Rate Adaptation for Slepian-Wolf Coding in Presence of Uncertain Side Information

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***Abstract*—In this paper, a rate adaptation framework is proposed to address the problem of Slepian-Wolf coding with uncertain side information at the encoder. The uncertainty arises due to the time-varying nature of the correlation between source and side information in settings such as wireless sensor networks and distributed video coding. The proposed framework is set up based on a multi-mode Slepian-Wolf coding scheme which is designed to minimize the average rate. The presented solution utilizes the feedback channel judiciously to select the best encoder mode and substantially reduces the delay and decoding complexity compared to the previous methods which rely on frequent retransmissions for successful decoding. The designs based on both practical and ideal Slepian-Wolf codes are considered, where the latter serves as the corresponding theoretical performance bound. Simulation results based on LDPC codes show that by using sufficient number of modes, a desirably small average rate gap from the theoretical bound with no uncertainty can be achieved.**

Keywords-Slepian-Wolf coding; rate adaptation; uncertain side information; rate-limited feedback channel.

I. Introduction

Distributed Source Coding (DSC) and especially Slepian-Wolf (SW) coding has been the subject of substantial research interest recently. This is mainly due to its applications in data compression for wireless sensor networks and distributed video coding.

The SW coding theorem as introduced in [1], states that the ultimate lossless compression rate for a source with a given correlated Side Information (SI) only available to the decoder, is the same as that when SI is also available to the encoder. Practical coding schemes appeared later and may be categorized as parity-based and syndrome-based coding approaches [2][3]. These schemes are constructed based on capacity approaching codes such as LDPC and Turbo codes [4][5]. As indicated in [1], the joint Probability Distribution Function (PDF) of source and SI must be available to the encoder for compression based on SW coding.

In many practical scenarios, the joint PDF of source and SI may not be available perfectly to the encoder. For instance, in distributed video coding [6] for wireless video sensor networks, correlation model cannot be estimated at the encoder due to its associated computational cost for the encoder, usually being a mobile device. Also, in wireless sensor networks for environmental monitoring, the correlation varies in time as a result of natural phenomena.

Simple feedback schemes are suggested to overcome this uncertainty problem in the joint PDF of source and SI. For instance, for distributed video coding based on Turbo or LDPC codes, decoding failure is first detected. Next, with requested additional syndrome or parity bits via a feedback channel, subsequent decodings are performed until a probability of error constraint is satisfied [6]. The aforementioned procedure has two major drawbacks; (1) the delay due to the number retransmissions and (2) the computational cost of multiple decodings each time more syndrome or parity bits are fed to the decoder. Different approaches have been proposed to tackle the inefficient use of feedback usually resulting in increasing encoder complexity. Recently in [7], the relay nodes in a wireless video sensor network are utilized to reduce the said retransmission delay by incorporating network coding. However, the delay is still non-negligible.

For flexible rate SW compression, [8] and [9] provide methods to construct multi-rate LDPC and serial and parallel concatenated convolutional codes from a parent code to efficiently handle possibly varying correlation of source and SI. However, these works are focused on code design and either do not present a code selection mechanism or rely on simple ACK/NACK feedback schemes discussed before.

The rate-distortion performance bounds for Gaussian Wyner-Ziv (WZ) coding (lossy DSC) with uncertain SI at the encoder is studied in [10]. In [11], for the case when the SI quality has two different states, unknown to both encoder and decoder, a two-layer WZ coding scheme using transform coding and ideal SW coding is presented.

In this work, we propose a framework to address the problem of SW coding with uncertain SI at the encoder. The framework consists of a multi-mode encoder working with a carefully designed feedback from the decoder. The proposed scheme utilizes the feedback channel effectively and consequently performs a one-time-only decoding for each data frame to substantially reduce decoding delay and computational cost of multiple decodings. It is assumed that the joint PDF of source and SI is fixed over each source frame but varies from frame to frame. In line with [10], we assume that this PDF is controlled by a single parameter σ, which is known to the decoder. We partition the range of σ and assign each interval to a unique mode of the encoder. For each frame, the decoder sends the mode index to the encoder

 ISBN: 978-1-61208-144-1

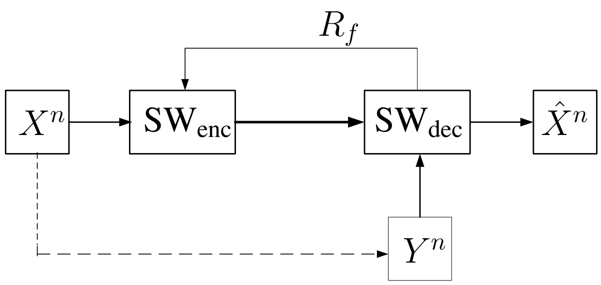

Figure 1. Rate-adaptive SW coding with a rate-limited feedback channel

using the feedback channel. To derive the corresponding performance bounds, we consider ideal (lossless) SW codes and obtain the optimized partitioning and code rates such that the average rate is minimized. Next considering practical SW codes with finite block length and discrete rates, we present an algorithm for the design of the optimized partitioning to minimize the average rate while a given probability of error constraint is satisfied. Simulation results based on LDPC SW codes using syndromes are provided which quantify the performance for different number of modes and demonstrate the effectiveness of the proposed algorithms and designs.

The rest of this paper is organized as follows. Section II discusses the preliminaries on SW coding and introduces the system model. In Section III, performance bounds for SW coding with uncertain SI are derived and in Section IV, effective solutions based on practical codes are presented. Section V is dedicated to simulations and numerical results and finally Section VI concludes the paper.

II. Preliminaries

In this section, we first briefly discuss the original SW coding theorem. We then present the proposed system model and introduce the parameters used in the rest of the paper.

A. Slepian-Wolf Coding and Related Limitations

Consider the i.i.d sequence $(X^n, Y^n) = \{x_i, y_i\}_{i=0}^{\infty}$ with joint probability distribution $P_{xy}(x, y) = P_x(x)P_{y|x}(y|x)$. It may be assumed that X^n and Y^n are correlated via a virtual innovation channel model $P_{y|x}(y|x)$. It was shown in [1] that the two sources X^n and Y^n may be separately encoded and jointly and losslessly decoded at a minimum sum rate of $H(X, Y)$, where $H(.)$ is the Shannon entropy function. This is referred to as distributed source coding in symmetric setting. Interestingly, the said rate coincides with that when both sources are collocated.

If Y^n is available to the decoder as SI, X^n can be encoded at the rate of $H(X|Y)$. This scenario is known as distributed source coding in asymmetric setting. The situation in which X^n is a discrete random variable but Y^n has a continuous alphabet, usually arises in compression of data in sensor networks [12] or compression of quantized indexes in WZ coding of correlated data [13]. A good model for these scenarios is that X^n is a sequence of binary uniform random variables, i.e., $\mathcal{X} = \{-1, 1\}$ and $P_{y|x}(y|x)$ represents an AWGN channel. If the variance of the AWGN channel is equal to σ^2, then the necessary rate for compression of X^n as given in [1] equals $R_{\text{SW}} = H(X|Y)$. This rate can be calculated as

$$R_{\text{SW}} = H(X|Y) = H(X) + \frac{1}{2}\log 2\pi e\sigma^2 - h(\sigma)\,, \quad (1)$$

where

$$h(\sigma) = -\frac{1}{2}\int_{-\infty}^{\infty}\Big(\frac{1}{\sigma\sqrt{2\pi}}\big(e^{\frac{-(y-1)^2}{2\sigma^2}} + e^{\frac{-(y+1)^2}{2\sigma^2}}\big)\Big) \log\Big(\frac{1}{\sigma\sqrt{2\pi}}\big(e^{\frac{-(y-1)^2}{2\sigma^2}} + e^{\frac{-(y+1)^2}{2\sigma^2}}\big)\Big)dy\,. \quad (2)$$

Clearly, the compression rate is a function of σ^2 and if the correct compression rate is not known at the encoder, lossless decoding is not possible at the decoder.

Practical SW coding is based on finite rate and finite length codes as follows. According to [3], if the parity check matrix for this code is H_p of size $(n-k)\times n$, first the syndrome S_l for a block X_l of source is created by calculating $S_l = H_pX_l$ and then S_l is sent to the decoder. This indicates a compression rate of $(n-k)/n$. At the decoder, exploiting SI, an estimate $\hat{X}_l$ is found such that $S_l = H_p\hat{X}_l$ and the Hamming distance between X_l and $\hat{X}_l$ is minimized. This is ideally done by Maximum-Likelihood decoding, but iterative decoding algorithms based on Turbo or LDPC codes are usually used in practice. In this paper, the method of [5] has been used for practical implementations in Section V.

B. System Model

Figure 1 shows the general system model in this paper. $X^n = \{x_i\}_{i=0}^{\infty}$ is the source to be encoded where x_i are binary random variables with uniform distribution. $Y^n = \{y_i\}_{i=0}^{\infty}$ is the correlated SI available to the decoder. To model the correlation, we have $y_i = x_i + z_i$, where z_i is Gaussian distributed with variance σ^2. The source is encoded in frames of length n_f and it is assumed that σ is constant for each separate frame, but varies from one frame to another. The random variable σ follows the PDF $g_\sigma(\sigma)$ and cumulative distribution function $\mathcal{G}(\sigma) = \int_0^\sigma g_\sigma(x)dx$, where the range of σ is denoted by Σ, usually including all non-negative real numbers.

Encoding is performed using a multi-mode encoder with a set $\mathcal{R}$ of pre-designed SW codes for compression, where $|\mathcal{R}| = M$. The decoder is assumed to estimate σ perfectly in each frame [10], and based on which select an encoder mode or equivalently the SW code to be used. The decoder then sends the index $i, i \in \{1, 2, \ldots, M\}$ of the selected mode to the encoder via a rate-limited feedback channel with the rate $R_f = \lceil\log_2(M)\rceil$ without delay or error. Therefore, Σ is partitioned into M disjoint intervals (modes) in an optimized

 ISBN: 978-1-61208-144-1

manner and each interval is associated with one code. The partitions are in the form $[T_{i-1}, T_i), i = 1, 2, \ldots, M$. The probability of each mode being used is then equal to $p_i = \int_{T_{i-1}}^{T_i} g_\sigma(x)dx$.

Given a set of SW codes $\mathcal{R}$ with rates $R_i, i \in \{1, \ldots, M\}$, the multi-mode SW design problem is to determine the partitioning of Σ such that the source coder average rate $R_{\text{avg}} = \sum_{i=1}^{M} p_i R_i$, is minimized (maximum compression). Without loss of generality, we assume that $R_i > R_{i-1}, 2 \leq i \leq M$. To the best of the authors' knowledge, this paper is the first in the context of Slepian-Wolf coding to formulate the rate control mechanism as the solution to an optimization problem. In Section III, performance bounds to this problem are obtained when ideal (lossless) SW codes are considered and their corresponding rates may also be designed. In Section IV, the design problem with practical SW codes and hence a practical lossless decoding constraint is addressed. This is accomplished by modeling the decoding error performance of each SW code by a function $f_i(.)$. Specifically, $f_i(\alpha)$ denotes the probability of bit error at the SW decoder, when $\sigma^2 = \alpha$ and α is known to both encoder and decoder.

III. Rate Adaptation for Ideal Slepian-Wolf Coding

Assuming ideal SW codes with infinite block length and zero error probability, one can obtain a lower bound for the average rate and use it as a benchmark for the performance of practical SW codes. In this direction, no constraint on the rate of the codes are assumed, i.e., $R_i \in [0, 1]$, and indeed, the code rates are design parameters. This is in contrast to practical SW codes with predetermined rates.

Formally stating the problem, the objective is to minimize the average rate (maximize the compression ratio) for lossless decoding. In this case, the probability of decoding error goes to zero as the block length goes to infinity.

Problem 1:

$$\min_{T_i \in \Sigma, R_i, i=1,2,\ldots,M} R_{\text{avg}} = \sum_{i=1}^{M} p_i R_i \quad \text{s.t.}\{\bar{P}_e \to 0 | n_f \to \infty\} \tag{3}$$

where $\bar{P}_e$ denotes the average probability of error and $R_i \in [0, 1]$. To solve this problem, we set

$$R_i = \mathcal{H}(T_i) = 1 + \frac{1}{2}\log(2\pi e T_i^2) - h(T_i)\,. \tag{4}$$

This is due to the fact that each T_i in fact corresponds to a value of σ and by definition of conditional entropy and as given in equation (1), $\mathcal{H}(T_i)$ is the minimum achievable rate for $\sigma = T_i$. It can be simply verified that if $R_i < \mathcal{H}(T_i)$ the probability of error cannot go to zero when $n_f \to \infty$. After setting R_i for given T_i, finding the optimal partitioning of Σ, or equivalently the values of T_i, completes the solution to problem 1. Algorithm 1 is proposed for this purpose.

Algorithm 1:

- Initialize $T_{i,0}$ for $i = 1, 2, \ldots, M-1$ as the interval thresholds at iteration 0 arbitrarily, but satisfying $T_{i,0} < T_{j,0}$ for any $i < j$. Set $T_{0,0} = \min(\Sigma)$ and $T_{M,0} = \max(\Sigma)$.
- Choose the values of η and $k_{\max}$ as predefined constants to control the number of iterations.

Now, for each iteration $k = 1, 2, \ldots, k_{\max}$, do the following,

1) For each $i = 1, 2, \ldots, M-1$, find $T_{i,k}$ such that

$$\frac{\partial R_{\text{avg},k}(T_{i,k})}{\partial T_{i,k}} = 0, \qquad T_{i-1,k} < T_{i,k} < T_{i+1,k}\,. \tag{5}$$

where

$$\frac{\partial R_{\text{avg},k}(x)}{\partial x} = g_\sigma(x)\big[\mathcal{H}(x) - \mathcal{H}(T_{i+1,k})\big] + \big[\mathcal{G}(x) - \mathcal{G}(T_{i-1,k})\big]\frac{\partial \mathcal{H}(x)}{\partial x}\,. \tag{6}$$

If there are multiple solutions for $T_{i,k}$, select the one which results in the smallest average rate $R_{\text{avg},k}$. Note that in calculating $T_{i,k}$ using (5), all other $T_{j,k}$, $j \neq i$ are kept fixed.

2) Calculate the average rate, $R_{\text{avg},k}$, using the final partitioning at iteration k. If $(R_{\text{avg},k-1} - R_{\text{avg},k}) \;/\; (R_{\text{avg},k-1}) < \eta$ or $k > k_{\max}$, stop.

More details on Algorithm 1 are presented below.

Proposition 1: The average rate computed using Algorithm 1 is reduced in each run of its step 1.

Proof: From (3) and (4), we have

$$R_{\text{avg}} = \sum_{i=1}^{M} \int_{T_{i-1}}^{T_i} g_\sigma(x)\mathcal{H}(T_i)dx \tag{7}$$
$$= \beta + \int_{T_{i-1}}^{T_i} g_\sigma(x)\mathcal{H}(T_i)dx + \int_{T_i}^{T_{i+1}} g_\sigma(x)\mathcal{H}(T_{i+1})dx\,,$$

where β is a constant independent of T_i. As evident in (6) for $x = T_i$, noting that $\mathcal{H}'(T_i) = \partial\mathcal{H}(T_i)/\partial T_i$ is differentiable for $T_i > 0$, $\partial R_{\text{avg}}/\partial T_i$ exists for all values of T_i and continuous $g_\sigma(x)$.

In the following, we show that Algorithm 1 is able to find at least one minimum for R_{avg} in each step. It can be verified that

$$R_{\text{avg}}|_{T_i \to T_{i-1}} = R_{\text{avg}}|_{T_i \to T_{i+1}} = \beta + \mathcal{H}(T_{i+1})(\mathcal{G}(T_{i+1}) - \mathcal{G}(T_{i-1})). \tag{8}$$

Also from (6), we have

$$\lim_{T_i \to T_{i-1}^+} \frac{\partial R_{\text{avg}}}{\partial T_i} = g_\sigma(T_{i-1})(\mathcal{H}(T_{i-1}) - \mathcal{H}(T_{i+1})) < 0\,, \tag{9}$$

$$\lim_{T_i \to T_{i+1}^-} \frac{\partial R_{\text{avg}}}{\partial T_i} = \mathcal{H}^{'}(T_{i-1})(\mathcal{G}(T_{i+1})-\mathcal{G}(T_{i-1})) > 0, \quad (10)$$

which follow as both $\mathcal{H}(.)$ and $\mathcal{G}(.)$ are increasing functions of their arguments or consequently $\mathcal{H}(T_{i-1}) < \mathcal{H}(T_{i+1})$, $\mathcal{H}^{'}(T_{i-1}) > 0$, and $\mathcal{G}(T_{i+1}) - \mathcal{G}(T_{i-1}) > 0$. Equations (8), (9) and (10) are sufficient conditions that $R_{\text{avg}}(T_i)$ has at least one minimum in the interval $[T_{i-1}, T_{i+1})$, which is found using (6). This indicates that R_{avg} is in fact reduced in each run of the step 1 of Algorithm 1. ■

Remark 1. $R_f \to \infty$ is associated with infinite number of encoder modes or having an ideal feedback channel with no rate limit. This eliminates any uncertainty at the encoder and the SW bound is then achieved for an equivalent encoder which is fully aware of SI statistics. This results in the minimum possible rate. In such case we have,

$$R_{\text{min}} = \int_0^\infty \mathcal{H}(x) g_\sigma(x) dx \quad (11)$$

which can be calculated numerically. This is referred to as SI-Aware SW Coding bound (SIA-SWC) and is used for comparison in Section V.

IV. RATE ADAPTATION FOR PRACTICAL SLEPIAN-WOLF CODING

The performance bounds of multi-mode rate-adaptation scheme for SW coding with rate-limited feedback was studied in Section III using ideal (lossless) SW codes. For practical SW coding and in presence of uncertain SI, the encoder may only use a certain number of pre-designed codes with predetermined rates. These codes are not ideal and may involve possible decoding error.

A. Rate-Adaptation Based on Practical Codes

The following design problem formulates the minimization of the average rate when finite length and discrete rate SW codes are used and their probability of error performance are taken into account.

Problem 2:

$$\min_{T_i \in \Sigma, i=1,2,\ldots,M} R_{\text{avg}} = \sum_{i=1}^{M} R_i \int_{T_{i-1}}^{T_i} g_\sigma(x) dx \quad (12)$$

$$\text{s.t.} \quad \{\bar{p}_i = \int_{T_{i-1}}^{T_i} g_\sigma(x) f_i(x) dx < p_0 \quad | \quad R_i \in \mathcal{R}\}$$

where $\bar{p}_i$ denotes the average probability of error in mode i. Note that in this design, practical lossless compression has been interpreted as the average probability of error per mode being limited to a small p_0. Satisfying the stricter per-mode average probability of error constraint also satisfies the overall average probability of error.

In the following, we present an algorithm to solve problem 2. The proposed solution is general in the sense that it is independent of the selected set of codes, their rates, and their error probability performance.

Algorithm 2:
Set $T_i = 0$ and select ϵ a small number. For $i = 1, 2, \ldots, M$ do the following,

1) Set $T_i = T_{i-1}$.
2) Increase T_i with a step size ϵ.
3) Calculate $\bar{p}_i$, if $\bar{p}_i < p_0$, go to (2). Else reduce T_i by ϵ.

Intuitively, with $R_i > R_{i-1}$, in order to reduce R_{avg}, the lower rates are to cover as much of the range of σ as possible. Thus, the thresholds are updated from T_1 to T_M. Each one is increased until the probability of error constraint for the corresponding mode is met with equality. To show that R_{avg} is decreased at each step of Algorithm 2, we present Proposition 2.

Proposition 2: Suppose that Σ is partitioned with the intervals $[T_{j-1}, T_j)$, $j = 1, 2, \ldots, M$. If all thresholds except T_i are fixed, in order for R_{avg} to be reduced, T_i must be increased.

Proof: Suppose that all thresholds $\{T_j\}_{j=1}^M, j \neq i$ are fixed and only T_i is to be updated. Also suppose that $T_i^{'}$ refers to the modified T_i value, and define $R_{\text{avg}}^{'}$ as the updated value for R_{avg} when T_i is modified. Now, we have

$$\begin{aligned}
&R_{\text{avg}}^{'} - R_{\text{avg}} = \\
&+ \Big(\gamma + \int_{T_{i-1}}^{T_i^{'}} R_i g_\sigma(x) dx + \int_{T_i^{'}}^{T_{i+1}} R_{i+1} g_\sigma(x) dx\Big) \\
&- \Big(\gamma + \int_{T_{i-1}}^{T_i} R_i g_\sigma(x) dx + \int_{T_i}^{T_{i+1}} R_{i+1} g_\sigma(x) dx\Big) \quad (13a) \\
&= \Big(\int_{T_{i-1}}^{T_{i+1}} R_i g_\sigma(x) dx + \int_{T_i^{'}}^{T_{i+1}} (R_{i+1} - R_i) g_\sigma(x) dx\Big) \\
&- \Big(\int_{T_{i-1}}^{T_{i+1}} R_i g_\sigma(x) dx + \int_{T_i}^{T_{i+1}} (R_{i+1} - R_i) g_\sigma(x) dx\Big) \\
&= \int_{T_i^{'}}^{T_i} (R_{i+1} - R_i) g_\sigma(x) dx \quad (13b)
\end{aligned}$$

where $\gamma = \sum_{j=1}^{M} \int_{T_{j-1}}^{T_j} R_j g_\sigma(x) dx \,, j \neq i, i+1$. The term $(R_{i+1} - R_i) g_\sigma(x)$ is always positive as the code rates are assumed ordered and $g_\sigma(x)$ is positive as a PDF. Therefore, it is a necessary and sufficient condition for R_{avg} to decrease that $T_i^{'} > T_i$. ■

B. Mode Selection

In practice, due to limitations for the feedback channel rate R_f and for reduced encoder/decoder complexity, only a limited number of modes equal to $N = 2^{R_f} < M$ may be allowed. For that reason in the following, we propose the Algorithm 3 to select a suitable subset of modes with the desired size N and their associated rates and

 ISBN: 978-1-61208-144-1

intervals from the original result obtained using Algorithm 2.

Algorithm 3:

- Initialization: Given the set $\mathcal{R}$, use Algorithm 2 to design a rate-adaptive multi-mode SW code. Consider the resulting thresholds in the sequel.
- Perform the followings for $m = 1, 2, \ldots, |\mathcal{R}| - N$.
- For $i = 1, 2, \ldots, |\mathcal{R}|$,
  1) Merge each two partitions in the form of $[T_{i-1}, T_i)$ and $[T_i, T_{i+1})$, into one single partition $[T_{i-1}, T_{i+1})$.
  2) Remove the code in $\mathcal{R}$ with the rate R_i, associated with $[T_{i-1}, T_i)$, temporarily from $\mathcal{R}$ and assign R_{i+1} to the partition $[T_{i-1}, T_{i+1})$.
  3) Calculate R_{avg} and store its value as R^i_{avg}.
- Select the merging of intervals corresponding to $\arg\min_i R^i_{\text{avg}}$.

Note that by selecting R_{i+1} for the merged partitions, the probability of error constraint is still satisfied because $R_{i+1} > R_i$ and SW code $(i+1)$ can be used for compression instead of code i without incurring more error. The presented algorithm provides a low complexity but suboptimal solution to select a subset of N codes out of M available codes for the multi-mode SW encoder. Using an exhaustive search to this end, involves running Algorithm 2 for each code subset, which equals $M!/(N!(M-N)!)$ subsets in total. However, Algorithm 3 has only $M-N$ steps (a small number). Also, the computational cost for each step, which only consists of merging partitions, is much smaller than cost of running Algorithm 2. Algorithm 3 is used for mode selection in simulations of Section V.

V. Simulations and Results

In this section, we present a simulation setting and corresponding results to validate and compare the designs of Sections III and IV. We used a set of high-performance LDPC codes from the DVB-S2 standard for SW compression. The selected set consists of 11 SW codes with discrete rates as in Table 1. The bit error rate performance of this set was obtained via extensive simulations. The probability of error function $f_i(\sigma^2)$ for the code with rate R_i is modeled by

$$f_i(\sigma^2) = \frac{1}{(1 + e^{c_i(1/\sigma^2 - b_i)})^{d_i}}\,, \tag{14}$$

for small (less than 10^{-2}) bit error rates, where b_i, c_i, d_i are constants that are obtained using fitting techniques as presented in Table 1. It is noteworthy that a similar error performance model as in (14) has been used to model channel decoding error in [14].

For the simulations, it is assumed that the parameter σ^2, as introduced in Section II, is Rayleigh distributed with parameter θ^2, i.e, if $u = \sigma^2$, then $f_u(u) = \frac{u}{\theta^2} e^{-\frac{u^2}{2\theta^2}}$, $u > 0$. A greater θ^2 implies more uncertain SI. We also set $p_0 = 1 \times 10^{-4}$.

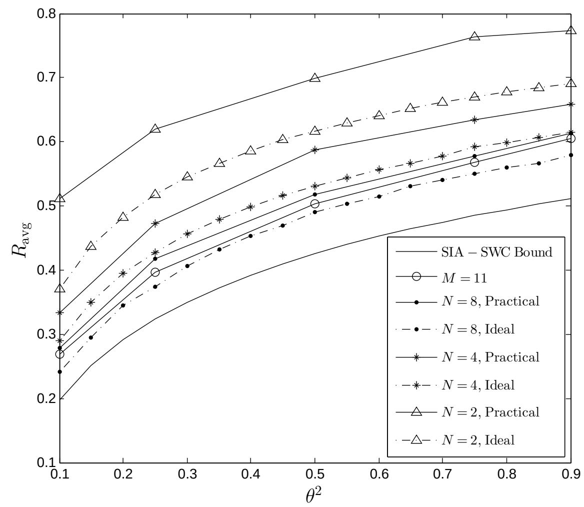

Figure 2. R_{avg} for different levels of SI uncertainty controlled by θ^2

Figure 2 depicts the performance of the proposed multi-mode SW coding quantified by R_{avg}, as a function of θ^2 for different number of modes. Using ideal SW codes and Algorithm 1, a lower bound for R_{avg} is presented. As expected, adding to the number of modes or equivalently the feedback rate, reduces the average rate. Using ideal SW codes with feedback rates $R_f = 1, 2, 3$ bps and for values of $\theta^2 > 0.2$ the gap from SIA-SWC bound (ideal feedback) approximately equals 0.188, 0.104, and 0.060 bps, respectively. For practical SW codes and using 2, 4, 8, and 11 modes, this gap is approximately 0.278, 0.145, 0.093, and 0.084 bits per symbol (bps), respectively.

The gap between ideal and practical SW code performance for $\theta^2 > 0.2$ equals 0.090, 0.047, 0.033 bps, respectively for $R_f = 1, 2, 3$ bps. For small values of θ^2, this R_{avg} gap is larger, e.g., for $\theta^2 = 0.1$ and $R_f = 1$ bps, it amounts to 0.141 bps. This small gap between practical and ideal code performance is more due to using finite length and discrete rate codes, and not to sub-optimality of Algorithm 3. This is supported by performance comparison of Algorithm 3 for mode selection and an exhaustive search solution as depicted in Figure 3. As evident, the incurred gap due to sub-optimality of the proposed Algorithm 3 is negligible over a wide range of values of R_f and θ. This is certainly outweighed by its much smaller complexity in comparison to an exhaustive search.

It is noteworthy that the gap between ideal and practical code performance decreases as R_f increases. This is due to the fact that as R_f and hence the number of modes (codes) increase, the set $\mathcal{R}$ approximately resembles a set of continuous rate codes. It is very interesting that the induced rate loss using 11 modes for compression is only 0.084 bps. This is comparable with the suggested SI aware

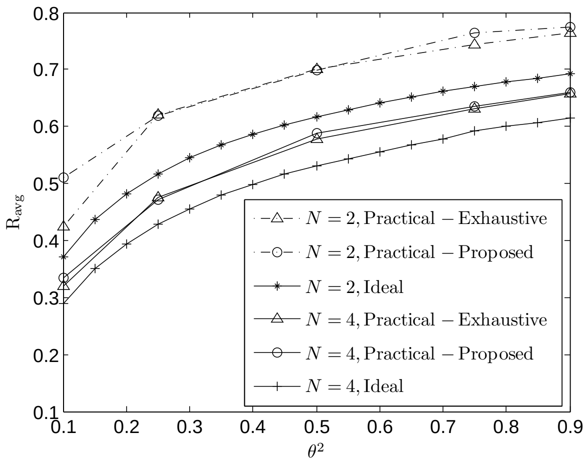

Figure 3. Performance comparison of the exhaustive search and the proposed method for mode selection (Algorithm 3)

Table I
FITTING PARAMETERS FOR MODELING THE PERFORMANCE OF LDPC-BASED SW CODES

R_i	b_i	c_i	d_i	R_i	b_i	c_i	d_i
1/10	4.249	4.18	11.9	2/5	1.898	9.22	90.4
1/9	3.803	4.04	6.8	1/2	1.183	40.78	9.5
1/6	3.124	14.24	7.0	3/5	0.923	28.35	50.1
1/5	2.820	11.73	11.5	2/3	0.680	50.11	12.2
1/4	2.474	12.45	15.5	3/4	0.505	60.18	16.0
1/3	2.042	14.59	31.4	-	-	-	-

schemes of [8] and [9] with LDPC and Turbo-based SW codes. In comparison to [6], the superiority of the proposed method is of course due to its judicious use of feedback and hence reduced decoding complexity and delay as discussed in section I.

VI. CONCLUSION AND FUTURE WORK

In this paper, a rate adaptation framework for the problem of Slepian-Wolf coding in presence of uncertain side information at the encoder is presented which uses a multi-mode encoder accompanied by a well-designed feedback scheme. SW compression rate and other mode parameters based on practical SW codes are designed such that the average rate is minimized. A lower bound is also derived considering ideal lossless codes. Simulation results based on LDPC codes show that by using sufficient number of modes, a desirably small average rate gap from the theoretical bound with no uncertainty can be achieved.

For the future work, the generalization of the proposed scheme for the context of Wyner-Ziv coding may be considered. This generalization requires introduction of distortion to the average rate minimization problem and the use of quantizers. The generalized scheme can then be adapted to be used in contexts such as distributed video coding and wireless sensor networks.

ACKNOWLEDGMENT

This project was funded in part by Iran Telecommunications Research Center (ITRC).

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