Preemptive Channel Allocations for Cellular Networks
with Multiple Sectors

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Abstract—This paper presents preemptive channel allocations (PCA) for multiple-sector cellular networks, where directional antennas are used to divide the coverage of a cell into a number of same-sized sectors. When traffic in a sector unexpectedly increases, call blocking probability will increase accordingly. To remedy channel insufficient problem in a single sector, two aspects of channel preemptions are utilized. First, to reduce the blocking probability of new calls, the proposed PCA allows a new call to preempt an ongoing call when the ongoing call is located in the overlapping regions of two adjacent sectors or two neighboring cells. Second, the reserved channels not only can be used by the handoff calls, but also by the preempted calls. For the purpose of performance evaluation, we build an analytical model with four-tuple Markov chains. Numerical results show that the proposed PCA scheme improve the system performance in terms of the blocking and preemption probabilities.

Keywords—Preemptive channel allocations; multiple sectors; cellular networks; blocking probability; Markov chains;

I. INTRODUCTION

Over the past decade, the rapid growth of the cellular technology (2G/3G/3.5G or even the upcoming 4G) has been proven that it can provide high reliability, stability, and ubiquity for personal communications [1]. A basic cellular network is composed of a base station (BS) and numerous mobile terminals (MTs). Channel capacity of a cellular network may become insufficient when MTs are attached to the network or moving between cells or sectors. There have been many previous works focused on the preemption mechanisms for cellular networks. A scheme called Adjusted Multimode Dynamic Guard Bandwidth (AM-DGB) [1] can temporarily block one or more lower-priority calls to guarantee longer connection time for higher-priority calls. A centralized and decentralized preemption algorithm was proposed by Lau et al. [3] for a connection-oriented network to minimize the service disruptions of ongoing calls. Recently, there were copious researches on sector-based cellular networks, such as WiMAX and LTE (Long Term Evolution)/LTE-A (Long Term Evolution-Advanced) networks. For example, to improve the throughput and capacity and to alleviate the inter-cell interference, numerous schemes on frequency reuse were proposed. Among them, Lei et al. [4] proposed a frequency reuse scheme to divide the available subcarriers into two groups, the super group used for the central region of a cell and the regular group used for the boundary of a cell. Similarly, Ali et al. [5] proposed the architecture of a dynamic fractional frequency reused (FFR) cell. Dynamic FFR scheme only partitions subcarriers into two physical groups. In our work, the main objective is to design a novel preemption scheme for ongoing calls residing in the overlapping areas of any two adjacent sectors. The remainder of this paper is organized as follows. Section II introduces the proposed channel preemption algorithms. Performance evaluation model is described in Section III. Section IV shows the analytical results along with discussions. Finally, Section V contains our concluding remarks.

II. PREEMPTIVE CHANNEL ALLOCATIONS

A. Sector-based Cellular Networks

Figure 1. Generalized topology of a sector-based cellular network

A sector-based cellular network consists of multiple sectors divided by directional antennas. Figure 1 shows a generalized topology of a sector-based cellular network, consisting of one central cell and six neighboring cells. In the sector-based cellular network, we assume the cell has radius $R$. Due to the different coverage areas, an MT in a sector may reside in the following four regions. Region 1 (R1): MT resides in the clockwise overlapping region of
two adjacent sectors (e.g., MT-A). Region 2 (R2): MT resides in the counterclockwise overlapping region of two adjacent sectors (e.g., MT-B). Region 3 (R3): MT resides in the overlapping regions of two adjacent cells (e.g., MT-C). Region 4 (R4): MT resides in a sector other than the Regions of 1, 2 and 3 (e.g., MT-D). We design two different types of handoffs; (i) Inter-sector handoff: an MT originally residing in the sector is moving out to the neighboring sectors by passing through R1 or R2, and (ii) Inter-cell handoff: an MT originally residing in a cell is moving out to one of its six neighboring cells by passing through R3. A cell is divided into $N_{S}$ sectors, and the $n$-th sector has channel capacity, $C_{n}$, among which certain amount of channels are purposely reserved for inter-sector/inter-cell handoff and preempted MTs, where $n=1,2,\ldots,N_{S}$ (counted in clockwise direction). Thus, the total channel capacity within a single cell is $C_{T} = \sum_{n=1}^{N_{S}} C_{n}$. Let $C_{SR}$ represent the channels reserved by the $n$-th sector for inter-sector handoff calls and preempted calls resides in the overlapping region of two adjacent sectors. Let $C_{CR}$ and $C_{NR}$ represent the channels reserved by the $n$-th sector of the central cell and that of the neighboring cell respectively for inter-cell handoff and preempted calls resides in the overlapping region of two adjacent cells. Accordingly, in a cell, the total number of $C_{SR} = \sum_{n=1}^{N_{S}} C_{SR}$, the total channels of $C_{CR} = \sum_{n=1}^{N_{S}} C_{CR}$, and the total channels of $C_{NR} = \sum_{n=1}^{N_{S}} C_{NR}$. As a result, in the central cell, the available channels of the $n$-th sector that can be assigned to new calls become $C_{A} = C_{n} - C_{SR} - C_{CR}$. Then the total available channels of the central cell are $C_{A} = \sum_{n=1}^{N_{S}} C_{n}$. 

### B. Channels for Odd/Even Sectors

To reuse the frequency spectrum, the total carriers in a cell can be divided into two subcarriers: $\{C_{T}/2, C_{T}/2\}$ for even number of sectors (e.g., $N_{S} = 2, 4, 6,\ldots$), and $\{C_{T}/3, C_{T}/3, C_{T}/3\}$ for odd number of sectors (e.g., $N_{S} = 3, 5, 7,\ldots$). Thus, for even sectors, $C_{n} = C_{T}/2$, and for odd sectors, $C_{n} = C_{T}/3$. Figure 2 illustrates the generalized cases of frequency reuse and channel allocations in a sector-based cellular network.

A preemptive channel allocations (PCA) algorithm is designed for the cellular network with multiple sectors. Under this assumption, three phases of channel preemption, $PCA-cws$, $PCA-ccs$, and $PCA-nbc$, could be invoked by an MT. They are explained one by one as below. $PCA-cws$: When the available channels in the $n$-th sector are used up, and there is no active MT residing in R1 or $C_{SR}^{n}$ are also used up, a new call generated in the sector can be blocked because $PCA-cws$ is not possible. However, $PCA-ccs$ can be invoked by the new call if: (i) one active MT residing in R2 is employing an available channel of the $n$-th sector, and (ii) at least one reserved channel of the counterclockwise neighboring sector $C_{SR}^{n+1}$ is free, where

$$n + 1 = \begin{cases} 1, & \text{if } n = N_{S} \\ 2, 3, \ldots, N_{S}, & \text{otherwise} \end{cases}$$

$PCA-nbc$: When the available channels in the $n$-th sector are used up, and $PCA-cws$ and $PCA-ccs$ are not possible, then a new call generated in the sector can be blocked. However, $PCA-nbc$ can be invoked by the new call if: (i) an active MT residing in R3 is employing an available channel of the $n$-th sector, and (ii) at least one channel of $C_{SR}^{n}$ is free. We define the following four types of ongoing calls which currently use the available channels in a sector according to the four regions; (i) $i$ is the number of ongoing calls which reside in R4 of a sector, (ii) $j$ is the number of ongoing calls which reside in R1 of a sector, (iii) $k$ is the number of ongoing calls which reside in R2 of a sector, and (iv) $l$ is the number of ongoing calls which reside in R3 of a sector. In addition, we use five variables, $C_{c}, C_{e}, C_{cc}, C_{nc}$, and $C_{c}$, to represent channel increment or decrement in $C_{SR}, C_{CR}, C_{SR}^{n+1}$, and $C_{SR}$, respectively.

III. PERFORMANCE EVALUATION MODEL OF PCA

In this section, we are interested in evaluating the proposed PCA algorithm on the sector-based cellular networks. Using 4-D $(i, j, k, l)$ in a Markovian state, we can capture the characteristics of the proposed PCA.

A. Model Assumptions

The following assumptions are made in the analytical model: (i) it needs one and only one channel for an MT to become active; and (ii) the co-channel interference is ignored when an active MT resides in the overlapping regions of two adjacent sectors or cells. To facilitate our analysis, as shown in Figure 3, we approximate a single hexagon cell with six overlapping areas into an equivalent topology with two concentric circles [6], the outer circle
with radius $a \times R_{eq}$ and the inner circle with radius $b \times R_{eq}$, where $R_{eq} = \sqrt{\frac{3\sqrt{3}}{2\pi}} R = 0.91 R$, $1 \leq a \leq 2$, and $0 \leq b \leq 1$.

Hence, $R_{eq}$ is the equivalent radius of the hexagon cell. By adjusting the parameters, $a$ and $b$, we can enlarge or shrink the handoff area. As compared to Figure 1, R3 is converted to the area between the outer and the inner circle, and R4 is converted to the area of the inner circle by excluding R1 and R2. If we let $A_{R1}$, $A_{R2}$, $A_{R3}$ and $A_{R4}$ denote the area ratio of R1, R2, R3 and R4 to the outer circle, respectively, we have

$$A_{R3} = \frac{(A_{360\theta}) (a^2 - b^2) R_{eq}^2 \pi}{A_3}$$

and

$$A_{R4} = \frac{(a^2 - b^2) R_{eq}^2 \pi}{A_3} - 1 - A_{R3} - A_{R1} - A_{R2}.$$

Notice that the coverage area of a directional antenna is $A_3 = (\theta_3/360\theta) (a R_{eq})^2 \pi$, the angle of a sector is $\theta_3 = 360\theta/N_\delta$, $\theta_1$ is the transmission angle of a directional antenna. If $\alpha$ denotes the angle of two overlapping sectors, then $\alpha = \theta_1 - \theta_3$.

Figure 3. Equivalent topology of a single cell

B. Markov Chains

| TABLE I. STATE TRANSITION RATES FOR ARRIVAL PROCESS |
|-----------------|-----------------|-----------------|-----------------|
| $A_1 = A_0 + A_4$ | $A_2 = A_0 + A_3$ | $A_3 = A_0 + A_3$ | $A_4 = A_0 + A_3$ |
| $A_0 = A_0$ | $A_1 = A_0$ | $A_2 = A_0$ | $A_3 = A_0$ |

A 4-D Markov chain model with states $(i, j, k, l)$ is built to analyze the proposed PCA algorithm on a sector-based cellular network. The transition rates for arrival and departure processes are listed as in Table I and Table II, respectively.

| TABLE II. STATE TRANSITION RATES FOR DEPARTURE PROCESS |
|-----------------|-----------------|-----------------|
| $D_{11} = i \times \times j \times k \times l$ | $D_{12} = i \times j \times k \times l$ | $D_{13} = i \times k \times l$ |

The call departure rates from one region to another region in a sector and the inter-sector/inter-cell handoff rates are listed in Table III.

| TABLE III. DEPARTURES RATES FOR CALLS MOVING BETWEEN REGIONS |
|-----------------|-----------------|-----------------|
| $f_1$ | $f_2$ | $f_3$ |

The call preemption probabilities under PCA-cws and PCA-ccs are listed in Table IV, and the call preemption probability under PCA-nbc is listed in Table V.

| TABLE IV. PREEMPTION RATES FOR NEW CALLS UNDER PCA-CWS AND PCA-CCS WHEN $i + j + k + l = C_\Lambda^n$ |
|-----------------|-----------------|-----------------|
| $A_{R2} = A_1$ | $A_{R3} = A_1$ | $A_{R4} = A_1$ |
| $A_{R4} = A_1$ | $A_{R5} = A_1$ | $A_{R6} = A_1$ |

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TABLE V. PREEMPTION RATES FOR NEW CALLS UNDER PCA-NBC

<table>
<thead>
<tr>
<th>i + j + k + l = C_a</th>
<th>Conditions</th>
<th>Regions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

To derive the state transition rates in Table I and Table II, first of all, we need to define the cell service rate and the handoff-area service rate by referring to [7] and [8]. In the model, we assume that the new-call arrival rate is a Poisson process with mean $\lambda_N$ and the call duration time, $T$, is exponentially distributed with mean $\mu$. Let $T_{d1}$, $T_{d2}$, $T_{d3}$, and $T_{d4}$ represent the dwell time of an ongoing call in R1, R2, R3 and R4, respectively. The service rates of four regions (denoted as $\mu_{d1}$, $\mu_{d2}$, $\mu_{d3}$ and $\mu_{d4}$) can be computed as shown in Eq. (1).

$$\mu_{d1} = \frac{2E[V]}{\pi \times 4R_{d1}^2} \times \frac{360^\circ}{\theta_d}, \quad \mu_{d2} = \frac{2E[V]}{(a-b) \times 4R_{d2}^2} \times \frac{360^\circ}{\theta_d}$$

When an MT resides in R1 or R2, the probability of moving out the overlapping region of two adjacent sectors is determined by the area ratio of R3 and R4. Thus, the inter-sector handoff rates of an MT residing in R1 and R2, denoted as $\mu_{d1}$ and $\mu_{d2}$, can be derived from $\mu_{d1}$ and $\mu_{d2}$ directly. Similarly, when an MT resides in R3, the inter-cell handoff rate of an MT, $\mu_{H}$, can be derived from $\mu_{d3}$ directly. That is,

$$\mu_{d1} = \frac{AR_1}{\theta_d} \times \mu_{d1} + \mu_{d2} = \frac{AR_2}{\theta_d} \times \mu_{d1} + \frac{2 \times \mu_{d1} + 2 \times \mu_{d2}}{\theta_d}$$

Let $S_{PCA-cws}$ be the successful-generation probability of a new call when available channels in the $n$-th sector and the reserved channels of the clockwise neighboring sector becomes zero, but at least one active MT resides in R1. Let $S_{PCA-ccs}$ be the successful-generation probability of a new call when available channels in the $n$-th sector are used up, PCA-cws and PCA-ccs cannot be invoked, and the reserved channels of the counterclockwise neighboring sector becomes zero, but at least one active MT resides in R2. We have

$$S_{PCA-cws} = \left( \frac{\mu_{1} \cdot \mu_{d1} \cdot \mu_{d3} \cdot \mu_{3}}{\mu_{d1} \cdot \mu_{d2} \cdot \mu_{d4} \cdot \mu_{4}} \right)$$

Likewise, let $S_{PCA-cws}$ be the successful-generation probability of a new call when available channels in the $n$-th sector are used up, PCA-cws and PCA-ccs cannot be invoked, and the reserved channels of the $n$-th sector in the neighboring cell becomes zero, but at least one active MT resides in R3. Let $S_{PCA-cws}$ be the successful-generation probability of a new call when available channels in the $n$-th sector are used up, PCA-cws and PCA-ccs cannot be invoked, and the reserved channels of a sector in the central cell becomes zero, but at least one active MT resides in R3. We have

$$S_{PCA-ccs} = \left( \frac{\mu_{1} \cdot \mu_{d1} \cdot \mu_{d3} \cdot \mu_{3}}{\mu_{d1} \cdot \mu_{d2} \cdot \mu_{d4} \cdot \mu_{4}} \right)$$
through R1, R2, R3, and R4 respectively. In this paper, we let \( n_1 = n_2 = n_3 = n_4 = 1 \). Finally, let \( \pi(i, j, k, l) \) be the steady-state probability in the 4-D Markov chain model. To analytically solve this model, we have to include the initial condition, as shown in Eq. (7), into the state-transition matrix, which can be derived from Tables II and III.

\[
\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \pi(i, j, k, l) = 1 \tag{7}
\]

### C. Performance Metrics

Let \( P_{nPCA} \) be the PCA preemption probability in the \( n \)-th sector. \( P_{nPCA} \) can be computed as shown in Eq. (8). \( P_{nPCA-cws} \), \( P_{nPCA-ccs} \), and \( P_{nPCA-nbc} \) respectively represent the preemption probability of new calls under the operation of PCA-cws, PCA-ccs, and PCA-nbc in the \( n \)-th sector as shown in Eq. (9), Eq. (10), and Eq. (11), respectively.

\[
P_{nPCA} = P_{nPCA-cws} + P_{nPCA-ccs} + P_{nPCA-nbc} \tag{8}
\]

Where

\[
P_{nPCA-cws} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \pi(C_i - j - k - l, j, k, l), \text{if } c_{py} > 0
\]

\[
P_{nPCA-ccs} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \pi(C_i - j - k - l, j, k, l) \times S_{PCA-cws}, \text{if } c_{py} = 0
\]

\[
P_{nPCA-nbc} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \pi(C_i - j - k - l, j, k, l) \times (1 - S_{PCA-cws}), \text{if } c_{py} = 0
\]

Let \( P_{nab} \) be the new-call blocking probability in the \( n \)-th sector as shown in Eq. (12). Basically, \( P_{nab} \) consists of two terms which describe PCA cannot be invoked. The first term represents the probability that there is no ongoing call residing in R3, and the second term represents the probability that the reserved channels of the neighboring cell become zero.

\[
P_{nPCA} \text{ is increased more rapidly as } \alpha \text{ increases when a cell is divided into five sectors. This is because that by referring to Eq. (1), low-speed MT will increase the dwell time in the sector. In other words, the channel occupancy time of low-speed MT (e.g., } \nu = 10 \text{ km/h}) \text{ is much longer than that of high-speed MT (e.g., } \nu = 50 \text{ km/h}).
\]

### IV. Numerical Results

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total channel capacity in a cell (( C_T ))</td>
<td>36</td>
</tr>
<tr>
<td>( C_{SR} ), ( C_{CR} ), ( C_{NR} )</td>
<td>1, 3</td>
</tr>
<tr>
<td>Call duration time (1/( \mu ))</td>
<td>500 sec</td>
</tr>
<tr>
<td>Distance from the hexagon center to any vertex (( R ))</td>
<td>1000 m</td>
</tr>
</tbody>
</table>

The parameters and values listed in Table VI were used when running the MATLAB tool. To investigate the impact of the traffic in the networks, we define traffic load as \( \rho = \lambda_c / C_{max} \). Figure 4 shows the new-call blocking probability as \( \rho \) increases from 0.3 to 1.2. It is interesting to notice that new-call blocking probabilities can be significantly reduced under the cell with four or five sectors due to the expanded available channels. There is another phenomenon worthy to observe that new-call blocking probability will be slightly increased when the speed of MT decreases from 10 to 50 km/h. The reason is because that by referring to Eq. (1), low-speed MT will increase the dwell time in the sector. In other words, the channel occupancy time of low-speed MT (e.g., \( \nu = 10 \) km/h) is much longer than that of high-speed MT (e.g., \( \nu = 50 \) km/h).

![Figure 4. New-call blocking probability versus traffic load](image-url)
with less sectors (e.g., \( N_S = 2 \)). Another interesting phenomenon is that \( P_{PCA}^n \) is continuously increased when the reserved channels become more (e.g., \( C_{SR}^n = 3 \)) in a sector, because the preempted calls have higher possibility to sustain their connections when PCA mechanism is invoked.

Figure 5. Preemption probability of PCA versus the angle of two overlapping sectors

Figure 6. Preemption probability of PCA versus the ratio \( a/b \)

Figure 7. Preemption probability of PCA versus speed of MT

V. CONCLUSIONS

This paper has presented an analytical model of adaptive channel preemption (PCA) for sector-based cellular networks. Three different preemption phases, \( PCA-cws, PCA-ccc, \) and \( PCA-nbc \) were proposed to fully utilize the capacity of the cellular networks with multiple sectors. One of the novelties presented in this paper is right in that the proposed PCA allows a new call to preempt an ongoing call when the latter is located in the inter-sector or inter-cell overlapping region. Analytical results have revealed two annotations: (i) the reserved channels can not only be used by the inter-sector/inter-cell handoff calls but also used by the preempted calls, and (ii) the low-speed MT makes more impact on the new-call blocking probability than the high-speed MT due to the longer channel occupancy time.