# Congestion Control of a Single Router With an Active Queue Management

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## Abstract

*Several works have shown the link between congestion control in communication networks and feedback control system. This paper is an extended version of [1] and proposes the design of an Active Queue Management (AQM) that ensures the congestion control stability. To this end, tools from control theory, and especially in a time delay systems framework, are considered. We aim at stabilizing the Transmission Control Protocol (TCP) as well as the queue length of the congested router. Furthermore, the control mechanism is then completed to deal with the stability issue under some non-responsive crossing traffic modeled as perturbation. Finally, a numerical example and simulations via the Network Simulator NS support our study.*
*Keywords: Active Queue Management, congestion control, control theory, time delay system, networks.*

## I. Introduction

Congestion control consists in adjusting data flow rates sent by end users into the network based on the network load status. Since the congestion avoidance algorithm of Jacobson [2], it has motivated a huge amount of work aiming at understanding the congestion phenomenon and achieving better performances in terms of *Quality of Service* (QoS). As a matter of fact, there has been a growing recognition that the network itself must participate in congestion control and ressource management [3], [4].

The AQM principle consists in dropping (or marking when ECN, *Explicit Congestion Notification* [5] option is enabled) some packets before buffer saturates. Hence, following the *Additive-Increase Multiplicative-Decrease* (AIMD) behavior of TCP, sources reduce their congestion window size avoiding then the full saturation of the router. Basically, AQM support TCP for congestion avoidance and feedback to the latter when traffic is too heavy. Indeed, an AQM drops/marks incoming packet with a given probability related to a congestion index (such as queue length or delays) allowing then a kind of control on the buffer occupancy at routers. Various mechanisms have been proposed in the literature such as Random Early Detection (RED) [6], Random Early Marking (REM) [7], Adaptive Virtual Queue (AVQ) [8] and many others [9]. Their performances have been evaluated in [9] and empirical studies have shown their effectiveness [4]. A study proposed by [10] have redesigned the AQMs using control theory and *PI* (Proportional and Integral) have been developed in order to address the packet dropping strategy issue. Then, using dynamical model of TCP developed by [11], many researches have been devoted to deal with congestion problem in a control theory framework (for example see [12], [13], [9], [14] and references therein). Nevertheless, most of these papers do not take into account the delay and ensure the stability in closed-loop for all possible delays which could be conservative in practice.

The study of congestion control in a time delay system framework is not new and has been successfully exploited (see for example [15], [16], [17], [18], [19]). The global stability analysis of TCP has been addressed in [20], [15] through the Lyapunov-Krasovskii theory. But no constructive algorithm is proposed to embed a control on routers. In [19], a delay dependent state feedback controller is provided by compensation of the delay with a memory feedback control. This latter methodology is interesting in theory but hardly suitable in practice. At last, all these papers deal with the congestion control stability considering constant delays requiring then restrictive assumptions.

In this paper, we focus in regulating buffer queue length of a congested router as well as rate at which TCP sources send data into the network. The proposed control has the objectives to ensure QoS, to avoid severe congestion and to maintain a prescribed *Round Trip Time* (RTT) with a low delay jitter. Stability of communications, guaranteed by an AQM, is proved through the Lyapunov method. It is worthy to note that unlike most of the studies in the literature, we take into account the time-varying nature of the RTT. The packet dropping strategy computed by the AQM is designed as a state feedback for time-varying delay systems based on a recently developed Lyapunov-Krasovskii functional [21]. Then, the methodology is extended to cope with additional non-responsive crossing traffics (like for example UDP and ICMP). Indeed, non-TCP traffics are not reactive to packet dropping and may affect the equilibrium of the communications. Hence, the second proposed control law stabilizes the TCP network (queue length and rates) to a desired equilibrium in spite of the presence of some non-responsive traffics, ensuring then a level of QoS.

The paper is organized as follows. The second part presents the model of a network supporting TCP and the time delay system representation. Section III is dedicated to the design of the AQM ensuring the stabilization of TCP. Section IV presents application of the exposed theory and simulation results using NS-2 [22].

*Notations:* For two symmetric matrices, $A$ and $B$, $A > (\geq) B$ means that $A-B$ is (semi-) positive definite. $A^T$ denotes the transpose of $A$. $1_n$ and $0_{m\times n}$ denote respectively the identity matrix of size $n$ and null matrix of size $m \times n$. If the context allows it, the dimensions of these matrices are often omitted. At last, for a given matrix $B \in \mathrm{R}^{m\times n}$ such that $rank(B) = r$, we define $B^{\perp} \in \mathrm{R}^{n\times(n-r)}$ the right orthogonal complement of $B$ by $BB^{\perp} = 0$.

## II. Problem statement

This second section is dedicated to the modeling of a network supporting TCP and the time delay system representation.

### A. The linearized fluid-flow model of TCP

We consider a network consisting of $N$ homogeneous TCP sources (i.e with the same propagation delay) connected to destination nodes through a single router (see Figure 1). The bottleneck link is shared by $N$ flows and TCP applies the well known congestion avoidance algorithm to cope with the phenomenon of congestion collapse [2]. Many studies have been dedicated to the modeling of TCP and its AIMD (*Additive-Increase Multiplicative-Decrease*) behavior (see [12], [13], [14] and references therein). We consider in this note the model (1) developed by [11] widely used for automatic control purpose [12]. This latter may not capture with high accuracy the dynamic behavior of TCP but its simplicity allows us to apply our methodology. Let us consider the following model

$$\left\{\begin{array}{rcl} \dot{W}(t) & = & \frac{1}{\frac{q(t)}{C}+T_p} - \frac{W(t)W(t-R(t))}{2\frac{q(t-R(t))}{C}+T_p} p(t-R(t)) \\ \dot{q}(t) & = & \frac{W(t)}{\frac{q(t)}{C}+T_p} N - C + d(t) \end{array}\right. \quad (1)$$

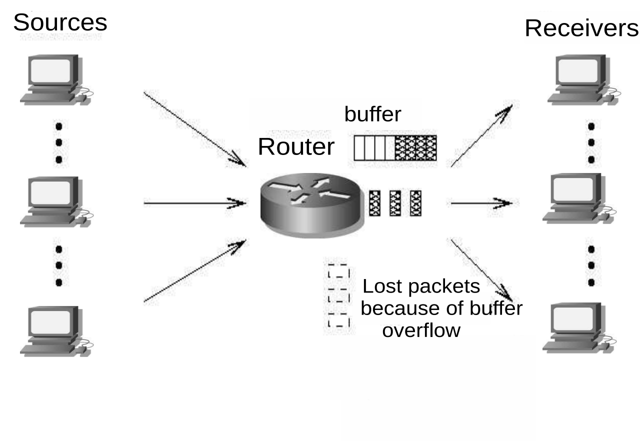


**Fig. 1. Network configuration**

where $W(t)$ is the TCP window size, $q(t)$ is the queue length of the router buffer, $R(t)$ is the round trip time (RTT) and can be expressed as $R(t) = q(t)/C + T_p$. $C$, $T_p$ and $N$ are parameters related to the network configuration and represent the transmission capacity of the router, the propagation delay and the number of TCP sessions respectively. The variable $p$ is the marking/dropping probability of a packet (depending whether the ECN option, is enabled, see [5]). In the mathematical model (1), we have introduced an additional signal $d(t)$ which models cross traffics through the router, filling the buffer. These traffics are not TCP based flows (not modeled in TCP dynamics) and can be viewed as perturbations since they are not reactive to packets dropping (for example, UDP traffic). Note that the model (1) is non linear and thus difficult to handle. Consequently, defining the set of equilibrium points $(W_0, q_0, p_0)$ by

$$\left\{\begin{array}{l} \dot{W} = 0 \;\Rightarrow\; W_0^2 p_0 = 2, \\ \dot{q} = 0 \;\Rightarrow\; W_0 = \frac{R_0 C}{N},\; R_0 = \frac{q_0}{C} + T_p, \end{array}\right. \quad (2)$$

TCP model can be linearized as follows

$$\left\{\begin{array}{l} \delta\dot{W}(t) = -\frac{N}{R_0^2 C}\Big(\delta W(t) + \delta W(t-R(t))\Big) \\ \qquad -\frac{1}{R_0^2 C}\Big(\delta q(t) - \delta q(t-R(t))\Big) - \frac{R_0 C^2}{2N^2}\delta p(t-R(t)) \\ \delta\dot{q}(t) = \frac{N}{R_0}\delta W(t) - \frac{1}{R_0}\delta q(t) + d(t) \end{array}\right. \quad (3)$$

where $\delta W \doteq W - W_0$, $\delta q \doteq q - q_0$ and $\delta p \doteq p - p_0$ are the signals variations around the operating point [10]. This approximation is valid as long as signals $\delta W(t)$, $\delta q(t)$ and $\delta p(t)$ remain small enough. Regulation is required in order to regulate quantities $\delta W(t)$ and $\delta q(t)$ around zero (and thus the congestion phenomenon), we have to control the dropping probability $\delta p(t)$. This probability is computed the help of an AQM, playing thus the role of a controller. In

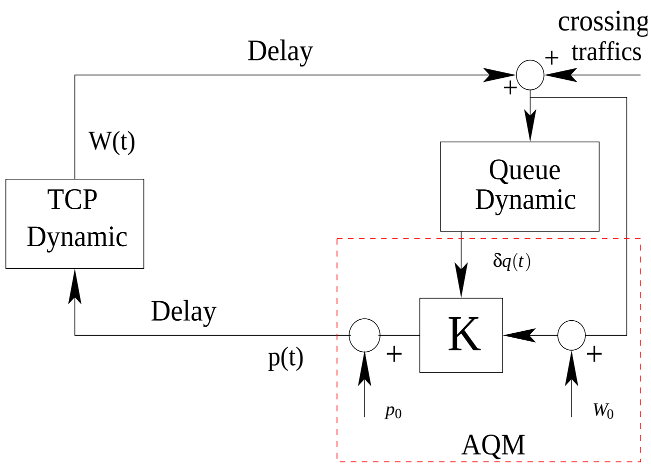


**Fig. 2. Design of an AQM as a state feedback**

this paper, this regulation problem is addressed in Section III with the design of a stabilizing state feedback for time-varying delay systems. Consequently, based on [23] and considering a state feedback, the queue management strategy of the drop probability will be expressed as (see Figure 2):

$$p(t) = p_0 + k_1 \delta W(t) + k_2 \delta q(t). \tag{4}$$

Scalars $k_1$ and $k_2$ are the components of the matrix gain $K$ which have to be designed to ensure the stability of the overall system.

## B. Time delay system model

The linearized fluid flow model of TCP (3) can be rewritten as a time-varying delay system of the general form:

$$\dot{x}(t) = Ax(t) + A_d x(t-R(t)) + Bu(t-R(t)) + B_d d(t) \tag{5}$$

with

$$A = \begin{bmatrix} -\frac{N}{R_0^2 C} & -\frac{1}{CR_0^2} \\ \frac{N}{R_0} & -\frac{1}{R_0} \end{bmatrix}, A_d = \begin{bmatrix} -\frac{N}{R_0^2 C} & \frac{1}{R_0^2 C} \\ 0 & 0 \end{bmatrix},$$
$$B = \begin{bmatrix} -\frac{C^2 R_0}{2N^2} \\ 0 \end{bmatrix}, B_d = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, x(t) = \begin{bmatrix} \delta W(t) \\ \delta q(t) \end{bmatrix}. \tag{6}$$

$x(t)$ is the state vector and represents the network variables status. A suitable control $u(t)$ (processed by the AQM) must be applied to system (5) in order to ensure a stable congestion control. Section 3 is devoted to the stability analysis of the interconnected system (system (5) + AQM). To this end, the Lyapunov Krasovskii method (see for example [24]) is used which is an extension of the traditional Lyapunov theory. It is an effective and practical method which provides LMI (Linear Matrix Inequalities, [25]) criteria easy to test.

# III. Stabilization: design of an AQM

In Section II, the model of TCP/AQM has been formulated in the general form of a time delay system. The stability of the congestion control requires the construction of a controller which regulates the buffer queue length as well as data flows. In this section, we are first going to present a delay dependent stability condition for time delay systems in general [21] (when $u(t)$ and $d(t)$ equal 0). Secondly, based on this criterion, a design method that provides a stabilizing state feedback is deduced. This control aims to ensure the convergence of $x(t)$ to 0 (thus the convergence of $W(t)$ and $q(t)$ to $W_0$ and $q_0$ respectively).

## A. Stability analysis of time delay systems

In this subsection, our goal is to derive a stability condition which takes into account an upperbound of the delay. The delay dependent case starts from a system asymptotically stable without delays and looks for the maximal delay that preserves stability. In this paper, we prove the stability property of (5) with the Lyapunov method which consists in looking for a positive function $V(x,t)$ such that its derivative $\dot{V}(x,t)$ along the trajectories of (5) is negative. This function $V(x,t)$ can be viewed as an energy function of the considered system which converges to zero proving then the stability of the system.

Usually, the method involves Lyapunov-Krasovskii functionals (see [24] and references therein), and more or less tight techniques to bound some cross terms. These choices of specific Lyapunov functionals and overbounding techniques are the origin of conservatism. In the present paper, we choose a recently developed Lyapunov-Krasovskii functional [21] which deals with the stability of time-varying delay systems and shows interesting results in terms of conservatism reduction. The key idea is to consider an extended state $z(t)$ (10) as it has been proposed in [26] in a robustness context. We make the following assumptions on the delay:

$$0 \leq h(t) \leq h_m \text{ and } |\dot{h}(t)| \leq r, \tag{7}$$

where $h_m$ and $r$ are upperbounds of the delay and the delay derivative respectively. Given a time delay system:

$$\dot{\varsigma}(t) = A\varsigma(t) + A_d \varsigma(t-h(t)) \tag{8}$$

where $\varsigma(t) \in \mathrm{R}^n$ is the state vector, $A, A_d \in \mathrm{R}^{n\times n}$ are known constant matrices. Differentiating the system (8), we get:

$$\ddot{\varsigma}(t) = A\dot{\varsigma}(t) + (1-\dot{h}(t))A_d \dot{\varsigma}(t-h(t)).$$

Consider now the artificially augmented system

$$\begin{cases} \dot{\varsigma}(t) = A\varsigma(t) + A_d\varsigma(t-h(t)) \\ \ddot{\varsigma}(t) = A\dot{\varsigma}(t) + (1-\dot{h}(t))A_d\dot{\varsigma}(t-h(t)) \end{cases} \quad (9)$$

composed of the original system (8) and its derivative. Introducing the augmented state

$$z(t) = \begin{bmatrix} \varsigma(t) \\ \dot{\varsigma}(t) \end{bmatrix} \quad (10)$$

and specifying the relationship between the two components of $z(t)$ with the equality $[1 \quad 0]\dot{z}(t) = [0 \quad 1]z(t)$, we get the new augmented system:

$$E\dot{z}(t) = \bar{A}z(t) + \bar{A}_d z(t-h(t)), \quad (11)$$

where

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \bar{A} = \begin{bmatrix} A & 0 \\ 0 & A \\ 0 & 1 \end{bmatrix},$$
$$\bar{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & (1-\dot{h}(t))A_d \\ 0 & 0 \end{bmatrix}. \quad (12)$$

Finally, we obtain a descriptor linear time delay and time varying system. To cope with the time-varying nature of system (11) a method consists in embedding the time varying parameters $h$ and $\dot{h}$ into an uncertain set, described by a polytopic set and employing quadratic stability framework (see [25] and [24]). The following Theorem is then proposed (see [21] for more details)

*Theorem 1:* Given scalars $h_m > 0$ and $r \geq 0$, the linear system (8) is asymptotically stable for any time-varying delay $h(t)$ satisfying (7) if there exists $2n \times 2n$ matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$ and $X \in \mathrm{R}^{8n\times 3n}$ such that the following LMI holds for $i = \{1,2\}$:

$$\Gamma^{(i)} + XS^{(i)} + S^{(i)^T}X^T < 0 \quad (13)$$

where $\Gamma^{(i)}$ and $S^{(i)}$ (defined in (18) and (19)) for $i = 1,2$ are the two vertices of $\Gamma(\dot{h}) \in \mathrm{R}^{8n\times 8n}$ ($S(\dot{h}) \in \mathrm{R}^{3n\times 8n}$ respectively), replacing the term $\dot{h}(t)$ by $r_i$. $r_i$, $i = \{1,2\}$ corresponding to the bounds of $\dot{h}(t)$: $r_1 = r$ and $r_2 = -r$.

**Proof :** We consider the following Lyapunov-Krasovskii functional associated with the augmented state vector $z(t)$:

$$\begin{aligned} V(z_t) = \; & z_t^T(0)Pz_t(0) + \int_{-h(t)}^{0} z_t^T(\theta)Q_1 z_t(\theta)d\theta \\ & + \int_{-h_m}^{0} z_t^T(\theta)Q_2 z_t(\theta)d\theta \\ & + \int_{t-h_m}^{t}\int_{\theta}^{t} \dot{z}^T(s)R\dot{z}(s)dsd\theta. \end{aligned} \quad (14)$$

Remark that since $P$, $Q_1$, $Q_2$, $R$ are positive definite, we can conclude that for some $\varepsilon > 0$, the Lyapunov-Krasovskii functional condition $V(x_t) \geq \varepsilon\|x_t(0)\|$ is satisfied [24]. The derivative along the trajectories of (11) leads to

$$\begin{aligned} \dot{V}(z_t) \;=\; & 2z^T(t)P\dot{z}(t) + z^T(t)Q_1 z(t) \\ & -(1-\dot{h}(t))z^T(t-h(t))Q_1 z(t-h(t)) \\ & +z^T(t)Q_2 z(t) - z^T(t-h_m)Q_2 z(t-h_m) \\ & +h_m\dot{z}^T(t)R\dot{z}(t) - \int_{t-h_m}^{t} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta. \end{aligned} \quad (15)$$

As noted in [27], the derivative of $\int_{t-h_m}^{t}\int_{\theta}^{t}\dot{z}^T(s)R\dot{z}(s)dsd\theta$ is often estimated as $h_m\dot{z}^T(t)R\dot{z}(t) - \int_{t-h(t)}^{t}\dot{z}^T(\theta)R\dot{z}(\theta)d\theta$ and the term $-\int_{t-h_m}^{t-h(t)}\dot{z}^T(\theta)R\dot{z}(\theta)d\theta$ is ignored, which may lead to considerable conservatism. Hence, the last term of (15) can be separated in two parts:

$$\begin{aligned} -\int_{t-h_m}^{t} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta \;=\; & -\int_{t-h_m}^{t-h(t)} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta \\ & -\int_{t-h(t)}^{t} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta. \end{aligned} \quad (16)$$

Using the Jensen's inequality [24], (16) can be bounded as follow:

$$\begin{aligned} & -\int_{t-h_m}^{t-h(t)} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta - \int_{t-h(t)}^{t} \dot{z}^T(\theta)R\dot{z}(\theta)d\theta \\ < \; & -v^T(t)\tfrac{R}{h_m-h(t)}v(t) - w^T(t)\tfrac{R}{h(t)}w(t) \\ < \; & -v^T(t)\tfrac{R}{h_m}v(t) - w^T(t)\tfrac{R}{h_m}w(t) \end{aligned}$$

with

$$\begin{aligned} v(t) &= z(t-h(t)) - z(t-h_m), \\ w(t) &= z(t) - z(t-h(t)). \end{aligned}$$

Therefore, we get $\dot{V}(z_t) < \psi^T(t)\Gamma(\dot{h})\psi(t)$ with

$$\psi(t) = \begin{bmatrix} \dot{z}(t) \\ z(t) \\ z(t-h(t)) \\ z(t-h_m) \end{bmatrix}. \quad (17)$$

$$\Gamma(\dot{h}) = \begin{bmatrix} h_m R & P & 0 & 0 \\ P & T & \frac{1}{h_m}R & 0 \\ 0 & \frac{1}{h_m}R & U & \frac{1}{h_m}R \\ 0 & 0 & \frac{1}{h_m}R & V \end{bmatrix} \quad (18)$$

and

$$\begin{aligned} T &= Q_1 + Q_2 - \tfrac{1}{h_m}R, \\ U &= -(1-\dot{h}(t))Q_1 - \tfrac{2}{h_m}R, \\ V &= -\tfrac{1}{h_m}R - Q_2. \end{aligned}$$

So, the system (11) is asymptotically stable if for all $\psi(t)$ such that $S(\dot{h})\psi(t)=0$ with

$$S(\dot{h}) = \begin{bmatrix} -E & \bar{A} & \bar{A}_d & 0 \end{bmatrix}, \quad (19)$$

the inequality $\psi(t)^T\Gamma(\dot{h})\psi(t) < 0$ holds. Using Finsler lemma [28], this is equivalent to

$$\Gamma(\dot{h}) + XS(\dot{h}) + S^T(\dot{h})X^T < 0. \quad (20)$$

At this stage, assume that $\dot{h}(t)$ is not precisely known but varies between a lower and upper bound, $\dot{h}(t) \in [-r,r]$. Since this uncertain parameter appears linearly in (20), the uncertain set can be described by a polytope [24]. The vertices of this set can be calculated by setting the parameter to either lower or upper limit. The inequality (20) can then be rewritten as follow:

$$\sum_{i=1}^{2}\alpha_i\Gamma^{(i)} + X\sum_{i=1}^{2}\alpha_i S^{(i)} + \sum_{i=1}^{2}\alpha_i S^{(i)^T}X^T < 0, \quad (21)$$

where $\alpha_i(t) \in [0,1]$, $\sum_{i=1}^{2}\alpha_i(t) = 1$ and $\Gamma^{(i)}$ ($S^{(i)}$), $i=1,2$ are the two vertices of the uncertain matrix $\Gamma(\dot{h})$ ($S(\dot{h})$ respectively) for $\dot{h}(t) \in [-r,r]$. Considering the quadratic stability framework [25], condition (21) is equivalent to

$$\Gamma^{(i)} + XS^{(i)} + S^{(i)^T}X^T < 0, \; \mathrm{i} = 1,2. \quad (22)$$

Thus, the inequality (20) has to be verified only on its vertices (22). Finally, the asymptotic stability of system (11) is guaranteed if the two LMI (22) are feasible at the same time. For any initial conditions, the whole state $z(t)$ converges asymptotically to zero. Its components $\varsigma(t)$ converge as well. The original system (8) is thus asymptotically stable.

*Remark 1:* Note that Theorem 1 provides a delay dependent stability condition. It means that if condition (13) holds for $h_{m_1}$ then it still holds for any $h_{m_2} \le h_{m_1}$ [21].

## B. A first result on synthesis

In the previous section, a general stability condition for time-varying delay systems has been introduced. We aim now at using this latter result for the considered issue, established in Section II. Applying the delayed state feedback (4) (AQM mechanism) on the system (5) (TCP+router), the resulting feedback system can be reduced to a system of the form (8). Hence, given the stability condition (13) and the interconnected system (5) combined with (4) (in this subsection, the disturbance is not taken into account $d(t)=0$), the following Theorem is obtained.

*Theorem 2:* Given scalars $h_m > 0$ and $r \ge$, if there exist symmetric positive definite matrices $P$, $R$, $Q_1$, $Q_2 \in \mathrm{R}^{2n\times 2n}$, a matrix $X \in \mathrm{R}^{8n\times 3n}$ and a matrix $K \in \mathrm{R}^{1\times n}$ such that

$$\Gamma^{(i)} + XS^{(i)} + S^{(i)^T}X^T < 0 \quad (23)$$

then, the system (5) (with $d(t)=0$) is stabilized by the control law $u(t) = Kx(t)$ for any time-varying delay R(t) satisfying $0 \le R(t) \le h_m$ and $\dot{R}(t) \le r$. $\Gamma^{(i)}$ and $S^{(i)}$ (defined in (18) and (25)) for $i=1,2$ are the two vertices of $\Gamma(\dot{R}) \in \mathrm{R}^{8n\times 8n}$ ($S(\dot{R}) \in \mathrm{R}^{3n\times 8n}$ respectively), replacing the term $\dot{R}(t)$ by $r_i$. $r_i$, $i=\{1,2\}$ corresponding to the bounds of $\dot{R}(t)$: $r_1 = r$ and $r_2 = -r$.

**Proof :** Consider system (5) with $d(t)=0$ and controlled by the state feedback (4), can be expressed as

$$\dot{x}(t) = Ax(t) + \check{A}_d x(t-R(t)), \quad (24)$$

where $\check{A}_d = A_d + BK$ and $A$, $A_d$ and $B$ are defined as (6). Then, Theorem 1 can be applied on the interconnected system (24). Following the same idea exposed in Section III-A Theorem 2 is derived considering now $S(\dot{R}(h))\xi(t) = 0$ where

$$S(\dot{R}(h)) = \begin{bmatrix} -E & \hat{A} & \hat{A}_d & 0 \end{bmatrix}, \quad (25)$$

with

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \hat{A} = \begin{bmatrix} A & 0 \\ 0 & A \\ 0 & 1 \end{bmatrix},$$

$$\hat{A}_d = \begin{bmatrix} A_d + BK & 0 \\ 0 & (1-\dot{R}(t))(A_d+BK) \\ 0 & 0 \end{bmatrix},$$

$$\xi(t) = \begin{bmatrix} \dot{z}(t) \\ z(t) \\ z(t-R(t)) \\ z(t-h_m) \end{bmatrix}.$$

Thus, the stability condition (23) of Theorem 2 enables the design of gains $k_1$ and $k_2$. This condition is formulated as a matrix inequality which can be systematically solved with an appropriate semi-definite programming solver in Matlab [29] and Yalmip [30].

*Remark 2:* Regarding the design problem, since $K$ is a decision variable condition (23) is bilinear with $X$ and a global optimal solution cannot be found. Nevertheless, the feasibility problem can still be tested to provide a solution by either using a BMI solver [31] or developping a relaxation algorithm based on LMI [32].

## C. State feedback with an integral action

The first proposed method, in Section III-B, for the design of an AQM ensures the stability of communications around an equilibrium point (2). However, this equilibrium may be perturbed when non-responsive crossing traffics are introduced. Indeed, these additional non-TCP (so non-modeled) flows fill up the buffer and the AQM may not control the congestion as expected. In order to cope with

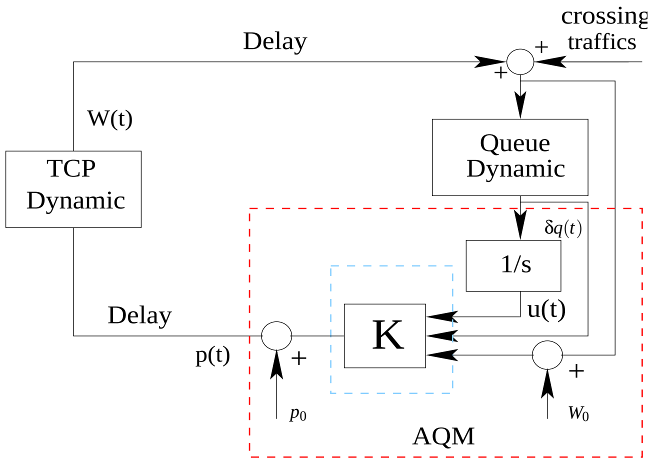


**Fig. 3. Design of an AQM as a dynamic state feedback**

this phenomenon, the AQM is completed with an integral action well known to be able to reject disturbances. The idea is to apply the same design methodology, exposed in Section III-B, over an augmented time delay system composed of the original system (5) and an integrator (see Figure 3). The augmented system has the following form

$$\dot{\eta} = \begin{bmatrix} A & & 0 \\ & & 0 \\ 0 & 1 & 0 \end{bmatrix} \eta(t) + \begin{bmatrix} A_d & & 0 \\ & & 0 \\ 0 & 0 & 0 \end{bmatrix} \eta(t-h) + \begin{bmatrix} B \\ 0 \end{bmatrix} \delta p(t-h) + \begin{bmatrix} B_d \\ 0 \end{bmatrix} d(t) \qquad (26)$$

with $\eta^T = [\delta W \quad \delta q \quad u]^T$ is the extended state variable. Then, the global control which correspond to our AQM, is a dynamic state feedback

$$\delta p(t) = K \begin{bmatrix} \delta W(t) \\ \delta q(t) \\ u(t) \end{bmatrix} = k_1 \delta W(t) + k_2 \delta q(t) + k_3 \int_0^t \delta q(t) dt. \qquad (27)$$

In our problem, non modeled crossing traffics $d(t)$ such as UDP based applications are introduced as exogenous signals (see Figures 2 and 3). The queue dynamic, the second equation of (3), is affected by this additional signals. Considering equations (27) and (3), we obtain the transfer function $T(s)$ from the disturbance $D(s)$ to the queue size (about the operating point) $\Delta Q(s)$:

$$T(s) = \frac{b(s)s}{(s + \frac{1}{R_0})sb(s) + c(s)}, \qquad (28)$$

with

$$a(s) = -\frac{R_0 C^2}{2N^2} e^{-hs},$$
$$b(s) = s + \frac{N}{R_0^2 C}(1 + e^{-hs}) + a(s)k_1$$
$$c(s) = \frac{N}{R_0} \left[ \frac{s}{R_0^2 C}(1 - e^{-hs}) + a(s)sk_2 + a(s)k_3 \right].$$

It can be easily shown that for a step type disturbance, the queue size still converges to its equilibrium. The proposed control law (27) is thus suitable to cope with the effects of non responsive traffics. Then, $k_1$, $k_2$ and $k_3$ are designed applying Theorem 2 to the augmented system (26).

### D. Estimation of the congestion window

In these last two parts, a state feedback synthesis has been performed for the congestion control of TCP flows and the management of the router buffer. So far we have considered that the whole state was accessible. However, although the congestion window can be measured in NS, it is not the case in reality. That's why, in this paper it is proposed to estimate this latter variable using the aggregate flow incoming to the router buffer. The sending rate of single TCP source can be approximated by congestion window size over the RTT. This latter approximation is valid as long as the model does not describe the communication at a finer time scale than few round trip time (see [12]). Consequently, the whole incoming rate observed by the router is $r(t) = NW(t)/R(t)$. The measure of the aggregate flow has already been proposed and successfully exploited in [8], [19] for the realization of the AVQ and a PID type AQM respectively. Moreover, other works have also developed tools that enable such measurements in anomaly detection framework (see for example [33], [34]). It is worth to note that queue-based AQMs like RED or PI can be assimilated as output feedbacks according to the queue length [35]. Conversely, AVQ can be viewed as an output feedback with respect to the aggregate flow, belonging thus to the rate-based AQM class. The proposed state feedbacks in Section III-B and III-C combine both informations and we expect thus to apply a relevant control on the communications.

## IV. NS-2 simulations

This section is devoted to elucidate the proposed methodology via an illustrative example and simulations with NS-2 [22]. Consider a network consisting of $N = 60$ communicating pairs through a congested router as illustrated on Figure 1. The transport protocol is TCP-New Reno without ECN marking and the propagation time is $T_p = 200$ ms. Since the link capacity $C = 3750$ packets/s (corresponds to a 15 Mb/s link with average packet size 500 bytes) at the router is shared among all users, there is congestion and the classical *drop tail* mechanism drops packets when buffer overflows (the maximal buffer size is set to 800 pkts). Hence, the queue length at the router shows large oscillations and reaches oftenly the buffer saturation (see Figure 4). Now, we aim at regulating the

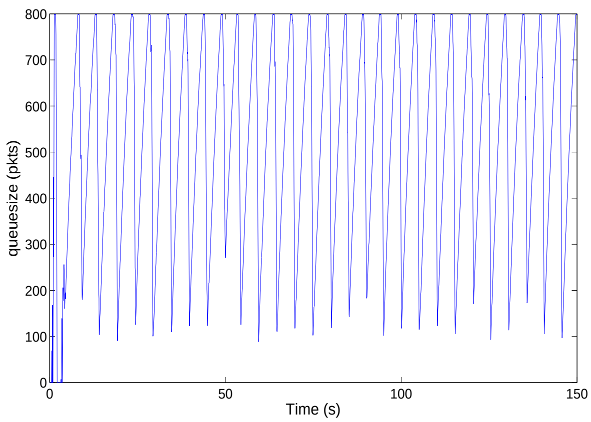


**Fig. 4. Time evolution of the queue length, dropping strategy is the traditional mechanism:** *DropTail*

queue length at the router to a desired level: $q_0 = 175$ packets. To this end, an AQM is embedded into the router in order to control the congestion phenomenon. Different AQMs have been simulated and their configuration parameters are shown in table I. Regarding to the design of the proposed AQM, given the network parameters ($N$, $C$ and $T_p$) and the specification on $q_0$, the equilibrium point (2) can be derived: $W_0 = 15$ packets, $p_0 = 0.008$ and $R_0 = 0.246$ seconds. Secondly, according to the design criteria presented in Section III, the state feedback matrices

$$K_{SF} = 10^{-3}\begin{bmatrix} 0.6103 \\ 0.0209 \end{bmatrix}$$
$$K_{SFI} = 10^{-4}\begin{bmatrix} 26.28 \\ 0.303 \\ 0.464 \end{bmatrix} \quad (29)$$

are calculated based on control laws (4) and (27) respectively.

On Figure 5, the time evolutions of the queue length for different AQMs are shown. In this first simulation, RED, REM, PI and our state feedback (SF: control law (4)) have been tested for congestion control under long-lived TCP flow such that ftp connections. It can be observed that our SF is able to regulate efficiently the buffer:

| | |
|---|---|
| RED | $min_{th}$=150,$max_{th}$=700, $w_Q$=13.3e-06,$max_p$=0.1, $f_s$=160Hz |
| REM | $\gamma$=0.001, $\Phi$=1.001,$q_{ref}$=175pkt |
| PI | a=1.822e-05,b=1.816e-05, $q_{ref}$=175pkt, $f_s$=160Hz |
| SF | gains $K$ (29), equilibrium point (2) |

**TABLE I. Adjustment of parameter setting of each AQM**

- the queue length reaches the steady state fastly,
- it maintains the size close to its equilibrium value $q_0$, ensuring then very low oscillations,
- this control which guarantees a stable queue length, allows to keep a queueing delay with very little variations, thus low delay jitter (see Figure 6).

Then, if non-responsive cross traffics are introduced, the queue is affected and may disturb the AQM control behavior. Considering the previous controller SF ($K_{SF}$ in (29)), we have carried out a new simulation (Figure 7) introducing additional traffics composed of 7 sources (CBR applications over UDP protocol) sending flows of 1Mbytes/s between $t = 50s$ and $t = 100s$. It appears that the queue length is still stable but not regulated at the desired level $q_0$ anymore. That's why, in Section III the first control law (4) has been completed with an integral action to tackle the steady state error in presence of non-responsive CBR traffics. Then, the same simulation is performed using the AQM $K_{SFI}$ (29) from the control law (27). Figure 8 shows the different results for each AQM. In addition, table II summarizes the benefits of the $K_{SFI}$ AQMs providing few statistical characteristics. These characteristics are mean, standard deviation ($Std$) and the square of the variation coefficient ($CV2 = (Std/mean)^2$). This latter calculation assess the relative dispersion of the queue length around its mean. The mean points out the control precision and the standard deviation shows the ability of the AQM to keep the queue size close to its equilibrium. In table II, we can observe that $K_{SFI}$ maintains a very good control on the buffer queue during the whole simulation. Hence, it enable to ensure QoS in terms of RTT (set to a desired value) and delay jitter (see Figure 9). Although PI reject the perturbation quite fast, extensive fluctuations appear during the steady state. Note that RED allows also a good regulation at the steady state but its response time is quite slow. Moreover, this latter is well known to be difficult to tune [36], [37] whereas the proposed AQM can be easily and systematically derived solving the inequality of Theorem 2 with an appropriate semi-definite programming solver (as penbmi [31], sedumi [38] or lmilab in Matlab [29]).

| AQM | RED | REM | PI | $K_{SFI}$ | Period |
|---|---|---|---|---|---|
| Mean | 235.7 | 177.4 | 178.8 | 176.7 | before additional traffic |
| Std | 112.4 | 144.74 | 89.83 | 71.19 | |
| CV2 | 0.227 | 0.665 | 0.252 | 0.162 | |
| AQM | RED | REM | PI | $K_{SFI}$ | Period |
| Mean | 270.3 | 212.4 | 199.4 | 178.3 | during additional traffic |
| Std | 57.39 | 101.5 | 79.05 | 40.42 | |
| CV2 | 0.045 | 0.228 | 0.1972 | 0.051 | |
| AQM | RED | REM | PI | $K_{SFI}$ | Period |
| Mean | 201.4 | 168.4 | 154.1 | 177.8 | after additional traffic |
| Std | 22.24 | 101.02 | 64.95 | 36.64 | |
| CV2 | 0.012 | 0.360 | 0.178 | 0.042 | |

**TABLE II. Statistical characteristics for different AQMs (units are pkts) at different periods (before, during and after the introduction of CBR traffic)**

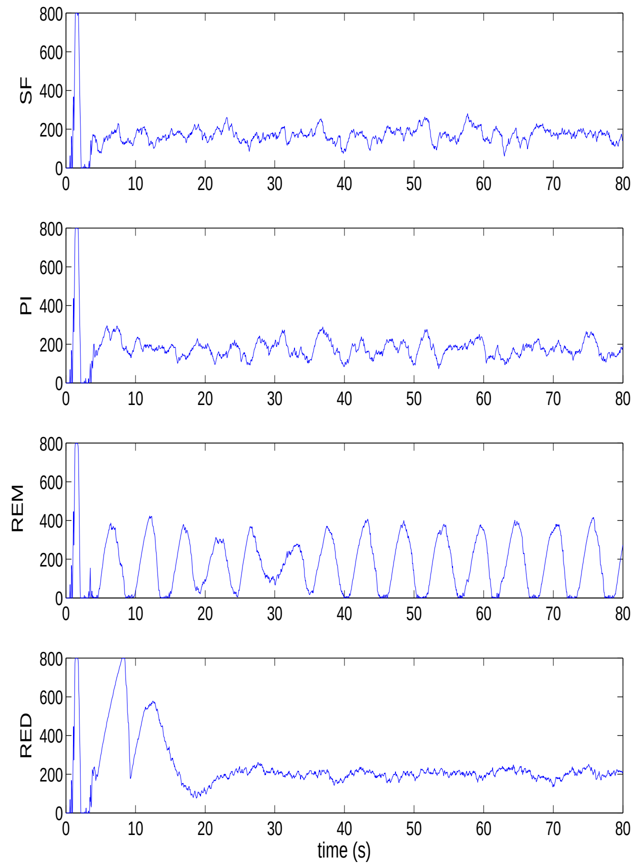


**Fig. 5. Time evolution of the queue length,** $AQM = \{K_{SF}, PI, REM, RED\}$

## V. Conclusion and Future Work

In this work, we have proposed the design of an AQM for the congestion control in communications networks. The developed AQM has been constructed using a state feedback control law. An integral action has been added to reject the steady state error in spite of disturbance, $d(t)$ (non-responsive crossing traffic). Finally, the AQM has been validated using NS simulator. Future works consist in the improvement of control laws and extension to larger networks (with TCP sources at varying hop-distances and using short-term/long-term flows). Validation on emulation platform (experimental part) will be also studied.

## References

[1] Y. Ariba, Y. Labit, and F. Gouaisbaut, "Design and performance evaluation of a state-space based aqm," in *IARIA International Conference on Communication Theory, Reliability, and Quality of Service (CTRQ 2008)*, Jul. 2008, pp. 89–94.

[2] V. Jacobson, "Congestion avoidance and control," in *ACM SIGCOMM*, Stanford, CA, Aug. 1988, pp. 314–329.

[3] B. Braden, D. Clark, and J. Crowcroft, "Recommendations on queue management and congestion avoidance in the internet," RFC 2309, Apr. 1998.

[4] L. Le, J. Aikat, K. Jeffay, and F. Donelson Smith, "The effects of active queue management on web performance," in *ACM SIGCOMM*, Aug. 2003, pp. 265–276.

[5] K. K. Ramakrishnan and S. Floyd, "A proposal to add explicit congestion notification (ecn) to ip," RFC 2481, Jan. 1999.

[6] S. Floyd and V. Jacobson, "Random early detection gateways for congestion avoidance," *IEEE/ACM Transactions on Networking*, vol. 1, pp. 397–413, Aug. 1993.

[7] S. Athuraliya, D. Lapsley, and S. Low, "An enhanced random early marking algorithm for internet flow control," in *IEEE INFOCOM*, Dec. 2000, pp. 1425–1434.

[8] S. Kunniyur and R. Srikant, "Analysis and design of an adaptive virtual queue (avq) algorithm for active queue management," in *ACM SIGCOMM*, San Diego, CA, USA, aug 2001, pp. 123–134.

[9] S. Ryu, C. Rump, and C. Qiao, "Advances in active queue management (aqm) based tcp congestion control," *Telecommunication Systems*, vol. 4, pp. 317–351, 2004.

[10] C. V. Hollot, V. Misra, D. Towsley, and W. Gong, "Analysis and design of controllers for aqm routers supporting tcp flows," *IEEE Trans. on Automat. Control*, vol. 47, pp. 945–959, Jun. 2002.

[11] V. Misra, W. Gong, and D. Towsley, "Fluid-based analysis of a network of aqm routers supporting tcp flows with an application to red," in *ACM SIGCOMM*, Aug. 2000, pp. 151–160.

[12] H. S. Low, F. Paganini, and J. Doyle, *Internet Congestion Control*. IEEE Control Systems Magazine, Feb 2002, vol. 22, pp. 28–43.

[13] R. Srikant, *The Mathematics of Internet Congestion Control*. Birkhauser, 2004.

[14] S. Tarbouriech, C. T. Abdallah, and J. Chiasson, *Advances in communication Control Networks*. Springer, 2005.

[15] A. Papachristodoulou, "Global stability of a tcp/aqm protocol for arbitrary networks with delay," in *IEEE CDC 2004*, Dec. 2004, pp. 1029–1034.

[16] C. Chen, Y. Hung, T. Liao, and J. Yan, "Design of robust active queue management controllers for a class of tcp communication networks," *Information Sciences.*, vol. 177, no. 19, pp. 4059–4071, 2007.

[17] S. Manfredi, M. di Bernardo, and F. Garofalo, "Robust output

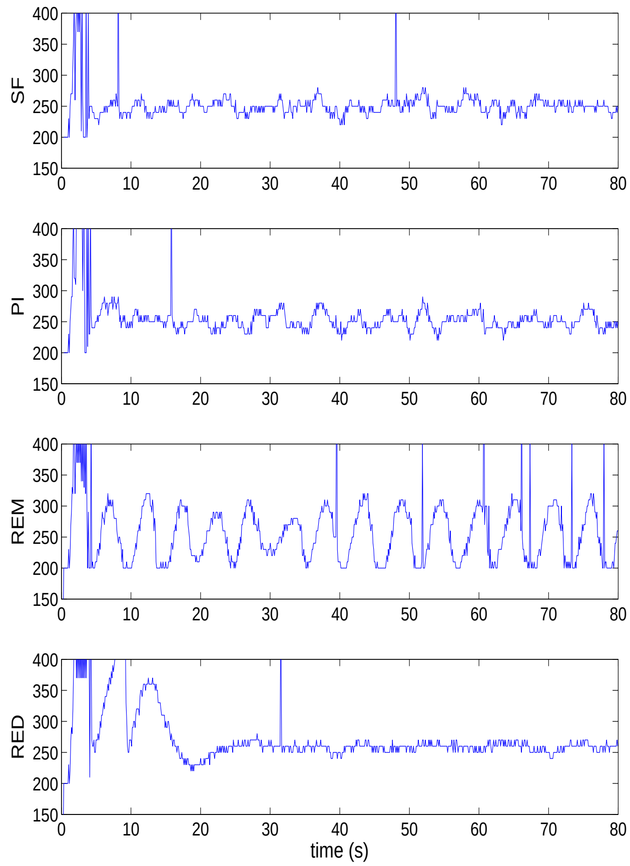


**Fig. 6. Time evolution of the RTT,** $AQM = \{K_{SF}, PI, REM, RED\}$

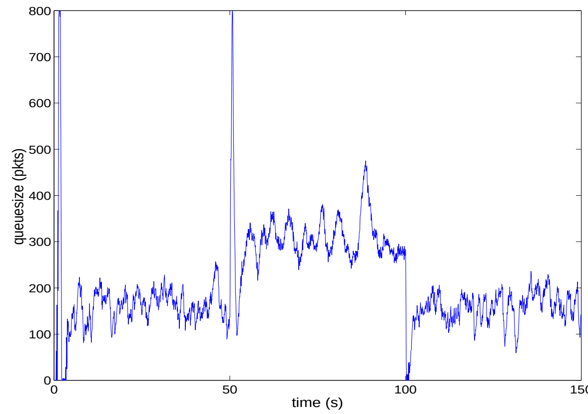


**Fig. 7. Time evolution of the queue length,** $AQM = K_{SF}$ **under UDP crossing traffic**

feedback active queue management control in tcp networks," in *IEEE Conference on Decision and Control*, Dec. 2004, pp. 1004–1009.

[18] D. Wang and C. V. Hollot, "Robust analysis and design of controllers for a single tcp flow," in *IEEE International Conference on Communication Technology (ICCT)*, vol. 1, Apr. 2003, pp. 276–280.

[19] K. B. Kim, "Design of feedback controls supporting tcp based on the state space approach," in *IEEE Trans. on Automat. Control*, vol. 51 (7), Jul. 2006.

[20] W. Michiels, D. Melchior-Aguilar, and S. Niculescu, "Stability analysis of some classes of tcp/aqm networks," in *International Journal of Control*, vol. 79 (9), Sep. 2006, pp. 1136–1144.

[21] Y. Ariba and F. Gouaisbaut, "Delay-dependent stability analysis of linear systems with time-varying delay," in *IEEE Conference on Decision and Control*, Dec. 2007, pp. 2053–2058.

[22] K. Fall and K. Varadhan, "The ns manual," notes and documentation on the software ns2-simulator, 2002, uRL: www.isi.edu/nsnam/ns/.

[23] C. V. Hollot, V. Misra, D. Towsley, and W. Gong, "On designing improved controllers for aqm routers supporting tcp flows," in *IEEE INFOCOM*, vol. 3, Apr. 2001, pp. 1726–1734.

[24] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*. Birkhäuser Boston, 2003, control engineering.

[25] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. Philadelphia, USA: SIAM, 1994, in Studies in Applied Mathematics, vol.15.

[26] Y. Ebihara, D. Peaucelle, D. Arzelier, and T. Hagiwara, "Robust performance analysis of linear time-invariant uncertain systems by taking higher-order time-derivatives of the states," in $44^{th}$ *IEEE Conference on Decision and Control and the European Control Conference*, Seville, Spain, Dec. 2005.

[27] Y. He, Q. G. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with time-varying delay," *Automatica*, vol. 43, pp. 371–376, 2007.

[28] R. Skelton, T. Iwazaki, and K. Grigoriadis, *A unified algebric approach to linear control design*. Taylor and Francis series in systems and control, 1998.

[29] Mathworks, "Matlab - the language of technical computing," http://www.mathworks.fr/products/matlab/.

[30] J. Lfberg, "Yalmip : A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004. [Online]. Available: http://control.ee.ethz.ch/ joloef/yalmip.php

[31] M. Kocvara and M. Stingl, "bilinear matrix inequalities: Penbmi," PENOPT GbR, http://www.penopt.com/.

[32] Y. Labit, Y. Ariba, and F. Gouaisbaut, "Design of lyapunov based controllers as tcp aqm," in *2nd IEEE Workshop on Feedback control implementation and design in computing systems and networks (FeBID'07)*, Munich, Germany, May 2007, pp. 45–50.

[33] P. Barford and D. Plonka, "Characteristics of network traffic flow anomalies," Nov. 2001.

[34] S. S. Kim and A. L. N. Reddy, "Netviewer: a network traffic visualization and analysis tool," in *LISA'05: Proceedings of the 19th conference on Large Installation System Administration Conference*. USENIX Association, 2005, pp. 185–196.

[35] D. Supratim and R. Srikant, "Rate-based versus queue-based models of congestion control," *IEEE Trans. on Automat. Control*, vol. 51, pp. 606–619, Apr. 2006.

[36] M. Christiansen, K. Jeffay, D. Ott, and F. Smith, "Tuning red for web traffic," in *ACM/SIGCOM*, 2000, pp. 139–150.

[37] T. Bonald, M. May, and J.-C. Bolot, "Analytic evaluation of red performance," in *IEEE INFOCOM*, vol. 3, Mar. 2000, pp. 1415–1424.

[38] J. Sturm, "Using sedumi 1.02, a matlab toolbox for optimization over symmetric cones," Optimization Methods and Software 625-653, Special issue on Interior Point Methods, 1999, http://sedumi.mcmaster.ca/.

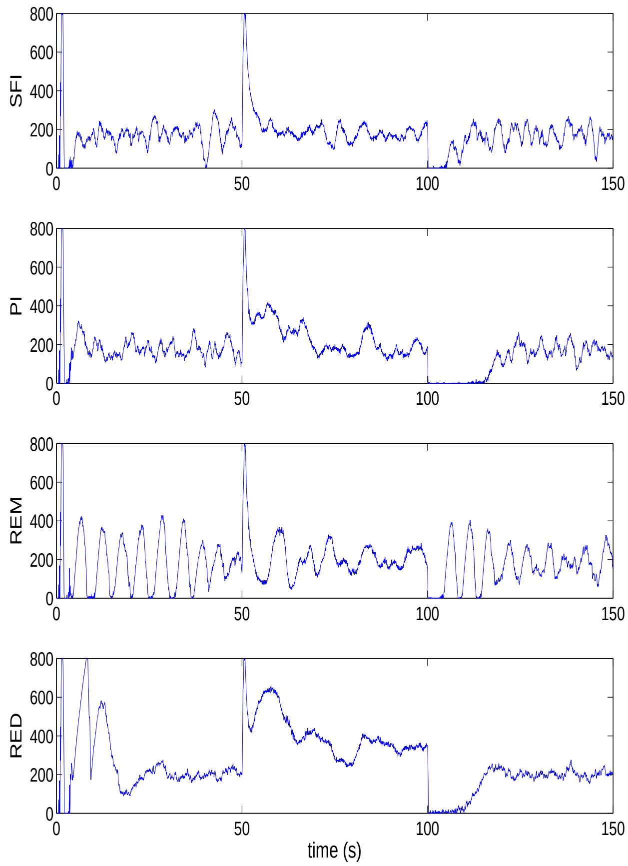


**Fig. 8. Time evolution of the queue length,** $AQM = \{K_{SFI}, PI, REM, RED\}$ **under UDP crossing traffic**

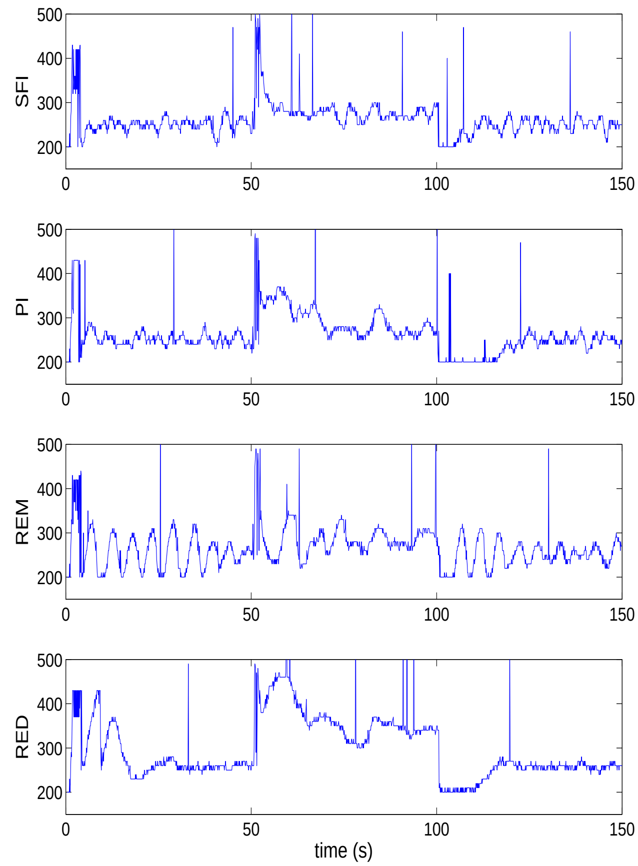


**Fig. 9. Time evolution of the RTT,** $AQM = \{K_{SFI}, PI, REM, RED\}$ **under UDP crossing traffic**