Throughput Analysis in Cognitive Radio Networks Using Slotted Aloha Protocol with Imperfect Sensing

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Abstract—This paper introduces an extension of a methodology for calculating the throughput in a cognitive radio network using the slotted aloha technique for medium access. This extension includes two new aspects: imperfect sensing and different transmission power levels in the secondary stations. The performance of the network is evaluated and numerical results are compared with the results for the original model. The new aspects considered in this paper make the model more realistic.

Keyword—Cognitive Radio; Multiple Access; Throughput; Performance Analysis.

I. INTRODUCTION

During the last decades, emerging applications in wireless communication networks gained attention due to the increasing demand of high transmission rates services [1]. According to [1] and [2], the majority of the available spectrum frequencies has already been fixed and licensed to primary user’s (PU’s). A study undertaken by Spectrum Policy Task Force (STPF), linked to Federal Communications Commission (FCC), of United States of America (USA), has shown that in certain locations and during certain periods of time, some spectrum bands are widely used by PU’s. However, in some other locations, there are many frequency bands that are partially occupied or not used at all [1]. According to the FCC, the usage of licensed spectrum bands varies between 15% and 85%. A way to overcome these limitations is to promote changes in the current licensing model, by allowing secondary users (SUs) to access the available spectrum, without causing harmful interference in the PU’s communications. The efficiency in the usage of the radio spectrum would be improved in order to support the required demands [2]. Cognitive Radio (CR) is a technology that can be used to implement this approach.

CR is defined as a radio that can change its transmission parameters based on the environment in which it is operating. The main functions of cognitive radio include spectral detection, spectrum management, spectral mobility and spectrum sharing [2]. Its paramount objective is to provide adaptability to wireless transmission systems through Dynamic Spectrum Access (DSA) in order to optimize the performance and improve the use of the spectrum [1].

The components of a Cognitive Radio Network (CRN) Architecture can be classified in two groups: primary and secondary networks. The first is the licensed network infrastructure, which is entitled to exploit a certain band of the frequency spectrum. The secondary network is not licensed to operate in that band and their radio stations access the spectrum done in an opportunistic manner, exploring the unused bands by PU’s, defined as a spectral holes.

Medium Access Control (MAC) is a key issue in CRN. In the primary network, the MAC protocols are important in order to organize the access to the channel of different PUs. In the secondary network, the MAC protocols have the responsibility to organize the access of SUs to the free channels of the primary network and prevent the licensed network from harmful interference. Several papers have analyzed the performance of MAC protocols in a CRN environment. Li et al. [3] analyzed the impact of imperfect sensing in a distributed multi-channel cognitive radio system. Moshksar et al. [4] analyzed the effect of power levels by SUs for achieving high rate per SU. Bayrakdar et al. [5] investigated the throughput of a slotted aloha-based random access CRN with capture effect in Rayleigh fading channels. In [6], the performance of the slotted aloha protocol, in both networks (primary and secondary), is also analyzed considering the capture effect and Rayleigh channels. The analyses presented in [5] and [6], however, do not consider the Packet Error Rate (PER) when two or more stations transmit simultaneously. This lack in the performance analysis has been solved by the extension published in [7]. However, the analyses presented in [6] and [7] do not consider two important aspects: imperfect sensing and different transmission power levels in SUs. The main goal of this paper is to extend the analysis presented in [6] and [7], considering these effects in the mathematical formulation.

The rest of this paper is organized as follows: in Section II, we present the model used in [6], and the extension presented in [7], called original model; the new model is proposed in Section III; numerical results and discussions are presented in Section IV; and conclusions are given in Section V.

II. SYSTEM MODEL

The analyzed architecture, which is illustrated in Figure 1, considers two networks (primary and secondary) that coexist in the same geographic region and frequency band. The secondary network contains a SAP and Ns users who compete for spectrum opportunities. The primary network has a primary access point (PAP) and Np PU’s. According to [6] and [7], during a time slot, there are \( \text{Ip} \) PU’s and \( \text{Js} \) SU’s attempting to transmit their data packets to their respective access points. During a time slot, each PU or SU, which is not in the packet transmission state can generate a new packet with probability \( \sigma_p \) or \( \sigma_s \), respectively. Thus, the probability of no new PU or SU packet generation is \((1 - \sigma_p)\) or \((1 - \sigma_s)\), respectively. If a PU or an SU generates a data packet on its network, the packet is transmitted in the next time slot. If an error occurs during the transmission of the packet, it is then retransmitted with
probability $\sigma_p$ or $\sigma_s$ in the following time slot, until the packet is received successfully. If a PU or SU is in the retransmission state, no new packet can be generated.

\[
\text{Figure 1. Primary and Secondary networks model using slotted aloha protocol}
\]

A. Fading Model for Primary and Secondary Network

In this paper, following [6] and [7], a quasi-static fading model is used, according to the Rayleigh statistical model, wherein the instantaneous power of the received signal has an exponential distribution, see (6).

B. Power Level Applied in the Network

Let $X_p$ and $X_s$ be the mean values of instantaneous power of the concerned packet from primary and secondary networks, respectively. Let $Y$ and $Z$ be the mean values of the interfering powers of one packet from primary and secondary network, respectively. In [6] and [7], we have $X_p = Y$ and $X_s = Z$. Keeping in mind that the SU's work with lower levels of transmission power, to minimize interference in the PUs, we define the following ratio:

\[
\gamma_i = \frac{X_p}{Z} = \frac{Y}{X_s}. \tag{1}
\]

C. Analysis of the Capture Effect

In slotted aloha systems, packets that arrive at the receiver have different power levels due to the distance between the transceivers, the transmission power level, the channel shadowing and the signal fading. If the difference between the power levels of a concerned packet in relation to other interfering packets is higher than a threshold, called capture ratio ($R$), then the concerned packet can be detected by the receiver [8].

In [6] and [7], the authors consider a capture model where the interfering power is the sum of all received interfering powers from SU's and PU's. If the power of the concerned packet is higher than the interfering power and it satisfies the capture ratio $R$, then the receiver captures the concerned packet. The analysis presented in [6] considers that the captured packet can be detected by the PAP or SAP without errors and the analysis presented in [7] considers the effects of the PER. Assuming that the concerned packet is coming from a PU, then the capture probability ($P_{\text{pcap-PAP}}$) can be calculated by [6]:

\[
P_{\text{pcap-PAP}}(I_p, J_s) = \Pr \left\{ \frac{X_p}{\sum_{i=1}^{I_p} y_i + \sum_{j=1}^{J_s} z_j} > R \right\}, \tag{2}
\]

where $I_p$ and $J_s$ are the respective numbers of PU’s and SU’s that can generate or retransmit a packet with probability $\sigma_p$ or $\sigma_s$ in a given time slot, $x_p$ is the instantaneous power of the concerned packet generated by a PU, $R$ is the capture ratio and $y_i$ and $z_j$ are the instantaneous powers of interfering packets generated in the primary and secondary networks, respectively [6].

If the concerned packet is coming from an SU, then the capture probability ($P_{\text{scap-SAP}}$) is calculated as [6]:

\[
P_{\text{scap-SAP}}(I_p, J_s) = \Pr \left\{ \frac{x_s}{\sum_{i=1}^{I_p} y_i + \sum_{j=1}^{J_s} z_j} > R \right\}, \tag{3}
\]

where $x_s$ is the instantaneous power of the concerned packet generated by an SU.

D. Packet Error Rate Analysis

The knowledge of the PER in communication systems is important, since most systems have data transmitted in packets [7]. Furthermore, the performance of the system is determined by PER instead of the Bit Error Rate (BER) or Symbol Error Rate (SER) [7][9]. The relation between PER and the system model is made by the Signal-to-Interference plus Noise Ratio (SNIR). According to [10], the Gaussian noise can be neglected in channels limited by the interference. Thus, in this paper, we consider the Signal-to-Interference Ratio (SIR).

In this article, following [7], the PER is calculated for a fading channel as a function of SIR, through the use of a fairly accurate upper bound for the considered system [9]. Knowing that $X_p = Y$ and $X_s = Z$ and applying (1), we obtain the mean value of SIR for the primary networks ($\Delta_p$) and secondary networks ($\Delta_s$), given by:

\[
\Delta_p = \frac{1}{(I_p - 1) + J_s / \gamma_i}, \tag{4}
\]

\[
\Delta_s = \frac{1}{\gamma_i I_p + J_s - 1}. \tag{5}
\]

Let $f(\delta)$ be a function that relates the PER with the instantaneous SIR at the reception ($\delta$), in an additive white Gaussian noise channel (AWGN), and $p(\delta)$ the probability density function of Signal-to-Interference Ratio (SIR) in the receiver, considering a Rayleigh channel, which has an exponential distribution given by:

\[
p(\delta) = \frac{1}{\Delta} e^{-\frac{\delta}{\Delta}}. \tag{6}
\]

According to [9], the PER represented by $P_{\text{ave}}(\Delta)$ can be calculated by the following expression:

\[
P_{\text{ave}}(\Delta) = \int_{0}^{\infty} f(\delta) p(\delta) d\delta, \tag{7}
\]
Calculating the Total Throughput of the Original Model

The Packet Success Rate (PSR) is then given by the following equation:

\[ \text{PSR}(\Delta) = e^{-w_0\Delta} \],

where \( w_0 \) is a constant value for Rayleigh channel and its value can be computed by the following expression [9]:

\[ w_0 = \frac{\gamma}{\int_0^\infty f(\delta) d\delta} \].

Not considering channel coding and considering \( n \)-bit packets, \( f(\delta) \) can then be obtained as follows [9]:

\[ f(\delta) = 1 - \left[1 - b(\delta)\right]^n \],

where \( b(\delta) \) is the BER in AWGN channels. Considering a BPSK modulation with coherent detection, \( b(\delta) \) can be calculated by [8]:

\[ b(\delta) = \frac{1}{2} \text{erfc}(\sqrt{\delta}) \].

Applying (12) in (11) and then (11) in (10), we can compute (using Mathcad software) \( w_0 \). Considering \( n = 127 \) bits per packet, the same value used in [7], we obtain \( w_0 = 3.4467 \).

Faria et al. [7] consider that a packet received with error is discarded and will be retransmitted, following the MAC protocol, until it is successfully received.

E. Calculating the Total Throughput of the Original Model

The total throughput \( S_{op} \) is defined as the total number of packets generated by PUs and SUs that are correctly received by the PAP and SAP, respectively, during a time slot. The primary throughput \( S_{op} \), secondary throughput \( S_{os} \), and total throughput \( S_{ot} \), are given as [7]:

\[ S_{op} = \sum_{i=0}^{N_{\text{p}}} \sum_{j=0}^{N_{\text{s}}} \left( N_{\text{p}}^i \right) \left( 1 - \sigma_p \right)^{N_{\text{s}} - i} \left( \gamma_{\text{p}} \right) \sigma_p^i \left( 1 - \sigma_p \right)^{N_{\text{s}} - j} \left( N_{\text{s}} \right) \left( 1 - \sigma_s \right) \left( 1 - \gamma_{\text{s}} \right)^{N_{\text{s}} - j} \left( \gamma_{\text{s}} \right), \]

\[ \left( \frac{1}{R+1} \right)^{N_{\text{s}}} e^{-n(\gamma_{\text{s}})} \left( \frac{1}{\gamma_{\text{s}}} \right) \left( \frac{1}{R+1} \right)^{N_{\text{s}}} e^{-n(\gamma_{\text{s}})} \left( \gamma_{\text{s}} \right). \]

\[ S_{os} = \sum_{i=0}^{N_{\text{p}}} \sum_{j=0}^{N_{\text{s}}} \left( N_{\text{p}}^i \right) \left( 1 - \sigma_p \right)^{N_{\text{s}} - i} \left( N_{\text{s}} \right) \left( 1 - \sigma_s \right) \left( 1 - \gamma_{\text{s}} \right)^{N_{\text{s}} - j} \left( \gamma_{\text{s}} \right), \]

\[ \left( \frac{1}{R+1} \right)^{N_{\text{s}}} e^{-n(\gamma_{\text{s}})} \left( \frac{1}{\gamma_{\text{s}}} \right) \left( \frac{1}{R+1} \right)^{N_{\text{s}}} e^{-n(\gamma_{\text{s}})} \left( \gamma_{\text{s}} \right). \]

\[ S_{ot} = S_{op} + S_{os}. \]

III. THE NEW MODEL

The expressions to compute the throughput presented in [7] do not consider imperfect sensing and different transmission power levels in the SUs. In this section, an extension of such model is presented taking into account these aspects, making the model more accurate.

We consider that there is an entity in the secondary network responsible for sensing and making decisions about the channel status. Depending on the decision made in a given time slotted, the SUs adapt their transmission power level in order to minimize the interference on the primary network.

A. Channel Sensing

The spectral sensing is one of the most critical parts of the CRN. We consider that a secondary user initially performs channel sensing, which can be modeled as a hypothesis testing problem. Various channel sensing methods, including energy detection, cycle stationary detection, and matched filtering, have been proposed and analyzed in the literature [11]-[14]. Regardless of which method is used, one common feature is that errors, in the form of missed detections and false-alarms, can occur.

We assumed that \( H_0 \) denotes the hypothesis that the primary users are inactive in the channel, and \( H_1 \) denotes the hypothesis that the primary users are active. Thus, \( H_1 \) denotes the decision that the PU is absent in the channel and \( H_1 \) denotes the decision that the PU is active in the channel. With the result of the decision and the true nature of the activity of the PU, we have four possible cases that are described below [12][13].

Case 1: correct detection probability, considering that PU is active

\[ P_d = \text{Pr}\left\{ H_1 | H_1 \right\}. \]

Case 2: false alarm probability, considering that PU is absent

\[ P_f = \text{Pr}\left\{ H_1 | H_0 \right\}. \]

Case 3: correct detection probability, considering that PU is absent

\[ P_{\text{f0}} = \text{Pr}\left\{ H_0 | H_1 \right\} = 1 - P_d. \]

Case 4: incorrect detection probability, considering that PU is active

\[ P_{\text{d0}} = \text{Pr}\left\{ H_0 | H_1 \right\} = 1 - P_f. \]
B. Power Level Applied in the Network

We considered that $X_{sk}$ is the mean value of the instantaneous power of the concerned packet from the secondary network and $Z_k$ is the mean value of the interfering power from one SU, where $X_{sk} = Z_k$.

In our notation, $k$ denotes the decision about the state of the channel, where $k = 0$ represents that the decision is channel free and $k = 1$ that the decision is channel busy.

The transmission power of the SU when $k = 0$ is greater than when $k = 1$, with the goal to reduce interference in the primary network. Now, the ratio between the power of a packet from the primary network and the power of one from the secondary network depends on the decision about the channel state and is given by:

$$ \gamma_k = \frac{X_p}{Z_k} = \frac{Y}{X_{sk}}. \quad (20) $$

C. Capture Effect for the New System

To compute the capture probability we consider that all SUs have the same decision about the state of the channel (free or busy). This approach is equivalent to considering that the decision process in the secondary network is centralized.

For the capture effect, if the concerned packet is generated by a PU, the probability of capturing this packet is given by (2), modified to consider that we have two different power levels in the secondary network:

$$ P_{\text{cap}, \text{PU}}(I_p, I_j) = \Pr \left\{ \frac{x_p}{\sum_{j=1}^{J_s} y_j + \sum_{j=1}^{J_s} z_k} > R \right\}, \quad (21) $$

If the concerned packet is generated by an SU, then the probability of capture is calculated modifying (3), resulting in:

$$ P_{\text{cap}, \text{SU}}(I_p, I_j) = \Pr \left\{ \frac{x_s}{\sum_{j=1}^{J_s} y_j + \sum_{j=1}^{J_s} z_k} > R \right\}, \quad (22) $$

where $x_{sk}$ is the instantaneous power of the concerned packet generated by an SU.

In (21) and (22), $x_p$, $x_s$, and $y_j$ have exponential distribution (see (6)), $\sum z_k$ represents the instantaneous interfering power from the secondary network, that is obtained by adding $J_s$ or $J_{s-1}$ independent identically distributed (iid) random variable with exponential distribution, which converges to an Erlang distribution, given by [15]:

$$ f(\sum z_k) = \left( \frac{C_j}{j^{(j-1)} e^{-z_k}} \right) \left( \frac{1}{(j-1)!} \right). \quad (23) $$

In (23), $j$ represents the number of interfering users ($j=J_s$ for primary networks and $j=J_{s-1}$ for secondary networks). The parameter $C_j$ is associated to the mean value of the interfering power from one SU and is given by:

$$ C_j = \frac{1}{Z_j}. \quad (24) $$

Considering that all PUs or SUs are mutually independent, the joint probability density functions for the received powers, for the primary and secondary networks, are given, respectively, by:

$$ f\left( x_p, y_1, y_2, \ldots, y_{j-1}, z_k \right) = \frac{1}{X_p} e^{-x_p} \prod_{j=1}^{J_s} \frac{1}{Y} e^{-y_j} \left( \frac{C_j}{j^{(j-1)} e^{-z_k}} \right) \left( \frac{1}{(j-1)!} \right). \quad (25) $$

$$ f\left( x_s, y_1, y_2, \ldots, y_{j-1}, z_k \right) = \frac{1}{X_s} e^{-x_s} \prod_{j=1}^{J_s} \frac{1}{Y} e^{-y_j} \left( \frac{C_j}{j^{(j-1)} e^{C_k}} \right) \left( \frac{1}{(j-1)!} \right). \quad (26) $$

Now, to compute the capture probability we need to solve the integrals given by (27) and (28), respectively for primary and secondary networks.

$$ \int_0^\infty \cdots \int_0^\infty \frac{1}{X_p} e^{-x_p} \prod_{j=1}^{J_s} \frac{1}{Y} e^{-y_j} \left( \frac{C_j}{j^{(j-1)} e^{-z_k}} \right) \left( \frac{1}{(j-1)!} \right) dx_p dy_1 \cdots dy_{j-1} dz_k, \quad (27) $$

$$ \int_0^\infty \cdots \int_0^\infty \frac{1}{X_s} e^{-x_s} \prod_{j=1}^{J_s} \frac{1}{Y} e^{-y_j} \left( \frac{C_j}{j^{(j-1)} e^{C_k}} \right) \left( \frac{1}{(j-1)!} \right) dx_s dy_1 \cdots dy_{j-1} dz_k. \quad (28) $$

Now we consider the value of $\gamma_k$ depending on the channel status decision. In this context, the decision can result in four values, as described below:

- $P_{ph}$, if the channel is busy and the decision is busy, we have: $k=1$ and $\gamma_k = \gamma_1$
- $P_{ph}$, if the channel is busy and the decision is free, we have: $k=0$ and $\gamma_k = \gamma_0$
- $P_{pf}$, if the channel is free and the decision is busy, we have: $k=1$ and $\gamma_k = \gamma_f$
- $P_{pf}$, if the channel is free and the decision is free, we have: $k=0$ and $\gamma_k = \gamma_f$

Thus, $\gamma_k$ represents the ratio (see (20)) when the decision about the channel is busy and $\gamma_k$ represents the ratio when the decision about the state of the channel is idle.

Now, we have the capture probability in the primary network, considering that PU is always active and the result of the decision can be right ($P_{ph}$) or wrong ($P_{pf}$), as given below:
\[ P_{\text{pcp}}(I_p, J_s) = \Pr \left[ \left( \frac{\gamma_i}{R + \gamma_i} \right) P_d + \left( \frac{\gamma_0}{R + \gamma_0} \right) P_{\text{ps}} \right]. \tag{29} \]

For the secondary network, we have the capture probability as given below:

\[ P_{\text{pcp}}(I_p, J_s) = \Pr \left[ \left( \frac{1}{R + 1} \right)^{J_s} \right] \tag{30} \]

\section{Packet Error Rate}

To compute the PER, we use the same approach as in Section II. However, now the SIR depends on the transmission power levels, which depends on the channel state decision. Thus, we have:

\[ \Delta_{pk} = \frac{1}{I_p - 1 + \frac{J_s}{\gamma_k}}. \tag{31} \]

\[ \Delta_{sk} = \frac{1}{\gamma_1 I_p + (J_s - 1)}. \tag{32} \]

The PER and PSR are computed for \( k=0 \) and \( k=1 \), using expressions similar to (8) and (9).

\section{Total Throughput of the New Model}

The total throughput of the network in the new model is defined as the total number of packets generated by PUs and SUs that are correctly received by the PAP and SAP, respectively, during a time slot.

Considering only correctly received packets, the primary network throughput, represented by \( S_{np} \), the secondary network throughput, represented by \( S_{ns} \), and the total throughput of the network, represented by \( S_{nt} \), are given, respectively, by (33), (34) and (35).

\[ S_{np} = \sum_{i=0}^{Np} \sum_{j=0}^{Np} \left( \sum_{i=0}^{Np} \frac{Np}{i} \sigma_i P_{ed} \right) \left( 1 - \sigma_i \right)^{Np-i} \left( 1 - \sigma_j \right)^{Np-j} \left( 1 - \sigma_j \right)^{Np-j} \left( 1 - \sigma_j \right)^{Np-j}. \]

\[ S_{ns} = \left( \frac{1}{R_{\gamma_p} + 1} \right) e^{-\gamma_0 P_{ed}} + \left( \frac{1}{R_{\gamma_1} + 1} \right) e^{-\gamma_1 P_{ed}}. \tag{34} \]

\[ S_{nt} = S_{np} + S_{ns}. \tag{35} \]

Based on (33) and (34) we can observe that the false alarm probability does not interfere with the throughput of the network.

\section{Numerical Results}

Let \( m \) be a relation among each network transmission probabilities [6][7]:

\[ m = \frac{\sigma_j}{\sigma_p}. \tag{36} \]

The curves presented in Figures 2, 3 and 4, show the throughput for the primary network, secondary network and total throughput. They are plotted considering: the original model introduced in [7] and the new model proposed here. We take into account imperfect sensing and different transmission power levels regarding the channel’s state decision. To compare the numerical results between the new model proposed and the original model, we considered the same parameters used in [7] i.e., \( Np = Ns = 30, \gamma = 10, \) and \( m \) alternating between 1, 2 and 5. For the new model, we set \( P_d = 0.8, \gamma_0 = 5, \) and \( \gamma_1 = 10. \) Observing Figures 2, 3 and 4, for the \( P_d \) value assumed in this paper, we can conclude that:

- The throughput in the primary network decreases when we consider the effects of the imperfect sensing.
- Regarding the secondary network, the throughput is not influenced by the imperfect sensing.

As future work, it is important to investigate the influence of the parameters \( P_d, \gamma_0, \gamma_1, Np, \) and \( Ns \) when imperfect sensing is considered.

Additionally, as future work, one can investigate the influence of channel coding techniques as a solution to increase the throughput of cognitive networks.

Also, as a future study, one can investigate adaptive modulation schemes and channel coding techniques as solutions to increase the throughput in the networks.
V. CONCLUSIONS

In this paper, we extended the analysis presented in [7], considering the effect of imperfect sensing in the throughput of a cognitive radio network that employs slotted aloha multiple access protocol in the primary and secondary networks. Also, we consider different transmission power levels in the secondary network as a function of the decision in this network about the state of the channel.

We observed that imperfect sensing and different transmission power levels reduce the throughput in primary network and have no influence in the secondary network.

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