Complexity Reduction in Graphs: A User Centric Approach to Graph Exploration

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***Abstract*—Human exploration of large graph structures becomes increasingly difficult with growing graph sizes. A visual representation of such large graphs, for example, social networks and citational networks, has to find a trade-off between showing details in a magnified view and the verall graph structure. Displaying these both aspects at the same time results in an overloaded visualization that is inaccessible for human users. In this paper, we present a new approach to address this issue by combining and extending graph-theoretic properties with community detection algorithms. Our approach is semi-automated and non-destructive. The aim is to retain core properties of the graph while–at the same time–hiding less important side information from the human user. We analyze the results yielded by applying our approach to large real-world network data sets, revealing a massive reduction of displayed nodes and links.**

Keywords*—*Complexity reduction; graph visualisation; big data exploration; graph metrics; community detection.

I. Introduction

Computing on data sets is becoming increasingly easier. With the rise of *big data* and powerful computing devices, the collection and processing of large data sets has become a common thing. Research as well as industry profits a lot from this capability to reveal new insights and connections that can only be detected by analyzing large amounts of data.

However, Moore's Law [1] does not apply to the ability of human users to understand and explore such big data sets. Making large data sets, for example, networks, accessible to human users is difficult and becomes increasingly more difficult with ever-increasing data sets. Human-centric data analysis techniques usually employ visualization of the data sets that the human user intends to work with. Visualizing real-world networks as connected nodes quickly results in an inaccessible chaos due to large amount of information to be shown. See Figure 1 for a visualization of a part of the Facebook social network [2]. In order to comprehend relations within such a network, a user needs to magnify the visualization a lot. This magnification implies a loss of overview, so that a user might be able to understand specific relations but loses track of the overall structure of the data set.

This issue of simultaneous visualization of details and overall structure complicates the process of exploring data sets when the outcome of the exploration is not pre-defined. Human users are often involved in exploring data sets when the aim is to find new insights, connections and cross-references. Predefined and well-specified analyses can be processed automatically without human involvement, and thus, no need for a visualization. However, exploratory data analysis usually does not allow for automated pre-filtering of data sets to obtain a human-centric, *decluttered* view on the data.

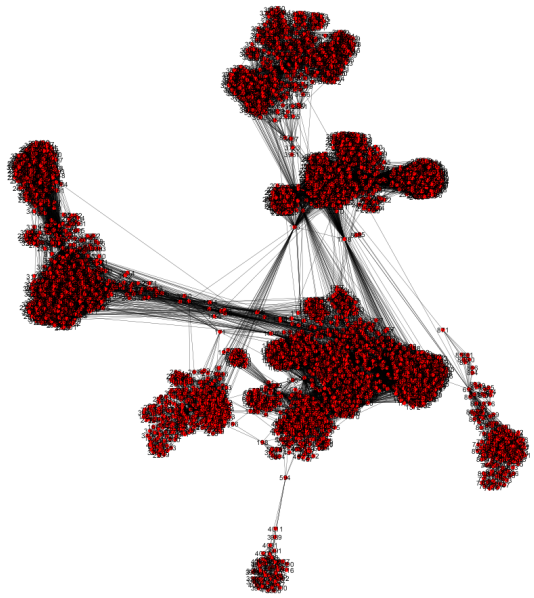

Fig. 1. User relations in a social network (data taken from Facebook).

The technique we present in this article combines and extends graph-theoretic properties with community detection. This technique aims at reducing the visual complexity of network data sets in order to render these more accessible to human users. For this reduction of visual complexity, we propose a semi-automated, non-destructive approach to identify core insights, while side-information is hidden.

Our results indicate a massive reduction of displayed nodes and links in every iteration of our proposed approach. Therefore, only one to three iterations effectively reduce graphs as the one shown in Figure 1 to a representation that is easy to comprehend for human users (see Figure 3 for the results of only three iterations).

The remaining paper is structured as follows: in Section II, we discuss relevant related work on complexity reduction in graphs; in Section III, we introduce fundamental background information to our contribution by discussing used graph theory measures and community detection mechanisms. We continue in Section IV with the introduction of our technique

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of reduce complexity and we present our findings in Section V. We conclude this paper in Section VI with a summary and future research directions.

II. Related Work

In this section, we discuss related work with respect to the field of complexity reduction in graphs.

Kimelman et al. [3] proposed techniques like ghosting, hiding and grouping of edges. The nodes and edges of the graph were removed based on various techniques like weights of edges, labels of nodes, etc., but these techniques were concerned with dynamic graphs. Differently, Holten et al. [4] reduced only the visual cluttering of edges by bundling them.

Fisheye techniques [5], [6] tend to concentrate only on the interesting regions of the user. The zooming feature in such techniques is only responsible for making a very small region of a graph appear larger. They do not remove any nodes or edges. So, the overall graph content remains the same. Fisheye views that retain structure are introduced by Furnas [7]. Abello et al. [8] introduced hierarchical clustering and depiction of a treemap in addition to a compound fisheye view technique but never concentrated on reducing the overall size of a network.

Various approaches towards creating communities in large graphs are presented in [9]–[11]. These techniques provide a significant level of understanding of the kind of nodes and their properties in large networks but never used the same to reduce the overall content in a large network and provide a simpler view.

Sundararajan et al. [12] introduced Rectangular Partitioning and Voronoi Partitioning techniques. The former involved partitioning the area of display into four quadrants while the latter involved the partition area being closer to the concerned node. This only reduces the distortion in the graphs.

Batagelj et al. [13] took a mathematical approach through the usage of matrix. Large graphs were reduced to *k-cores*. Later, the graphs are represented as an adjacency matrix or a contextual matrix based on their size. But when the graphs grows really large, managing the matrix becomes a humongous task.

All before mentioned techniques reduce complexity based on a global perspective, disregarding the current interests of a user. Thus, these techniques may maintain and highlight information that is not relevant the user's current situation while rejecting important information from the user's slant.

III. Background

In this section, we introduce our terminology that is used to model data sets using graphs. After this terminology description, we introduce the foundation of various graph-theoretic properties and discuss them in the context of our complexity reduction. Additionally, we introduce those community detection algorithms that we use to reduce the visual complexity.

A. Terminology: Graph-based Data Representation

Many data sets can be represented by a Graph $G = (V, E)$ that is formed by a set of *vertices* $v_i \in V$, which represents the pieces of data. Relations and *connections* between these information are represented by *edges* $E \subseteq V \times V$, i.e., connections are represented by pairs of vertices (v_x, v_y). Considering our exemplary application scenario of a social network, each vertex represents a user and edges represent the connections between users. Another example is the model of (research) citations as citation graph where publications are represented as vertices, and a citation is represented by a directed edge.

A graph G can be *directed* or *undirected*. In a directed graph, an edge e_k can exist between a vertex v_i and v_j while the other direction (e_l: v_j to v_i) may or may not exist, independent of the existence of e_k. In an undirected graph, the existence of before-mentioned edge e_k also implies the existence of e_l. A social network like Facebook applies undirected connections, thus, if user Alice is connected to user Bob, Bob is also connected to Alice (Alice and Bob are "friends"); in opposite to that, Twitter applies directed connections. Thus, Alice may be connected to Bob (Alice "follows" Bob), but Bob may not be connected to Alice.

The number of connections of a vertex v_i is noted as the *degree* d_{v_i} of v_i. In case of a directed graph, the degree of a vertex has to be specified for outgoing connections: the *out-degree* and for incoming connections: the *in-degree*.

A connection between any two vertices is called a path p. A path between adjacent vertices has the length 1; yet, a path may include intermediate vertices to connect them. A path p is represented by an sequence of edges $p = (e_0, e_1, ..., e_n)$. The *shortest path* between two vertices is the path with least edges.

B. Selection Criteria Values

Vertex selection is one of the fundamental steps in reducing the complexity of a graph, i.e., the selection of an appropriate subgraph. Selecting a set of vertices based on specific parameters helps in creating a subgraph that retains its inherent properties and reflects the user's interests. The vertices are identified based on *selection criteria values* (SCV). A SCV acts as the basis of vertex selection in our framework. For a vertex to be selected as part of a graph, its SCV must be greater than the SCV of the user's interest. Here, we express the user's interest by selecting a start vertex whose SCV is compared with every other vertex in the network while reducing the graph. The SCVs include graph-theoretic properties and centrality measures but are not limited to just these presented measures. We focus this paper on the usage of importance, connectivity, and distance measures as these appeared most promising in our literature research. The importance measures are PageRank and Betweenness Centrality; the connectivity measures are Clustering Coefficient and Degree Centrality; the distance measures is Closeness Centrality. In

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the following sections, we discuss every SCV to understand their significance to our idea of reducing visual complexity.

1) Closeness Centrality: Closeness Centrality determines the closeness of vertices in a network, i.e., it determines the distance of vertices in a graph. The vertices that have high closeness centrality values are considered to be closer to other vertices in the graph. For example, in a network where information flows from one vertex to another, transmission of information takes place quickly due to their high closeness centrality values.

According to Freeman [14], closeness centrality of a vertex is defined as:

"The sum of graph-theoretic distances from all other vertices, where the distance from a vertex to another is defined as the length of the shortest path from one to the other."

Closeness is generally attributed to the quickness in flow of information in the network. The quicker the information arrives at or departs from a vertex, the closer is the destination or the source vertex respectively [15].

In a social network scenario, this could mean when an information is shared between two individuals, the chances of such information propagating in the network is higher with those who are immediate friends or neighbors. For a given vertex p_i, closeness centrality [16] is calculated using (1), where V denotes the total number of vertices in the graph, i and k are integers where $i \neq k$, $d(p_i, p_K)$ denotes the number of edges in the shortest path between p_i and p_k.

$$C_C(p_i) = \frac{V - 1}{\sum_{k=1}^{V} d(p_i, p_k)} \tag{1}$$

2) Betweenness Centrality: Betweenness Centrality depicts influence or powerfulness of a vertex in a network. The vertices that have high betweenness centrality values are *highly influential* as they act as bridges for interactions between several pairs of vertices. In a social network scenario, where individuals form friendships with individuals who in turn have a large group of friends tend to have an influential clout due to their connections with such individuals.

According to Freeman [14], the betweenness centrality of a vertex is defined as:

"The share of times that a vertex i needs a vertex k (whose centrality is being measured) in order to reach a vertex j via the shortest path."

Betweenness centrality [16] can be calculated using (2), where g_{jk} denotes the total number of shortest paths between p_i and p_k and $g_{jk}(p_i)$ denotes the number of shortest paths containing p_i.

$$C_B(p_i) = \sum_{i \neq j \neq k \in V} \frac{g_{jk}(p_i)}{g_{jk}} \tag{2}$$

3) PageRank: PageRank [17] makes use of the idea that was conceived to compute citations to a web page and also add its own Midas Touch by ensuring that all web pages are not treated equally but by applying normalization to the web page depending on the total number of links present on that page. In this way, PageRank provides a rank to every web page.

A transition matrix is created based on the transfer of importance from one to another. Initially, we apply uniform distribution based on the initial grade structure. Then, depending upon the incoming links we re-calculate the PageRank value [18].

The vertices that have high PageRank values are considered more important due to their high incoming links. For example, in the world wide web network, a web page that is referenced in many other web pages will have a high PageRank value. In a social network scenario, individuals with higher PageRank are generally highly powerful due to a lot of connections whose betweenness centrality value is also high. Since the concept of PageRank is based on Betweenness Centrality, the PageRank value of a vertex depends on its betweenness centrality value and its neighbors.

4) Degree Centrality: Degree Centrality portrays the level of connectivity of a vertex in a network. The degree centrality is computed as presented in (3) and given by the number of adjacent neighbors. The vertices that have high degree centrality values are more influential or important. For example, in a social network, a vertex with a high number of connections is powerful and highly visible when compared to others [10].

$$C_D(p_i) = deg(p_i) \tag{3}$$

5) Clustering Coefficient: Clustering Coefficient gives an indication of clusters in a network. The vertices that have high clustering coefficient values are more closely knit together. For example, in a social network, vertices with high clustering coefficients have a desire to form close bonds or friendships with their neighbors [19].

According to Zafarani et al. [20], the clustering coefficient can be defined as shown in (4).

$$CC = \frac{3 \times (Number\ of\ triangles)}{Number\ of\ Connected\ Triples\ of\ Vertices} \tag{4}$$

C. Community Detection Algorithms

Community detection allows to group vertices that share similar properties. The properties vary depending on the type of the used community detection algorithm (CDA). For example, Louvain algorithm optimizes communities with respect to maximizing their modularity while direct K-means community detection optimizes on the spread of communities.

We use the found communities to reduce complexity in larger steps, being able to remove larger portions of the graph in the first few complexity reduction cycles. CDAs form one of the core evaluation criteria in our implementation. The four

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algorithms used in our framework are Hierarchical Clustering, Original Louvain, Louvain with Multilevel Refinement, and Direct Clustering. In the following sections, we will discuss the CDAs that help us in understanding the way in which various clusters are formed.

1) Hierarchical Clustering: The hierarchical clustering method by Jain and Dubes [21] involves obtaining a series of partitions that are nested by transforming a proximity matrix. We utilize the complete-link clustering method in an agglomerative setting in our implementation. One of the main advantages of this method is the avoidance of chaining-effects. Also, the clusters are balanced and smaller when applying this method.

2) Original Louvain: The Original Louvain algorithm by Blondel et al. [22] aims to increase the graph property *modularity* [23] due to reassigning vertices from one community to another one [24]. A local moving heuristic is used to optimize the modularity graph property in iterative steps, i.e., each vertex evaluates if moving to a neighbor's community can increase the modularity and reassigns itself to the modularity maximizing community. After that, a reduced size graph is established, where the vertices are the previously established communities.

3) Louvain with Multilevel Refinement: Louvain with Multilevel Refinement algorithm was formulated and conceived by Rotta et al. [25]. The algorithm is similar to Original Louvain except for the fact that the local moving heuristic is applied twice. Louvain with Multilevel Refinement generates community assignments yielding higher modularity values compared to the Original Louvain.

4) Direct K-means: The direct K-means algorithm by Alsabti et al. [26] partitions a data set into k communities where k is fixed. First, k vertices are selected randomly, and the remaining vertices are assigned to the community of the closest of the k selected vertices. After assigning all vertices, the means of the communities are computed and assigned to the new k selected start nodes for the next iteration. This process is repeated until stabilization.

IV. Complexity Reduction Approach

In this section, we present our framework to reduce the data complexity in graphs.

A. Subjectivity of Visual Complexity

Visual complexity is a subjective measure. Every user has a different understanding of visual complexity of a data set, depending on previous experience, expert knowledge, and familiarity with the respective subject. Hence, the challenge of visual complexity reduction cannot be solved with an "one for all" approach.

To overcome this subjectivity challenge, we provide a round-based complexity reduction method that allows to reduce the visual complexity in succeeding iterations until the user is able to understand the data set well enough to collect the desired information.

```
1: function ReduceVisualComplexity(g, r, s, id, type_g, c_alg)
   g: Input graph
   r: Number of iterations
   s: Selection criteria
   id: Start node (picked by user or at random)
   type_g: Graph Type (directed or undirected)
   c_alg: Community detection algorithm
   g_reduced: Output graph with reduced complexity

2:     g_reduced ← new Graph();
3:     for i ← 1 to r do
                                   ▷ loc: List of Communities
4:         loc ← getCommunities(id, g, algo, g_type)
5:         for j ← 1 to sizeOf(loc) do
6:             community ← loc.get(j)
7:             for k ← 1 to sizeOf(community) do
8:                 v_k ← community[k]
9:                 if getSCV(v_k, s) > getSCV(id, s) then
                                       ▷ compute shortest path
10:                    sp ← getSP(v_k, id)
11:                    addToReducedGraph(sp, g_reduced)
                   ▷ break if applied w/ community detection
12:                if loc > 0 then
13:                    break loop
14:                end if
15:            end if
16:        end for
17:    end for
18:    i++
19:    end for
20:    return g_reduced
21: end function
```

Fig. 2. Iterative Reduction of Visual Complexity

B. Visual Complexity Reduction

Our method to reduce the visual complexity is a two-step process, which also allows to parallelize the computation to reduce computation time faced by the user. We give an algorithmic description in Algorithm 2. In the first step, we apply a community detection algorithm to group similar information. According to lines 3-6 of Algorithm 2, we compute the disjoint communities and proceed with each of them individually. At this point, we can easily parallelize the computation, having the communities being handled concurrently.

In the second step, we have to differentiate whether community detection was applied in the first place or not. If community detection is not applied, we calculate the SCV of each vertex and compare it with the SCV of the vertex representing the user's interest. When the SCV of the inspected vertex is higher than a threshold, which is in our case the SCV of the user's interest, the vertex is retained in the reduced graph. To preserve the relation and connection of the user's interest and inspected vertex, we retain not only the vertex but

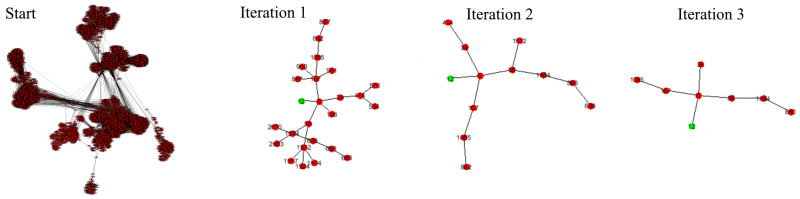

Fig. 3. Reduction of graphs by applying Algorithm 2 (first four iterations shown).

also the vertices on the shortest path. In our social network scenario, these vertices represent the chain of friends between two subjects and, thus, may be relevant to the user.

If community detection was applied in the first place, we only retain a single representative of each community and hide the remaining vertices of a community. This way, we can easily achieve a massive complexity reduction in only a few iterations. Yet, the user can select the representative of a community as next interest, restarting the process from this new perspective and gain new insights into the data set.

These two steps are then repeated until the user is able to sufficiently comprehend the graph. Our algorithmic description uses a predefined number of iterations, yet an application would delegate this decision to the user, who might request further iterations.

In Figure 3, we visualize the iterative complexity reduction process. On the left-hand side, the original graph, which fed into the visual complexity reduction mechanism, is visualized. In three steps, visualized towards the right-hand side, we reduce the visual complexity of the data set considering a randomly chosen vertex to reflect an otherwise user-selected interest. The user interest is marked by the green vertex with the ID 12 in Figure 3. The original data set, labeled "Start", is the Facebook data set already presented in the introduction in Figure 1.

V. Evaluation

In a simulation study, we evaluated the performance and effectiveness of our proposed technique to reduce the visual complexity of data sets. In the next section, we provide insights into our experimental setup and details of our simulation study. After that, we present and discuss our results for different network classes.

A. Experimental Setup

In this section, we describe and discuss our selection of data sets, followed by the description of our simulation setup.

a) Networks: To provide useful insights, we decided to perform our simulation based on samples of real-world networks. We imagine the usage of our technique in applications based on social networks (with respect to to social ties), in (scientific) libraries (with respect to to co-authorship), biological data (with respect to protein interactions), and computer networks (with respect to to the logical interconnections) as described in Table I. We restrict our simulation to the largest connected component of the data set, as accounting for importance and relationship of non-related information is out of scope of this paper.

TABLE I
Large Network Data Sets

Class	data set	Vertices	Connections
Social	Facebook (FB)	4,029	88,234
Library	Arxiv gr-qc (CA)	4,158	13,428
Biological	Vidal (VI)	3,133	6,726
	Moreno (MO)	1,870	2,277
Computer	p2p-Gnutella08 (P8)	6,301	20,777

As the representative for a social network, we decided to use a sample of Facebook's social graph [2]. This data set (FB) consists of 4,029 vertices and 88,234 connections, representing users of Facebook and their respective interconnections.

As the representatives for library-based and collaboration-based data sets, we selected a co-authorship data set [2] that reflects the research collaborations and co-authorships in the Arxiv gr-qc (General Relativity and Quantum Cosmology) area. This data set (CA) consists of 4,158 vertices and 13,428 connections in their largest connected component.

As the representatives of biological networks, we selected the protein interaction networks vidal [27] and moreno [28]. These data sets reflect interactions of proteins on a molecular level. The vidal data set (VI) consists of 3,133 vertices and 6,726 connections; the moreno data set (MO) consists of 1,870 vertices and 2,277 connections.

As the representative of computer networks, we selected a sample of the Gnutella network [2] that reflects the interconnections of users in the Gnutella network. The sample (P8) is sampled on August 8, 2002 (p2p-Gnutella08) and consists of 6,301 vertices and 20,777 connections.

b) Simulation Setup: We perform a reduction of visual complexity on each of the aforementioned data sets and calculate graph-theoretic properties that are also used as SCVs, namely closeness centrality (C_C), betweenness centrality (C_B), PageRank (PR), degree centrality (C_D), and clustering coefficient (CC).

We perform the complexity reduction on each of the data sets with each pairwise combination of selection critera values (SCV) and community detection algorithms (CDAs) been described in Sections III-B and III-C and compare and interpret

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TABLE II
SIMULATION DETAILS

Property	Value(s)
SCV	Closeness Centrality (C_C) Betweenness Centrality (C_B) PageRank (PR) Degree Centrality (C_D) Clustering Coefficient (CC)
CDA	Hierarchical Clustering (HC) Direct Clustering (DC) Original Louvain (OL) Louvain w/ multilevel refinement (LM)
Repetitions	15
Iterations	3
User Interest	V.getRandomVertex()

the results represented by the statistical mean values and number of removed vertices.

We need a user interest expressed as one "preselected" piece of information from the data set to apply our technique to reduce visual complexity. To account for this, we selected a random vertex as user interest and repeat this whole process 15-times.

In Table II, we summarize the details of our simulation study and used abbreviations.

B. Results

In this section, we present the results of our simulative study. The results are structured according to the class of networks as aforementioned. Detailed measurements are shown in Table III and IV.

a) Social Networks: Social networks, here the FB data set, exhibit a strongly connected core, the so-called *rich club*. These nodes are connectors between various clusters. This structural behavior leads to comparable short paths, i.e., the closeness centrality is comparably high, and core vertices exhibit a high betweenness and degree centrality. Moreover, due to the "clustering" of vertices, the clustering coefficient is also high (in the FB data set: 0.6055).

Our complexity reduction technique reduces up to 4,035 (99.9%) vertices in the three iterations. All combinations but one of the SCV and CDA combinations preserve the core of the data set which is indicated by the high C_B, C_C, and C_D values. These indicate that the retained information is representative for their clusters. The drop of PR and CC indicate that the preserved structure roughly follows a star-topology, which is also shown in Figure 3.

Using CC as only SCV leads to a different result where different information is retained, yet, this is expected. The core vertices have by definition a lower CC value and are, thus, not likely to be above the threshold. These vertices are connected to a multitude of different other vertices that are forming their own, smaller clusters.

b) Library/Collaboration Data Set: A collaboration, or library, data set yields a similar structure to a social network. Collaborators establish clusters and an inner core of highly influential people and publications. The gr-qc data set reveals a high CC (0.5296) and a strongly connected core. While the diameter of the data set suggests otherwise, the core of the data set still shows a high connectivity and tendency towards short paths. That is evident when comparing the diameter (17) and the diameter when only 90% of vertices have to be connected (0.9-percentile effective diameter: 7.6).

Our complexity reduction technique removes up to $4,196$ vertices in the three performed iterations. We can see that most combinations of SCV and CDA perform similarly, producing star-topology-like results retaining relevant information of the core of the data set as indicated by higher C_B and C_D. The drop of the CC supports the star-topology again.

However, using CC or C_D as SCV produces different results. Using one of these properties as SCV retains information revealing the significantly higher CC, C_D, and PR and lower C_D values. Thus, retaining representatives of smaller but better and tighter connected clusters.

c) Biological Data Set: Protein interaction networks reveal a different structure, the spread between average and maximum degree is between factor 23 and 30; this factor is by a magnitude smaller than in the previously discussed social networks. This drop in degree goes hand in hand with increased diameter (13–19) and a massive decline in the CC (0.035–0.055 compared to 0.6055 on the Facebook data set).

Our complexity reduction technique removes up to 3,125 (99.74%) vertices on the vidal protein interaction network and up to 1,860 (99.47%) vertices on the moreno protein interaction network. Our technique performs similarly on both data sets, and produces star topologies regardless of the used combination of SCV and CDA. Thus, the resulting C_B is comparably high, and the C_C is very low < 0.0001. The CC also drops to 0.01–0.0004 and endorses the star topology as the remaining vertices are hardly forming local clusters. The PR is similarly low. Yet, the CO-DC combination retains local clusters, which is indicated by higher C_C and CC values.

d) Computer Networks: Computer networks, in this case a sample from the Gnutella network, are resembling either AS-networks with social-like structures on a large scale or, if consisting of only "a few" computers (compared to the whole population), random networks. As the Gnutella network was comprised of only a small subset of all Internet users, the Gnutella sample is akin to a random network. Yet, the degree distribution reveals on the base data set exponentially distributed degrees. Nonetheless, the PageRanks reveal the lack of a highly connected inner core; this core is essential to a "social" network.

Our complexity reduction techniques removed up to 6,299 (99.97%) vertices. The consistently high C_B shows that some well-connected vertices are retained, but the retained vertices are distant from each other, thus, resulting in very low C_C values; only PR used as SCV preserves closer vertices. Using C_C, CC, or PR as SCV results in retaining local clusters in the results while the other SCVs do not keep clusters in their

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reduced data sets – this is visible by the high CC and PR values for these SCVs.

C. Destructive vs. Non-Destructive Complexity Reduction

The complexity reduction can be performed in two variations: a destructive and a non-destructive one.

First, the execution can be *destructive* by removing vertices that are not matching the user's interest, i.e., removing vertices when their SCV is lower than the threshold and if they are not selected to be a representative. During execution, the data set will shrink and, thus, free memory and reduce the computational complexity of the calculation of SCVs with each performed complexity reduction iteration. However, the user cannot revert the reduction process to earlier stages of complexity reduction.

Second, the execution can be *non-destructive* by, e.g., concealing "removed" vertices only without deleting them from the actual graph. This variation allows moving back and forth between different stages of complexity reduction. Moreover, this also allows a user to adapt her interest, i.e., she can use gained knowledge and re-run the visual complexity reduction with a different focus/interest.

The main functionality of our proposed approach to visual complexity reduction remains unaltered by this design decision as it only affects the ability to move through the different stages of complexity reduction and the visualization for the human user.

Our proposed technique to reduce the visual complexity is capable of removing the vast majority of information in just a few iterations as we have shown before. In order to overcome unwated loss of information during the process of data reduction, our approach always allows the user to track back and select different nodes as user interest. This, however, is infeasible for dynamic data sets, which can be adressed, for example, by caching results from earlier iterations.

VI. Conclusion

The collection and processing of large data sets got common with the rise of *big data* and powerful computing devices. Human users are hardly able to keep up with the increasing pace of collecting and accruing data. Accessing and–more important–understanding these data sets becomes difficult.

In this article, we introduced a combination of graph-theoretic properties and community detection to reduce the visual complexity to render the visualizations of data sets more usable and useful for human users.

Our simulation study has shown that our combination of graph-theoretic properties, measuring the importance of data in the data set, and community detection, grouping similar data in the data set, is able to reduce the visual cluttering of information efficiently. If performed in a non-destructive setting, i.e., if discarded data is only concealed and not removed from the data set, human users can shift their focus when inspecting a data set to account for new insights gained

TABLE III
Properties of Complexity-reduced Data Sets of Facebook, Collaboration, and Gnutella. Protein Interaction Networks.

	Technique	Mean Values				
		C_B	C_C	CC	C_D	PR
FB	C_C & DC	2211000	0.3457	0.00038	298.1	0.00215
	C_C & HC	2356000	0.3478	0.00033	311.9	0.00230
	C_C & OL	1925000	0.3425	0.00037	260.3	0.00191
	C_C & LM	1925000	0.3425	0.00037	260.3	0.00191
	C_B & DC	1453000	0.3327	0.00148	205.9	0.00163
	C_B & HC	1685000	0.3182	0.00604	219.5	0.00209
	C_B & OL	2112000	0.3397	0.00291	251	0.00197
	C_B & LM	2253000	0.3389	0.00356	262.5	0.0021
	PR & DC	1520000	0.3308	0.00268	203.9	0.0017
	PR & HC	1754000	0.3356	0.00208	223.5	0.00178
	PR & OL	1588000	0.3332	0.00284	206.2	0.00163
	PR & LM	1809000	0.3339	0.00336	232.6	0.00185
	C_D & DC	1539000	0.3304	0.0024	219.7	0.00175
	C_D & HC	1810000	0.3291	0.00492	236.4	0.002
	C_D & OL	1943000	0.3311	0.0063	236.2	0.00194
	C_D & LM	2100000	0.3356	0.00451	262.3	0.00211
	CC & DC	70010	0.2638	0.07401	14.13	0.00013
	CC & HC	79330	0.2639	0.08139	15.11	0.00014
	CC & OL	30171	0.27314	2.42765	36.62	2.0282
	CC & LM	83490	0.2641	0.08111	15.71	0.00015
CA	C_C & DC	73470	0.00005	0.02978	16.36	0.00041
	C_C & HC	71590	0.00005	0.04217	8.94	0.00031
	C_C & OL	74020	0.00005	0.02445	17.66	0.00044
	C_C & LM	73470	0.00005	0.02978	16.36	0.00041
	C_B & DC	233900	0.00005	0.00592	16.95	0.00058
	C_B & HC	224500	0.00005	0.00579	16.47	0.00057
	C_B & OL	158000	0.00005	0.01006	14.12	0.00049
	C_B & LM	151600	0.00005	0.00884	13.93	0.00047
	PR & DC	222000	0.00005	0.0015	20.85	0.00067
	PR & HC	98330	0.00005	0.00811	14.92	0.00045
	PR & OL	215300	0.00005	0.00445	19.91	0.00063
	PR & LM	226500	0.00005	0.00266	20.64	0.00066
	C_D & DC	28418	4.44	1.57434	8.38	0.88412
	C_D & HC	28261	4.44	1.57331	8.37	0.88485
	C_D & OL	28370	4.44	1.57288	8.37	0.88744
	C_D & LM	97370	0.00004	0.00991	12.61	0.00039
	CC & DC	22449	4.17	0.84129	4.83	2.43779
	CC & HC	22449	4.17	0.84129	4.83	2.43779
	CC & OL	21060	4.14	0.80191	4.63	2.58936
	CC & LM	43790	0.00003	0.1903	7	0.0003
P8	C_C & DC	38492	0.00023	1.42208	9.55	3.59775
	C_C & HC	39680	0.00023	0.00003	9.53	0
	C_C & OL	38492	0.00023	1.42208	9.55	3.59775
	C_C & LM	38666	0.00023	1.43795	9.54	3.59132
	C_B & DC	82970	0.00023	0	9.92	0
	C_B & HC	88810	0.00023	0.00003	9.72	0
	C_B & OL	162900	0.00023	0	9.84	0
	C_B & LM	137000	0.00023	0	9.83	0
	PR & DC	20920	0.61077	0.58976	3.77	4.02269
	PR & HC	20920	0.61077	0.58976	3.77	4.02269
	PR & OL	18941	0.61688	0.57661	3.69	4.09170
	PR & LM	21934	0.60621	0.59507	3.81	4.14545
	C_D & DC	119200	0.00023	0	12.68	0
	C_D & HC	122900	0.00023	0	12.55	0
	C_D & OL	47420	0.00023	0.00002	10.18	0
	C_D & LM	47420	0.00023	0.00002	10.18	0
	CC & DC	46280	0.00393	2.44391	9.69	3.76677
	CC & HC	46304	0.00393	2.44341	9.69	3.76735
	CC & OL	46280	0.00393	2.44391	9.69	3.76677
	CC & LM	46576	0.00388	2.43269	9.69	3.75996

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TABLE IV
PROPERTIES OF COMPLEXITY-REDUCED DATA SETS OF PROTEIN INTERACTION NETWORKS.

	Technique	Mean Values				
		C_B	C_C	CC	C_D	PR
VI	C_C & DC	57420	0.00009	0.00196	15.28	0.00107
	C_C & HC	57420	0.00009	0.00196	15.28	0.00107
	C_C & OL	58990	0.00009	0.00154	15.73	0.0011
	C_C & LM	54320	0.00009	0.00232	14.61	0.00102
	C_B & DC	61750	0.00009	0.00045	15.6	0.0011
	C_B & HC	58410	0.00009	0.00044	15.06	0.00106
	C_B & OL	57790	0.00009	0.00046	14.98	0.00106
	C_B & LM	56350	0.00009	0.00046	14.78	0.00104
	PR & DC	62850	0.00009	0.00600	15.86	0.00112
	PR & HC	138900	0.00009	0.00133	29.01	0.00203
	PR & OL	49980	0.00009	0.00686	13.65	0.00097
	PR & LM	85260	0.00006	0.01049	18.23	0.00136
	C_D & DC	31410	0.00009	0.02806	9.053	0.00064
	C_D & HC	156200	0.00009	0.00314	31.63	0.00221
	C_D & OL	83280	0.00007	0.04043	18.82	0.00136
	C_D & LM	111300	0.00007	0.04704	24.04	0.00172
	CC & DC	23410	8.61693	1.67212	7.98	0.00318
	CC & HC	30480	0.00008	0.1194	8.65	0.00063
	CC & OL	23590	0.00009	0.07014	7.94	0.00057
	CC & LM	24450	0.00009	0.08001	8.03	0.00058
MO	C_C & DC	33615	0.00005	0.00681	6.31	0.00115
	C_C & HC	31886	0.00005	0.00596	6.16	0.00114
	C_C & OL	33467	0.00005	0.00668	6.33	0.00116
	C_C & LM	42355	0.00005	0.0124	7.32	0.00132
	C_B & DC	22275	0.00004	0.00152	4.99	0.00096
	C_B & HC	21245	0.00004	0.00153	4.91	0.00094
	C_B & OL	62989	0.00004	0.00306	9.87	0.00178
	C_B & LM	61556	0.00004	0.00312	9.7	0.00175
	PR & DC	33375	0.00004	0.00595	6.67	0.00125
	PR & HC	30472	0.00004	0.00604	6.34	0.0012
	PR & OL	32441	0.00004	0.0062	6.51	0.00123
	PR & LM	24884	0.00004	0.00618	5.70	0.00109
	C_D & DC	25683	0.00004	0.00277	5.82	0.00109
	C_D & HC	23793	0.00004	0.0028	5.64	0.00106
	C_D & OL	24978	0.00004	0.00284	5.80	0.00108
	C_D & LM	61071	0.00004	0.00376	10.53	0.00189
	CC & DC	20061	0.00004	0.0867	5.28	0.00099
	CC & HC	20061	0.00004	0.0867	5.28	0.00099
	CC & OL	20503	0.00004	0.0857	5.33	0.001
	CC & LM	21185	0.00004	0.0845	5.39	0.00101

by the visual inspection of data sets. The complexity reduction can then be performed again with a focus on new, shifted interests.

Our proposed technique opens an additional direction of research to support user-centric systems in rising exposure to big data and accruing amounts of data. Future research may include user studies to select the–per application field–appropriate graph-theoretic properties in order to achieve an appropriate visual complexity reduction.

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