A Novel Unambiguous BOC Acquisition Scheme for Global Navigation Satellite Systems

Youngseek Lee and Seokho Yoon*
College of Information and Communication Engineering
Sungkyunkwan University
Suwon, Korea
e-mail: {fortrtwo and syn}@skku.edu

Abstract—This paper addresses the problem of ambiguity in the acquisition of binary offset carrier (BOC) signals employed in global navigation satellite systems, which is caused by the multiple side-peaks of the BOC autocorrelation function. We first observe that the side-peaks arise due to the fact that the BOC autocorrelation is made up of the sum of the sub-correlations shaped irregularly, and then, propose a novel unambiguous BOC acquisition scheme based on a recombination of the sub-correlations. The proposed scheme is demonstrated to remove the side-peaks completely for any type of BOC signals and to provide a performance improvement over the conventional scheme in terms of the receiver operating characteristic and mean acquisition time.

Keywords—Acquisition; ambiguity problem; binary offset carrier

I. INTRODUCTION

Recently, new global navigation satellite systems (GNSSs) such as Galileo and global positioning system (GPS) modernization are being developed to satisfy the increasing demand for GNSS-based services such as location-based service (LBS) and emergency rescue service (ERS) and complement the existing GNSSs such as GPS [1]-[3]. Currently, new GNSSs are designed to use the same frequency band of the existing GNSSs: for example, the E1 and E5 bands of Galileo are overlapped with the L1 and L5 bands of GPS, respectively [1], [4]. Thus, if a Galileo signal is modulated by a conventional scheme such as phase shift keying (PSK) used in GPS, it would suffer from co-channel interference. To overcome these problems, binary offset carrier (BOC) modulation has been proposed, where a high degree of spectral separation between the BOC-modulated signals and the others is achieved by shifting the signal energy from the band center [5]. The BOC signal is generated through the product of a spreading pseudo random noise (PRN) code and a sine-phased or cosine-phased square wave sub-carrier, and denoted by BOC_{sinc}(kn,n) or BOC_{cos}(kn,n) depending on which of the sine-phased and cosine-phased sub-carriers are used, where k and n are the ratios of the PRN code chip period to the sub-carrier period and the PRN code chip rate to 1.023 MHz, respectively [4], [6]. For larger values of k, more separated spectrums are obtained, reducing the co-channel interference more effectively. However, the BOC signal has multiple side-peaks on both sides of the main-peak of its autocorrelation function, causing an ambiguity problem in the BOC signal synchronization. Moreover, the number of side-peaks increases as the value of k becomes larger. The synchronization process of BOC signals consists of two stages: Acquisition and Tracking. The acquisition process aligns the locally generated BOC signal with the received signal within a chip duration, and then, the tracking process performs fine synchronization and maintains the synchronized lock point. To solve the ambiguity problem, several unambiguous acquisition schemes [7]-[11] and tracking schemes [12]-[14] have been proposed. In [7]-[9], sideband filtering was used to deal with the ambiguity problem in the BOC signal acquisition; however, these schemes destroy the sharpness of the main-peak of the BOC autocorrelation function, degrading the BOC signal tracking performance severely. In [10], an interesting unambiguous acquisition scheme that maintains the sharp main-peak of the BOC autocorrelation function was proposed combining the correlation between the BOC and PRN signals with the BOC autocorrelation; however, this scheme is applicable to only BOC_{sinc}(kn,n) signals. In [11], an extended unambiguous acquisition scheme including the scheme in [10] as a special case was proposed. This scheme is applicable to generic BOC_{sinc}(kn,n) signals; however, its extension to generic BOC_{cos}(kn,n) signals is not straightforward since it uses the designed local signals for BOC_{sinc}(kn,n) signals. In [12]-[14], on the other hand, unambiguous tracking schemes have been proposed by employing side-peak cancellation techniques in a delay lock loop, focusing on the tracking process of the BOC signals, and they are applicable to BOC_{sinc}(kn,n) and BOC_{cos}(kn,n) signals. In this paper, we focus on acquisition of the BOC signals.

In this paper, a novel unambiguous acquisition scheme applicable to both BOC_{sinc}(kn,n) and BOC_{cos}(kn,n) signals is proposed based on a recombination of the sub-correlations making up the BOC autocorrelation, which is found to remove the side-peaks of the BOC autocorrelation completely, while keeping the sharp shape of the main-peak, and also, to offer a performance improvement over the scheme in [11] in terms of the correlation function, receiver operating characteristic (ROC) curves (which is the probability of detection P_{D}), and mean acquisition time (MAT) (which is the time that elapses prior to acquisition on the average
and the ultimate performance metric of interest for signal acquisition).

II. ANALYSIS OF BOC AUTOCORRELATION FUNCTION

The BOC signal \( b(t) \) can be expressed as:

\[
b(t) = \sqrt{P} \sum_{i=-\infty}^{\infty} c_i p_c(t-iT_c) d_{i[T_c,T]}(t) s(t),
\]

where \( P \) is the signal power, \( c_i \in \{-1,1\} \) is the \( i \)th chip of a PRN code with a period of \( T_c \), \( T_c \) is the PRN code chip period, \( p_c(t) \) is the PRN code waveform defined as a unit rectangular pulse over \([0,T_c)\), \( d_{i[T_c,T]}(t) \) is the navigation data, where \( d_i(t) \) is the \( x \)th navigation data and \([x]\) is the largest integer not larger than \( x \), and

\[
s(t) = \begin{cases} 
\sum_{k=0}^{2k-1} (-1)^k p_c(t-kT_c-uT_c), & \text{for } \text{BOC}_{2n}(k,n), \\
\sum_{k=0}^{4k-1} (-1)^{\frac{k}{2}} p_c(t-k\frac{T_c}{2}-\frac{uT_c}{2}), & \text{for } \text{BOC}_{2n+1}(k,n)
\end{cases}
\]

is the square wave sub-carrier, where \( T_c \) is the sub-carrier pulse duration of \( T_c / 2k = 1/(2kn \times 1.023 \text{ MHz}) \), \( p_c(t) \) is the unit rectangular sub-carrier pulse waveform over \([0,T_c)\), and \([x]\) is the smallest integer not less than \( x \). In this paper, focusing on the problem of ambiguity due to side-peaks, we assume that there is a pilot channel for acquisition [15] so that no data modulation is present during acquisition (i.e., \( d_{i[T_c,T]}(t)=1 \) for all \( i \)), and do not consider the effect of the secondary code. Then, considering that the PRN code period \( T \) is generally much larger than the PRN code chip period \( T_c \) and the out-of-phase autocorrelation of a PRN code is designed to be as low as possible for easy signal acquisition, we can obtain the correlation (normalized to the signal power) between the received and locally generated BOC signals as [16]:

\[
R_{2n}^R(\tau) = \frac{1}{PT} \int_0^T (b(t-\tau) + w(t))b(t)dt \\
\approx \sum_{k=0}^{2k-1} (-1)^{\frac{k}{2}} \Lambda_{2n}(T_c - T) + w_{2n}^w
\]

for BOC\(_{2n}(k,n)\) and

\[
R_{2n+1}^R(\tau) \approx \sum_{k=0}^{4k-1} (-1)^{\frac{k}{2}} \Lambda_{2n+1}(T_c - T) + w_{2n+1}^w
\]

for BOC\(_{2n+1}(k,n)\), where \( \tau \) is the phase difference between the received and locally generated BOC signals, \( w(t) \) is the additive white Gaussian noise (AWGN) process with mean zero and one-sided noise power spectral density \( N_0 \),

\[
w_{2n}^w = \frac{1}{\sqrt{2PT}} \int_0^T \int_0^{\infty} (-1)^{\frac{k}{2}} c_i p_c(t-iT_c-uT_c)w(t)dt, \\
w_{2n+1}^w = \frac{1}{\sqrt{2PT}} \int_0^T \int_0^{\infty} (-1)^{\frac{k}{2}} c_i p_c(t-i\frac{T_c}{2}-\frac{uT_c}{2})w(t)dt,
\]

\[
\Lambda_{2n}(\tau) = \begin{cases} 
|x|, & |x| \leq x, \\
0, & \text{otherwise}
\end{cases}
\]

is the triangular function of height \( x \) and area \( x^2 \). Denoting the terms \( \sum_{k=0}^{2k-1} (-1)^{\frac{k}{2}} \Lambda_{2n}(T_c - T) + w_{2n}^w \) and \( \sum_{k=0}^{4k-1} (-1)^{\frac{k}{2}} \Lambda_{2n+1}(T_c - T) + w_{2n+1}^w \) in (3) and (4) by \( R_{2n}^{wR}(\tau) \) and \( R_{2n+1}^{wR}(\tau) \), respectively, we can re-write \( R_{2n}^R(\tau) \) and \( R_{2n+1}^R(\tau) \) as:

\[
R_{2n}^{wR}(\tau) = \frac{1}{T_c} \sum_{k=0}^{4k-1} (-1)^{\frac{k}{2}} \Lambda_{2n}(T_c - T) + w_{2n}^w \\
= \sum_{k=0}^{2k-1} (-1)^{\frac{k}{2}} \Lambda_{2n}(T_c - T) + w_{2n}^w
\]
and similarly,

$$R_{\text{in}}^{k,\omega}(\tau) = \sum_{\nu=-\infty}^{\infty} \frac{1}{\nu T_s} R_{\text{in}}^{\nu,\omega}(\tau),$$

(7)

where $r(t) = b(t - \tau) + w(t)$ and $L$ is a correlation length and would be generally limited to be equal to or less than the PRN code period (normalized to $T_s$) due to some constraints such as the frequency error, data modulation, and secondary code. From (6) and (7), we can see that $R_{\text{in}}^{k,\omega}(\tau)$ and $R_{\text{in}}^{k,\omega}(\tau)$ are sub-correlations making up the correlations (3) and (4), respectively, and which are shown for $k = 1$ and $k = 2$ in the absence of noise in Fig. 1. From the figure, we can see that the main-peaks of the sub-correlations are coherently combined through the summation of the sub-correlations, thus forming the sharp main-peak of the BOC autocorrelation, and on the other hand, the sub-peaks of the sub-correlations are irregularly spread around the main-peaks, and thus, the summation of the sub-correlations results in the multiple side-peaks of the BOC autocorrelation. Another important observation is that the number of the side-peaks increases as $k$ increases. From this observation, it is expected that the accuracy performance is degraded as $k$ increases.

In the next section, we propose a novel unambiguous acquisition scheme, removing the side-peaks completely through a recombination of the sub-correlations.

III. PROPOSED UNAMBIGUOUS ACQUISITION SCHEME

From Fig. 1, we can clearly observe that $R_{\text{in}}^{k,0}(\tau)$ and $R_{\text{in}}^{k,2\omega-1}(\tau)$ and $R_{\text{in}}^{k,4\omega-1}(\tau)$ are symmetric with respect to $\tau = 0$ and have only one single overlapped peak at $\tau = 0$ for BOC$_{\text{in}}(kn,n)$ and BOC$_{\text{in}}(kn,n)$, respectively. Thus, if the two sub-correlations are summed, a main-peak with a larger magnitude (than that of the main-peak of a sub-correlation) is obtained without increasing the magnitudes of the side-peaks, and on the other hand, the difference between the two sub-correlations yields side-peaks only, whose magnitudes and positions are the same as those of the side-peaks in the sum of the two sub-correlations. Thus, the difference between the two sub-correlations might be used to remove the side-peaks in the sum of the two sub-correlations, leaving only the main-peak. This observation is the key motivation of the proposed scheme.

Since the side-peaks in the sum and difference of the two sub-correlations are out-of-phase and in-phase at $\tau < 0$ and $\tau > 0$, respectively, however, we cannot remove the side-peaks in the sum of the two sub-correlations completely through the subtraction between the sum and difference of the two sub-correlations. To align the phases of the side-peaks in the sum and difference of the two sub-correlations, thus, we use the sum of the absolute values of the two sub-correlations, obtaining the side-peaks with the same slopes as those of the side-peaks in the absolute difference of the two sub-correlations. Fig. 2 shows that the subtraction of the absolute difference of the two sub-correlations from the sum of the absolute values of the two sub-correlations yields an unambiguous correlation function with a single main-peak and no side-peak.

Since the unambiguous correlation function is generated with several absolute operations, it may suffer from undesired noise enhancement. To alleviate the noise enhancement while maintaining the signal part (i.e., the main-peak), we first multiply the unambiguous correlation function with each of the sub-correlations, and then, sum the product results together. Since the noise random variables $\{w_{\text{in}}^{k,0}\}_{k=0}^{2\omega-1} \{w_{\text{in}}^{k,2\omega-1}\}_{k=0}^{2\omega-1}$ in $\{R_{\text{in}}^{k,0}(\tau)\}_{k=0}^{2\omega-1} \{R_{\text{in}}^{k,2\omega-1}(\tau)\}_{k=0}^{2\omega-1}$ are independent from each other under the AWGN-limited satellite environment, the product sum of $\{R_{\text{in}}^{k,0}(\tau)\}_{k=0}^{2\omega-1} \{R_{\text{in}}^{k,2\omega-1}(\tau)\}_{k=0}^{2\omega-1}$ will average out the noise, and moreover, the main-peak magnitude of the unambiguous correlation function can be maintained after the product sum, since all sub-correlations have an equal magnitude of $1 / (2k)$ and $1 / (4k)$, the inverse of the number of the sub-correlations, for BOC$_{\text{in}}(kn,n)$ and BOC$_{\text{in}}(kn,n)$, respectively.

From the above discussions, the proposed unambiguous correlation function can be expressed as:

$$R_{\text{in}}^{k,\text{proposed}}(\tau) = \sum_{\nu=0}^{2\omega-1} R_{\text{in}}^{k,\nu,\omega}(\tau) \left| R_{\text{in}}^{k,\nu,\omega}(\tau) + R_{\text{in}}^{k,2\omega-1,\nu}(\tau) \right|$$

$$- \left| R_{\text{in}}^{k,\nu,\omega}(\tau) - R_{\text{in}}^{k,2\omega-1,\nu}(\tau) \right|$$

for BOC$_{\text{in}}(kn,n)$ and

$$R_{\text{in}}^{k,\text{proposed}}(\tau) = \sum_{\nu=0}^{2\omega-1} R_{\text{in}}^{k,\nu,\omega}(\tau) \left| R_{\text{in}}^{k,\nu,\omega}(\tau) + R_{\text{in}}^{k,4\omega-1,\nu}(\tau) \right|$$

$$- \left| R_{\text{in}}^{k,\nu,\omega}(\tau) - R_{\text{in}}^{k,4\omega-1,\nu}(\tau) \right|$$

for BOC$_{\text{in}}(kn,n)$.
for $\text{BOC}_{\text{cos}}(kn,n)$.

Fig. 3 shows the baseband receiver structure of the proposed unambiguous acquisition scheme for $\text{BOC}_{\text{sin}}(kn,n)$ and $\text{BOC}_{\text{cos}}(kn,n)$. The received BOC signal $r(t)$ is first multiplied with the locally generated PRN code and sub-carrier, and then, integrated and sampled every $T_s$ and $T_s/2$ seconds for $\text{BOC}_{\text{sin}}(kn,n)$ and $\text{BOC}_{\text{cos}}(kn,n)$, respectively. Subsequently, sub-correlation values are obtained by summing $L$ samples per sub-correlation, and then, combined according to (8) and (9) to produce a decision variable based on $R_{\text{proposed}}^{k_{\text{sin}}}(\tau)$ and $R_{\text{proposed}}^{k_{\text{cos}}}(\tau)$, respectively. Finally, decision variables corresponding to possible phases in the uncertainty region are collected and the process is transferred to the tracking stage with the phase corresponding to the largest of the decision variables. It should be noted that the proposed scheme needs only a single correlator since each sub-correlation is sequentially obtained by sampling the single correlator output every $T_s/2$ seconds and every $T_s/4k$ seconds for $\text{BOC}_{\text{sin}}(kn,n)$ and $\text{BOC}_{\text{cos}}(kn,n)$ signals, respectively, as shown in Fig. 3.

IV. NUMERICAL RESULTS

In this section, the proposed unambiguous acquisition scheme is compared with the unambiguous acquisition scheme in [11] called the general removing ambiguity via side-peak suppression (GRASS) in terms of the correlation function, ROC curves, MAT. In comparisons, we assume the following parameters: Galileo E1-C PRN code of $T = 4092$ chips [4], correlation length of $L = 1023$, and a search step size of $T_s$ and $T_s/2$ for the sine-phased and cosine-phased BOC signals, respectively.

Fig. 4 shows the normalized correlation functions of the proposed, GRASS, and traditional BOC schemes for (a) $\text{BOC}_{\text{sin}}(kn,n)$ and (b) $\text{BOC}_{\text{cos}}(kn,n)$ when $k = 1$ and $k = 2$ in the absence of noise.

Fig. 5. ROC curves of the proposed, GRASS, and traditional BOC schemes for (a) $\text{BOC}_{\text{sin}}(kn,n)$ and (b) $\text{BOC}_{\text{cos}}(kn,n)$ when $k = 1$ and $k = 2$.
GRASS and traditional BOC schemes in the CNR range 20 ~ 40 dB-Hz of practical interest.

In this paper, we have proposed a novel unambiguous BOC acquisition scheme for GNSSs. First, we have analyzed that the BOC autocorrelation is made up of the sum of several sub-correlations, and then, observed that the irregular shapes of the sub-correlations cause the side-peaks in the BOC autocorrelation. Then, we have proposed an unambiguous correlation function with no side-peak via a recombination of the sub-correlations. From numerical results, it has been observed that the proposed scheme removes the side-peaks completely providing a performance improvement over the GRASS and traditional BOC schemes in terms of the ROC curves and MAT.

ACKNOWLEDGMENT
This research was supported by the National Research Foundation (NRF) of Korea under Grant 2012R1A2A2A01045887 with funding from the Ministry of Science, ICT & Future Planning (MSIP), Korea, by the Information Technology Research Center (ITRC) program of the National IT Industry Promotion Agency under Grant NIPA-2013-H0301-13-1005 with funding from the MSIP, Korea, and by National GNSS Research Center program of Defense Acquisition Program Administration and Agency for Defense Development.

REFERENCES