Physarum Syllogistic L-Systems

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Abstract—One of the best media for studying natural computing is presented by the behavior of Physarum Polycyphalum plasmodia. Plasmodium has active zones of growing pseudopodia and these zones interact concurrently and in a parallel manner. This behavior can be stimulated by attractants and repellents. In the paper, different syllogistic systems are proposed for simulating the plasmodium’s behavior. While Aristotelian syllogistic may describe concrete directions of Physarum spatial expansions, pragmatic syllogistic proposed in this paper may describe Physarum simultaneous propagations in all directions. It is a more suitable system for applying syllogistic models in designing logic gates in plasmodia.

Keywords—plasmodium; L-system; Aristotelian syllogistics; non-Aristotelian syllogistics.

I. INTRODUCTION

There are many approaches to biological computing as a kind of unconventional computing; one of them is presented by systems invented by Aristid Lindenmayer [4]. They are called L-systems and allow us to simulate the growth of plants by formal grammars [6][13]. In the project [2], we are going to develop another approach to biological computing, assuming a massive parallelism of biological behavior. In this paper, we will show that we can implement two syllogistics in the biological behavior: the Aristotelian syllogistic [5] and a non-Aristotelian syllogistic constructed in [10][12]. The first is implementable within standard trees of appropriate L-systems. The second is massive-parallel and contain cycles and, therefore, can be implementable just within non-standard trees of some rigorous extensions of L-systems. This means that Physarum Polycyphalum, the medium of computations, which we have studied in the project, embodies complex extensions of L-systems.

Let us recall that Physarum Polycyphalum is a one-cell organism that behaves according to different stimuli and can be considered the basic medium of simple actions that are intelligent in the human meaning [1][2][7][8][9]. It behaves by plasmodia which can have the form either waves or protoplasmic tubes (arches). Hence, it is a system that is being spatially extended, as well as standard L-systems. This extension can be described as an extension of L-system called Physarum L-system (Section 2). Within this system we can implement (i) Aristotelian syllogistic in the Physarum media (Section 4), as well as non-Aristotelian syllogistic defined in [10][12] (Section 5).

In our project [2], we obtained a basis of new object-oriented programming language for Physarum polycyphalum computing [11]. Within this language we are going to check possibilities of practical implementations of storage modification machines on plasmodia and their applications to behavioral science such as behavioral economics and game theory. The point is that experiments with plasmodia may show fundamental properties of any intelligent behavior. The language, proposed by us, can be used for developing programs for Physarum Polycyphalum by the spatial configuration of stationary nodes. Some preliminary results of computational models on plasmodia are obtained in [1]. In this paper, we consider possibilities to implement syllogistic models as logic gates for Physarum Polycyphalum, which can be programmable within our language. In Section 2, we define Physarum L-systems. In Section 3, we consider their particular case presented by Aristotelian trees. In Section 4, we show how we can implement Aristotelian syllogistic in the Physarum behavior. In Section 5, we show how we can implement non-Aristotelian syllogistic defined in [10][12].

II. PHYSARUM L-SYSTEM

The behavior of Physarum plasmodia can be stimulated by attractants and repellents. We have the following entities which can be used in programming plasmodia:

- The set of active zones of Physarum \( \{V_1, V_2, \ldots\} \), from which any behavior begin to carry out.
- The set of attractants \( \{A_1, A_2, \ldots\} \): they are sources of nutrients, on which the plasmodium feeds, or pheromones which chemically attract the plasmodium. Any attractant is characterized by its position and intensity.
- The set of repellents \( \{R_1, R_2, \ldots\} \). Plasmodium of Physarum avoids light and some thermo- and salt-based conditions. Thus, domains of high illumination (or high grade of salt) are repellents such that each repellent is characterized by its position and intensity, or force of repelling.
- The set of protoplasmic tubes \( \{T_1, T_2, \ldots\} \). Typically, plasmodium spans sources of nutrients with protoplasmic tubes/veins. The plasmodium builds a planar graph, where nodes are sources of nutrients or pheromones, e.g., oat flakes, and edges are protoplasmic tubes.

Plasmodia grow from active zones. At these active zones, according to Adamatzky's experiments [2][3], the following
three basic operations stimulated by nutrients (attractants) and some other conditions can be observed: fusion, multiplication, and direction operations (see Fig. 1):

Figure 1. The stimulation of the following operations in Physarum automata: (a) fusion, (b) multiplication, and (c) direction, where \( A_1, A_2, A_3 \) are active zones, \( N_1, N_2, N_3 \) are attractants, \( \alpha \) is a protoplasmic tube, \( R \) is a repellent.

(1) The fusion, denoted \( \text{Fuse} \), means that two active zones \( A_1 \) and \( A_2 \) either produce new active zone \( A_3 \) (i.e., there is a collision of the active zones) or just a protoplasmic tube \( \alpha \): \( \text{Fuse}(A_1, A_2) = A_3 \) or \( \text{Fuse}(A_1, A_2) = \alpha \).

(2) The multiplication, \( \text{Mult} \), means that the active zone \( A_1 \) splits into two independent active zones \( A_2 \) and \( A_3 \), propagating along their own trajectories: \( \text{Mult}(A_1) = \{ A_2, A_3 \} \) or \( \text{Mult}(\alpha) = \{ A_2, A_3 \} \).

(3) The direction, \( \text{Direct} \), means that the active zone \( A \) is not translated to a source of nutrients but to a domain of an active space with certain initial velocity vector \( v \): \( \text{Direct}(A, v) \).

These operations, \( \text{Fuse, Mult, Direct} \), can be determined by the attractants \( \{ A_1, A_2, \ldots \} \) and repellents \( \{ R_1, R_2, \ldots \} \).

On the basis of active zones \( \{ V_1, V_2, \ldots \} \), attractants \( \{ A_1, A_2, \ldots \} \), repellents \( \{ R_1, R_2, \ldots \} \), and protoplasmic tubes \( \{ T_1, T_2, \ldots \} \), we can define a Physarum \( L \)-system. Let us remember that an \( L \)-system consists of (i) an alphabet of symbols that can be used to make strings, (ii) a collection of production rules that expand each symbol into some larger or shorter string of symbols, and (iii) an initial string from which we move. These systems were introduced by Lindenmayer [4][6][13] to describe and simulate the behavior of plant cells.

The Physarum \( L \)-system is defined as follows: \( G = \langle G, \omega, Q \rangle \), where (i) \( G \) (the alphabet) is a set of symbols containing elements that can be replaced (variables), namely they are active zones \( \{ V_1, V_2, \ldots \} \), which can be propagated towards attractants \( \{ A_1, A_2, \ldots \} \) by protoplasmic tubes and avoid repellents \( \{ R_1, R_2, \ldots \} \), i.e., \( G = \{ V_1, V_2, \ldots \} \cup \{ A_1, A_2, \ldots \} \cup \{ R_1, R_2, \ldots \} \); (ii) \( \omega \) (start, axiom or initiator) is a string of symbols from \( G \) defining the initial state of the system, i.e., \( \omega \) always belongs to \( \{ V_1, V_2, \ldots \} \); (iii) \( Q \) is a set of production rules or productions defining the way variables can be replaced with combinations of constants and other variables, i.e., production rules show a propagation of active zones by protoplasmic tubes towards attractants with avoiding repellents.

Let \( A, B, C \) are called primary strings, their meanings run over symbols \( V_1, V_2, \ldots, A_1, A_2, \ldots \). Production rules allow us to build composite strings from primary strings. So, a production \( A \rightarrow_0 B \) consists of two strings, the predecessor \( A \) and the successor \( B \). Some basic cases of productions are as follows: (i) the fusion, denoted \( AB \rightarrow_0 C \), means that two active zones \( A \) and \( B \) produce new active zone \( C \) at the place of an attractant denoted by \( C \); (ii) the multiplication, \( A \rightarrow_0 BC \), means that the active zone \( A \) splits into two independent active zones \( B \) and \( C \) propagating along their own trajectories towards two different attractants denoted then by \( B \) and \( C \); (iii) the direction, \( A \rightarrow_0 B \), means that the active zone \( A \) is translated to a source of nutrients \( B \).

\( L \)-systems can generate infinite data structure. Therefore it is better to define some production rules, denoted by \( A \rightarrow B \), recursively like that: \( A \rightarrow BA \), producing an infinite sequence \( BABABABA \ldots \) from \( A \), or \( A \rightarrow BCA \), producing an infinite sequence \( BCABCABCABC \ldots \) from \( A \). In the Physarum \( L \)-system, the rule \( A \rightarrow BA \) means that we will fulfill the direction, \( A \rightarrow_0 B \), infinitely many time, the rule \( A \rightarrow BCA \) means that we will fulfill the multiplication, \( A \rightarrow_0 BC \), infinitely many time. Let us consider an example of recursive production rules. Let \( G = \{ A, B \} \) and let us start with the string \( A \). Assume \( (A \rightarrow BA) \) and \( (B \rightarrow B) \). Thus, we obtain the following strings:

<table>
<thead>
<tr>
<th>Generation n</th>
<th>String</th>
</tr>
</thead>
<tbody>
<tr>
<td>n = 0</td>
<td>A</td>
</tr>
<tr>
<td>n = 1</td>
<td>BA</td>
</tr>
<tr>
<td>n = 2</td>
<td>BBA</td>
</tr>
<tr>
<td>n = 3</td>
<td>BBBA</td>
</tr>
<tr>
<td>n = 4</td>
<td>BBBBBA</td>
</tr>
<tr>
<td>n = 5</td>
<td>BBBBBA</td>
</tr>
</tbody>
</table>
In an appropriate *Physarum* L-system, these generations are represented as an infinite tree by permanent additions new attractants before the plasmodium propagation. In other words, we obtain the binary tree labeled with *s* and *t*, and whose interior nodes are either one unary node labeled with *B* or one binary node labeled with *A* (Fig. 2).

![Figure 2. Example of labels for binary trees.](image)

To sum up, we obtain the infinite binary tree of Fig. 3.

![Figure 3. Example of infinite binary tree.](image)

Let \( \text{Tr} \) be the set of trees that we have been defined. Then our definition introduces a coalgebra \([14]\):

\[
\text{Tr} = \{x, y, \ldots\} \cup (\{u_1, u_2, \ldots\} \times \text{Tr}) \cup (\{b_1, b_2, \ldots\} \times \text{Tr} \times \text{Tr}).
\]

Thus, within L-systems, we can obtain complex structures including infinite structures defined coalgebraically. In some cases, it is better to deal with infinite structures (infinite trees), assuming that all strings are finite.

### III. Aristotelian Trees

Let us consider Aristotelian syllogistic trees, which can be large, but their strings are only of the length 1 or 2. An *Aristotelian syllogistic tree* is labeled with \(x, y, \ldots\), its interior nodes are \(n\)-ary nodes labeled with \(b_1, b_2, \ldots\), and it is defined as follows: (1) the variables \(x, y, \ldots\) are Aristotelian syllogistic trees whose single descendants are underlying things (\*hypokeimenon, ὑποκείμενον*) such that for each \(x, y, \ldots\), parents are supremums of descendants (notice that all underlying things are mutually disjoint); (2) if \(t_1, t_2, \ldots\) are Aristotelian syllogistic trees such that their tops are concepts which are mutually disjoint and their supremum is \(b_1 \in \{b_1, b_2, \ldots\}\), then adding a single node labeled with \(b_1\) as a new root with \(t_1, t_2, \ldots\) as its only subtrees gives an Aristotelian syllogistic tree; (3) an Aristotelian syllogistic tree is finite.

The idea of hypokeimenon allowed Aristotle to build up finite trees. He starts with underlying things as primary descendants of trees in constructing syllogistic databases. Now, let us define syllogistic strings of the length 1 or 2 by means a *Physarum* L-system. Let each \(b_i \in \{b_1, b_2, \ldots\}\) be presented by an appropriate attractant and underlying things by initial active zones of *Physarum*. So, first trees \(x, y, \ldots\), whose single descendants are underlying things, are obtained by fusion or direction. Their supremums are denoted by attractants which were occupied by the first plasmodium propagation. These trees are considered subtrees for the next plasmodium propagation by fusion or direction. At the end, we can obtain just one supremum combining all subtrees. Let \(a_1, a_2, a_3, \ldots\) be underlying things. Then they are initial strings, i.e., they can be identified with active zones of plasmodia. Their meanings are as follows: “there exists \(a_1\)”, “there exists \(a_2\)”, “there exists \(a_3\)”, … Assume that in the tree structure the supremum of \(a_1\) and \(a_2\) is \(b_1\), the supremum of \(a_2\) and \(a_3\) is \(b_2\), … These supremums are fusions of plasmodia. Then, we have the strings \(a_1b_1, a_2b_2, a_2b_3, \ldots\). Their meanings are as follows: “\(a_1\) is \(b_2\)”, “\(a_2\) is \(b_3\)”, “\(a_2\) is \(b_3\)”, “\(a_3\) is \(b_2\)”, … Further, let \(b_0\) be a supremum for \(b_1\) and \(b_2\). It denotes an attractant that was occupied by the plasmodium at the third step of the propagation. Our new strings are as follows: “\(b_0\) is \(b_1\)”, “\(b_0\) is \(b_2\)”, etc. Now we can appeal also to the following new production rule: if “\(x\) is \(y\)” and “\(y\) is \(z\)”, then “\(x\) is \(z\)”. Thus, we have the strings: \(a_1b_1b_0, a_2b_2b_0, a_3b_3b_0, \ldots\)

### IV. Aristotelian Syllogistic

The symbolic system of Aristotelian syllogistic can be implemented in the behavior of *Physarum* plasmodium. Let us design cells of *Physarum* syllogistic which will designate classes of terms. We can suppose that cells can possess different topological properties. This depends on intensity of chemo-attractants and chemo-repellents. The intensity entails the natural or geographical neighborhood of the set’s elements in accordance with the spreading of attractants or repellents. As a result, we obtain Voronoi cells \([3][11]\). Let us define what they are mathematically. Let \(P\) be a nonempty

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\[ \text{Tr} = \{x, y, \ldots\} \cup (\{u_1, u_2, \ldots\} \times \text{Tr}) \cup (\{b_1, b_2, \ldots\} \times \text{Tr} \times \text{Tr}). \]

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finite set of planar points and \(|P| = n\). For points \(p = (p_1, p_2)\) and \(x = (x_1, x_2)\), let

\[
d(p, x) = \sqrt{(p_1 - x_1)^2 + (p_2 - x_2)^2}
\]
denote their Euclidean distance. A planar Voronoi diagram of the set \(P\) is a partition of the plane into cells, such that for any element of \(P\), a cell corresponding to a unique point \(p\) contains all those points of the plane which are closer to \(p\) in respect to the distance \(d\) than to any other node of \(P\). A unique region

\[
\text{vor}(p) = \bigcap_{m \in P, m \neq p} \{z \in \mathbb{R}^2 : d(p, z) < d(m, z)\}
\]

assigned to the point \(p\) is called a Voronoi cell of the point \(p\). Within one Voronoi cell, a reagent has a full power to attract or repel the plasmodium. The distance \(d\) is defined by intensity of reagent spreading like in other chemical reactions simulated by Voronoi diagrams. A reagent attracts or repels the plasmodium and the distance on that it is possible corresponds to the elements of a given planar set \(P\). When two spreading wave fronts of two reagents meet, this means that on the board of meeting the plasmodium cannot choose its one further direction and splits (see Fig. 5). Within the same Voronoi cell, two active zones will fuse.

Now, we can obtain coordinates \((x, y)\) for each Voronoi center. The number \((x, y)\) can be assigned to each concept as its character. If a Voronoi center with the coordinates \((x_0, y_0)\) is presented by an attractant that is activated and occupied by the plasmodium, this means that in an appropriate Physarum syllogistic model there exists a string \(a\) with the coordinates \((x_0, y_0)\). This string has the meaning “\(a\) exists”. If a Voronoi center with the coordinates \((x_0, y_0)\) is presented by a repellent that is activated and avoided by the plasmodium, this means that in an appropriate Physarum syllogistic model there exists a string \([a]\) with the coordinates \((x_0, y_0)\). This string has the meaning “\(a\) does not exist”. If two neighbor Voronoi cells with the coordinates \((x_0, y_0)\) of centers contain activated attractants which are occupied by the plasmodium and between both centers there are protoplasmic tubes, then in an appropriate Physarum syllogistic model there exists a string \(ab\) and a string \([ba]\) where \(a\) has the coordinates \((x_0, y_0)\) and \([b]\) has the coordinates \((x_0, y_0)\). The meaning of those strings is the same and it is as follows: “\(ab\) exist”, “\(ba\) exist”, “some \(a\) is \(b\)”, “some \(b\) is \(a\)”. If one neighbor Voronoi cell with the coordinates \((x_0, y_0)\) of its center contains an activated attractant which is occupied by the plasmodium and another neighbor Voronoi cell with the coordinates \((x_0, y_0)\) of its centre contains an activated repellent which is avoided by the plasmodium, then in an appropriate Physarum L-system there exists a string \([a]\) and a string \([b]\) where \(a\) has the character \((x_0, y_0)\) and \([b]\) has the character \((x_0, y_0)\). The meaning of those strings is the same and it is as follows: “\(ab\) do not exist, but \(a\) exists without \(b\)”, “there exists \(a\) and no \(a\) is \(b\)”, “no \(b\) is \(a\) and there exists \(a\)”, “\(a\) exists and \(b\) does not exist”.

If two neighbor Voronoi cells with the coordinates \((x_0, y_0)\) and \((x_0, y_0)\) of their centers contain activated repellents which are avoided by the plasmodium, then in an appropriate Physarum L-system there exists a string \([ab]\) and a string \([ba]\) where \([a]\) has the character \((x_0, y_0)\) and \([b]\) has the character \((x_0, y_0)\). The meaning of those strings is the same and it is as follows: “\(ab\) do not exist together”, “there are no \(a\) and there are no \(b\)”, “no \(b\) is \(a\)”, “no \(a\) is \(b\)”. Hence, existence propositions of Aristotlean syllogistic are spatially implemented in Physarum L-systems.

Let \(y\) denote all neighbor Voronoi cells for \(x\) which differ from \(y\). Now, let us consider a complex string \(xy\&x[\prime]\). The sign & means that we have strings \(xy\) and \(x[\prime]\) simultaneously and they are considered the one complex string. The meaning of the string \(xy\&x[\prime]\) is a universal affirmative proposition “all \(x\) are \(y\)”.

As a consequence, each Physarum L-system is considered a discourse universe verifying some propositions of Aristotelian syllogistic.

V. NON-ARISTOTELIAN SYLLOGISTIC

Let us propose now the syllogistic system formalizing performative propositions of the form ‘\(A\) is \(P\)’ (see [10][12]), i.e., propositions with context-based meanings. This system is said to be synthetic (pragmatic) syllogistic, while we are assuming that Aristotelian syllogistic is analytic (informative). The basic logical connectives of pragmatic syllogistic are as follows: \(a\) (‘every + noun + is + adjective’), \(i\) (‘some + noun + is + adjective’), \(e\) (‘no + noun + is + adjective’) and \(o\) (‘some + noun + is not + adjective’) that are defined in the following way:

\[
\text{SaP} := \exists A \ (A \text{ is } S) \land (\forall A (A \text{ is } S \land A \text{ is } P)). \tag{1}
\]
\[
\text{SiP} := \forall A (\neg (A \text{ is } S) \land \neg (A \text{ is } P)). \tag{2}
\]
\[
\text{SoP} := \neg (\exists A (A \text{ is } S) \land (\forall A (A \text{ is } S \land A \text{ is } P))), \text{ i.e.,} \tag{3}
\]
\[
\forall A \neg (A \text{ is } S) \lor (\exists A (A \text{ is } S) \lor \neg (A \text{ is } P)).
\]
\[
\text{SeP} := \neg (\forall A (\neg (A \text{ is } S) \land \neg (A \text{ is } P))), \text{ i.e.,} \tag{4}
\]
\[
\exists A (A \text{ is } S \lor A \text{ is } P).
\]

Now, let us formulate axioms of pragmatic syllogistic:

\[
\text{SaP} \Rightarrow \text{SeP}. \tag{5}
\]
\[
\text{SaP} \Rightarrow \text{PaS}. \tag{6}
\]
\[
\text{SiP} \Rightarrow \text{PiS}. \tag{7}
\]
\[
\text{SaM} \Rightarrow \text{SeP}. \tag{8}
\]
\[
\text{MaP} \Rightarrow \text{SeP}. \tag{9}
\]
\[(MaP \land SaM) \Rightarrow SaP \quad (10)\]
\[(MrP \land SiM) \Rightarrow SiP. \quad (11)\]

In pragmatic syllogistic, we have a novel square of opposition that we call the synthetic square of opposition (see Fig. 4), where the following theorems are inferred from (1) – (11):

\[
\begin{array}{|c|c|}
\hline
SaP & SiP \\
\hline
SeP & SoP \\
\hline
\end{array}
\]

Figure 4. The synthetic square of opposition.

\[
\begin{align*}
SaP \Rightarrow \neg(SeP), & \quad \neg(SeP) \Rightarrow SaP, \\
SiP \Rightarrow \neg(SeP), & \quad \neg(SeP) \Rightarrow SiP, \\
SeP \Rightarrow \neg(SiP), & \quad \neg(SiP) \Rightarrow SeP, \\
SoP \Rightarrow \neg(SaP), & \quad \neg(SaP) \Rightarrow SoP, \\
\neg(SeP) \Rightarrow SoP, & \quad \neg(SeP) \Rightarrow SeP, \\
\neg(SeP) \Rightarrow SeP, & \quad \neg(SeP) \Rightarrow SiP, \\
\neg(SeP) \Rightarrow SeP, & \quad \neg(SeP) \Rightarrow SiP.
\end{align*}
\]

For more details, see [10][12].

In the implementations within Physarum L-systems, the four basic syllogistic propositions of non-Aristotelian syllogistic defined above are understood as follows:

- 'All S are P': there is a string AS and for any A which is a neighbor for S and P, there are strings AS and AP. This means that we have a massive-parallell-occupation of region, where the cells S and P are located.

- 'Some S are P': for any A which is a neighbor for S and P, there are no strings AS and AP. This means that the plasmodium cannot reach S from P or P from S immediately.

- 'No S are P': there exists A which is a neighbor for S and P such that there is a string AS or there is a string AP. This means that the plasmodium occupies S or P, but surely not the whole region, where the cells S and P are located.

- 'Some S are not P': for any A which is a neighbor for S and P there is no string AS or there exist A which is a neighbor for S and P such that there is no string AS or there is no string AP. This means that the plasmodium does not occupy S or there is a neighbor cell which is not connected with S or P by a protoplasmic tube.

Thus, the pragmatic syllogistic allows us to study different zones containing attractants for Physarum if they are connected by protoplasmic tubes homogenously.

VI. CONCLUSION

We constructed two syllogistic versions of storage modification machine in Physarum Polycephalum: Aristotelian syllogistic and pragmatic syllogistic (non-Aristotelian syllogistic of Section 5). While Aristotelian syllogistic may describe concrete directions of Physarum spatial expansions, pragmatic syllogistic may describe Physarum simultaneous propagations in all directions. Therefore, while for the implementation of Aristotelian syllogistic we need repellents to avoid some possibilities in the Physarum propagations, for the implementation of pragmatic syllogistic we do not need them. Hence, the second syllogistic can simulate massive-parallel behaviors, including different form of propagations such as processes of public opinion formation.

In our opinion, the general purpose of Physarum computing covers many behavioural sciences, because the slime mould’s behaviour can be considered the simplest natural intelligent behaviour. Thus, our results may have an impact on computational models in behavioural sciences in general.

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Figure 5. The Voronoi diagram for Physarum, where different attractants have different intensity and power.