparse network scenario (Figure 2), we see that multilateration performs better than MDS. However, multilateration cannot localize all nodes for traversal times of less than 4 hours. At higher AUV speeds the nodes that can be localized by multilateration also have fewer ranging

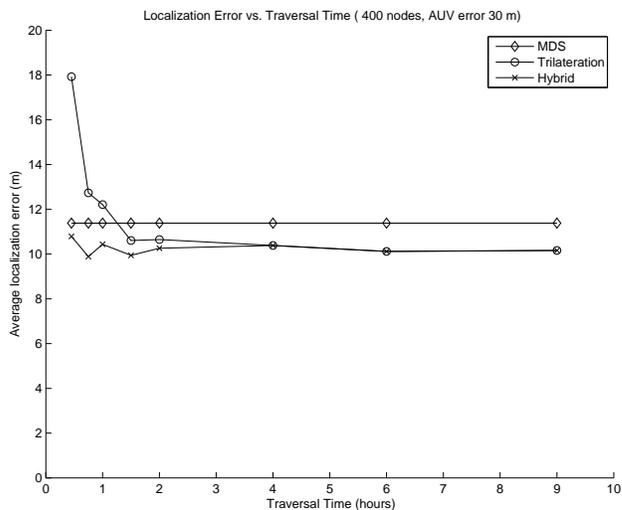


Figure 2. Localization Error (sparse networks)

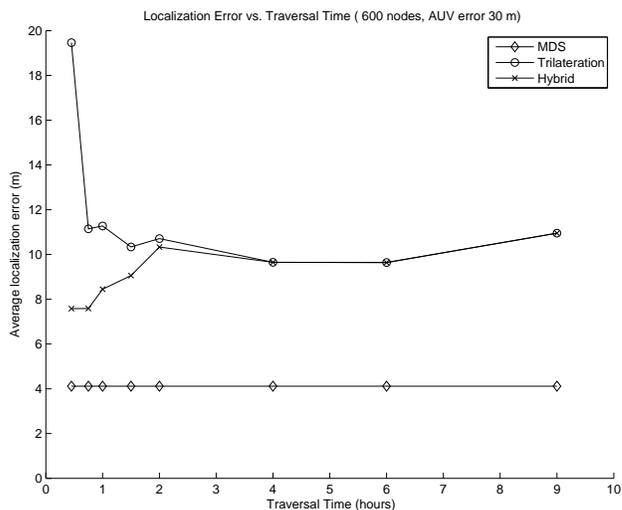


Figure 3. Localization Error (dense networks).

measurements with the AUV, than at lower speeds. This decrease in the number of measurements accounts for a larger multilateration error at higher speeds. The hybrid approach is able to localize all nodes, since it uses MDS. In addition, it is able to provide improved localization performance when the AUV is at both high and low speeds. So, using the position estimates from multilateration decreases the error of the MDS position estimates for that scenario.

For the dense network scenario (Figure 3), the MDS approach always outperforms the multilateration approach. The error from estimating inter-node distances with the shortest path algorithm is lower than the error due to the uncertainty of AUV's location. For higher speeds, the hybrid approach can be thought of as a MDS refinement of the

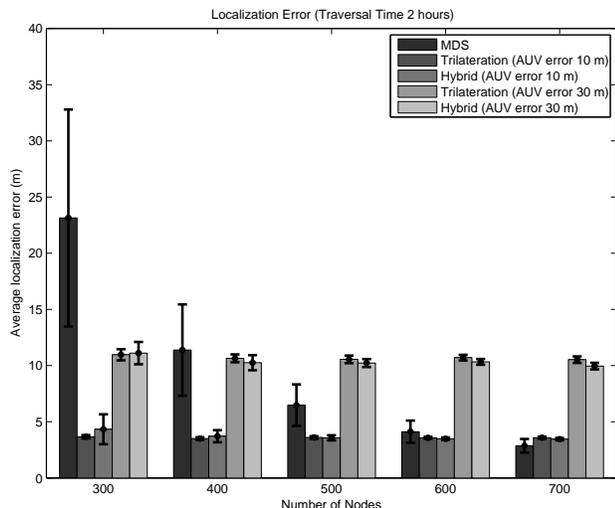


Figure 4. Localization Error (dense networks).

multilateration approach. At lower speeds, the error in the multilateration position estimates affects the performance of the hybrid approach.

Figure 4 compares the performance of the three algorithms for different network sizes and AUV errors, when the AUV traversal time is set to 2 hours. The error bars show the standard deviation of the localization error. The figure shows that MDS performs better in dense networks and that the hybrid approach can improve MDS performance in sparse networks. However, if the error introduced by the AUV is too large, MDS should be used by itself.

V. CONCLUSION AND FUTURE WORK

We proposed and analyzed three approaches for localization in EM-based underwater sensor networks. The first approach uses an AUV and a multilateration algorithm. One drawback of this approach is that it may not be able to localize all nodes in the network when the AUV moves at high speeds. The second approach uses MDS to localize nodes based on their neighbourhood inter-node distance measurements. While the MDS approach localizes all nodes in the network, it may suffer from localization errors in sparse networks where not all inter-node distance measurements are available. The performance of MDS degrades in sparse networks because the missing inter-node distance measurements are estimated with a shortest path algorithm. The third approach is a hybrid approach that aims at improving the performance of the MDS by using the position estimates from multilateration to calculate the missing inter-node distances. Our simulations show that the hybrid approach improves the performance of MDS in sparse networks.

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