Theoretical Performance of Transform Domain Communication Systems

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***Abstract*—Transform Domain Communication System (TDCS) is a cognitive-radio technology that avoids frequency underutilization by doing spectrum-scavenging. Although TDCSs' is well-known when dealing with interferers, theoretical limits of TDCS in terms of spectrum efficiency remain unknown. Based on the TDCS' multidimensional property, we detail them in terms of spectrum efficiency and Bit Error Rate (BER). It is shown that most of previous TDCSs had suboptimal performances. Guidelines are given to improve these metrics without much increase in system's complexity.**

***Index Terms*—TDCS, spectral efficiency, CSK, cognitive radio, multidimensional modulation**

I. Introduction

Introduced by Mitola in [1], Cognitive-Radio was presented to overcome the problem of spectrum under-utilization [2], [3]. Technologies were proposed to counter this waste, such as Transform Domain Communication Systems (TDCSs) that generate interference-free waveforms.

TDCS's major contributions are recent, and a significant part was achieved by the Airforce Institute of Technology [4]–[7]. However these studies mainly focused on the BER performance when bypassing jammers. More recently Han et al. proposed a phase scrambling to improve the BER of TDCS signals based on contiguous spectrum [8]. TDCS is therein described as a low bit-rate communication system, while in another article, Budiarjo et al. [9] mention they can significantly improve the BER performance and the spectrum utilization of TDCS by using simultaneously two modulations.

Thus, if we know well TDCSs' benefits in terms of cognitive radio and interference-avoidance property, its BER and spectral efficiency performance remain unknown, though these are essential performance metrics.

By means of the dimensionality property [10, pp 227-229] that applies to TDCS, we propose to fill this missing piece of literature in order to give insights on the effective speed and reliability that TDCS can achieve. We demonstrate that the granularity of the spectrum mask does impact the spectral efficiency and show that taking benefit of the whole dimensionality leads to great BER improvements while increasing the spectral efficiency. Via this approach we stress that previous TDCS implementations were suboptimum with regard to these metrics.

In this article, we first recall the definition of the dimensionality and its impacts on the BER and spectrum efficiency. Then, our TDCS system is introduced and key properties related to its dimensionality are given. In the last section, a particular care is taken on spectrum efficiency of previously studied TDCS systems, we enhance them by using system's whole dimensionality to ensure maximum spectral efficiency and minimum BER. Finally, a specific focus is done on the modulation proposed in [9] and a new result is presented by means of the dimensionality and the orthogonality point of view. We explain in which extent the spectrum efficiency and the BER of their system is improved and why.

II. Signal Dimensionality and its Impacts on System Performance

In this section, one recalls the notion of dimensionality before studying its impact on the system in terms of BER.

A. General principles, and relations with spectrum efficiency

The dimensionality N of a signal $x(t)$, which lasts T seconds and occupies a bandwidth W_u, defines the number of orthogonal signals such that $x(t)$ can be expressed by a linear combination of these orthogonal signals. In [10, pp. 227-229], it is stated that the number of dimensions N of the space of a signal $x(t)$ is well approximated by Eq. (1):

$$N \approx 2W_uT \tag{1}$$

Although this formula looks very simple, the information it contains is of the greatest importance since it has direct consequences on the spectral efficiency η_{eff} of a multidimensional system. Indeed, it is shown that [10, pp. 227-229] :

$$\begin{aligned} \eta_{\text{eff}} &= \frac{R_{bit}}{W_u} = \frac{\log_2(M_{mod})R_{symb}}{\frac{N}{2T}} \\ &= \frac{2\log_2(M_{mod})}{N} \end{aligned} \tag{2}$$

where M_{mod} is the constellation size, R_{bit} is the bit-rate, and $R_{symb} = \frac{R_{bit}}{\log_2(M_{mod})} = \frac{1}{T}$ is the symbol rate.

As a consequence, the higher the signal dimensionality is, the lower the spectrum efficiency gets. Thus, to improve the latter, M_{mod} must be increased.

Let us now briefly summarize the effects of high dimensionality systems on the achievable BER. To do so, we will take the example of the widely studied M-Ary Orthogonal

 ISBN: 978-1-61208-123-6

Signaling [10](MOS), where one symbol fills one dimension ($M_{mod} = N$), before seeing how it applies to TDCS.

B. Bit Error Probability for MOS and TDCS

While in PSK, the bigger the constellation size M_{PSK} is, worse gets the BER (following the well known "waterfall curves"), in MOS, where each symbol is represented by a waveform orthogonal to every other one, the waterfall curves' order is reversed : the higher the dimensionality, the better the BER gets, as developed in [10, pp. 203-206]. It comes from the fact that the more dimensions a signal has, the more bits are sent with the same amount of energy, while keeping a constant symbol error probability, since the distance between two orthogonal signals does not change. One can transmit information with high reliability at very low SNR, but with a low spectrum efficiency. MOS systems are in a word "power-efficient".

In the case of a given TDCS system, N is set by the spectrum mask definition as further stated in Section III-D. Thus, it does not come into question to increase or to decrease the system's dimensionality. However the chosen modulation defines the number of used dimensions. And similarly as the phenomenon described above, the more dimensions are used, the better gets the BER. Note that meanwhile, because more dimensions are used, the constellation size of the modulation increases, and thus, spectrum efficiency also increases. This phenomenon is fully described in [10], [11].

In a word, when a telecommunication system's dimensionality is $N \gg 1$, it is of paramount importance to use every possible dimensions. Otherwise it could lead to BER and spectrum efficiency leakages.

III. TDCS System Model

Hereafter, the receiver and transmitter's model are described. The waveform modulation is also detailed.

A. Transmitter's Side

The typical TDCS transceiver model presented in [7] is simple and cited in many publications, but does not well describe how the modulation stage can be implemented. We decide thus to present in Fig. 1 a model that takes into account the two main modulations used in TDCS.

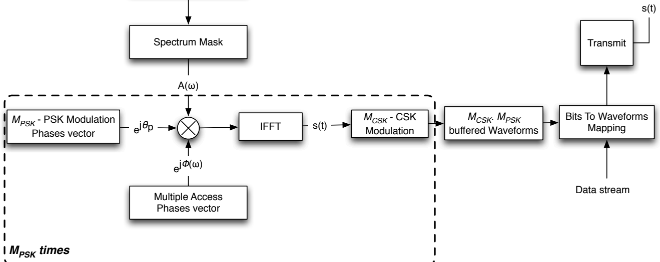


Figure 1. TDCS Transmitter

The system described in Fig. 1 starts first with sensing the available spectrum to ensure an interference-free transmission. This topic has been widely investigated through the use of different techniques to better detect the interferences [6]. From this estimate, a spectrum mask of bandwidth W_u is chosen, which the transmitted signal will have to respect. An Inverse Fourier Transform of this spectrum is then done, and the generated waveforms are stored in a buffer. These waveforms are then modulated before being sent.

B. Possible waveform modulations

As stated, two modulation processes are mainly used in TDCS: a Cyclic Shift of the waveform and a Phase Shift Keying modulation that occur in two different stages of the transmitter [4], [7]. The first one is the M_{PSK}-Ary PSK modulation and consists of generating several waveforms respecting the same spectrum mask, having the same random phase vector, but with a $\theta_{m_{\text{PSK}}} = 2\pi \frac{m_{\text{PSK}}}{M_{\text{PSK}}}$ phase offset on every component, with $m_{\text{PSK}} \in \{1, \cdots, M_{\text{PSK}}\}$ the PSK symbol index, and M_{PSK} the number of possible PSK symbols. The signal's spectrum can thus be written as in Eq. (3):

$$\begin{aligned} S_{m_{\text{PSK}}}^{(v)}(f) = \sum_{k=1}^{K} A_k^{(v)} \delta\left(f - k\Delta_f\right) e^{+j\left(\phi_k^{(v)} + \theta_{m_{\text{PSK}}}^{(v)}\right)} + \\ A_k^{(v)} \delta\left(f + k\Delta_f\right) e^{-j\left(\phi_k^{(v)} + \theta_{m_{\text{PSK}}}^{(v)}\right)} \end{aligned} \tag{3}$$

with $A_k \in \{0, 1\}$ the amplitude of the frequency components (determined by the spectrum sensing), K the number of frequency components, Δ_f the spectrum sample spacing, $\theta_{m_{\text{PSK}}}^{(v)}$ the data phase modulation. $\phi_k^{(v)}$ is the multiple-access phase for the k-th frequency component of the v-th user and is usually described in the literature as a random phase. This causes the signal to be noise-like and also enables multiple-access capability, as exploited in [4]. In the time domain, a PSK symbol can be written as in Eq. (4):

$$\begin{aligned} s_{\text{PSK}_{m_{\text{PSK}}}}^{(v)}(t) &= \mathcal{F}^{-1}\left(S_{m_{\text{PSK}}}^{(v)}(f)\right) \\ s_{\text{PSK}_{m_{\text{PSK}}}}^{(v)}(t) &= 2\sum_{k=1}^{K} A_k \cos(2\pi k \Delta_f t + \phi_k^{(v)} + \theta_{m_{\text{PSK}}}^{(v)}) \end{aligned} \tag{4}$$

To alleviate the notation we decide not to specify the multiple-access index (v).

Since the Eq. (4) is computed from the Fourier transform of a sampled spectrum, $s_{\text{PSK}_{m_{\text{PSK}}}}(t)$ is T-periodic with $T = 1/\Delta_f$. But in the rest of this document we consider $s_{\text{PSK}_{m_{\text{PSK}}}}(t)$ as a symbol whose duration is T.

Although the M_{CSK}-Ary CSK modulation can be applied over the PSK one, in a matter of clarity we decide to focus on CSK and omit thus the subscript $_{\text{PSK}_{m_{\text{PSK}}}}$. When present, it means the symbol also carries PSK-modulated information.

First, a waveform s_{CSK_0} is generated following the Eq. (4). Then s_{CSK_0} is shifted in time to produce different symbols, as written in Eq. (5).

ISBN: 978-1-61208-123-6

$$s_{\text{CSK}_{m_{\text{CSK}}}}(t) = s_{\text{CSK}_0}\left(t - \frac{m_{\text{CSK}}T}{M_{\text{CSK}}}\right)_T \quad (5)$$

with T the waveform duration, and $s\left(t - \frac{T}{m_{\text{CSK}}}\right)_T$ the notation introduced in [4] for a $\frac{T}{m_{\text{CSK}}}$ circular shift. $m_{\text{CSK}} \in \{0 \cdots M_{\text{CSK}} - 1\}$ is the CSK symbol index.

Depending on the narrowness of the autocorrelation function of s_{CSK_0}, it can be shown that the set of shifted waveforms $s_{\text{CSK}_0} \cdots s_{\text{CSK}_{(M_{\text{CSK}}-1)}}$ is pseudo-orthogonal [12].

C. Receiver's side

A Maximum Likelihood (ML) receiver, known to be optimum in an AWGN channel, is used and showed in Fig.2.

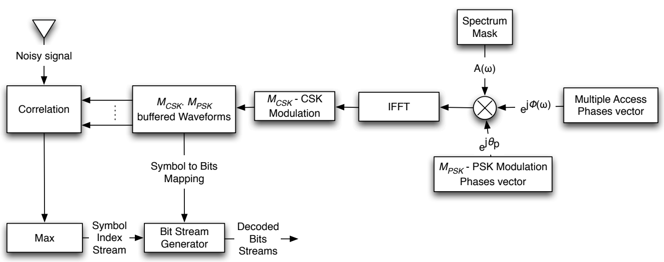


Figure 2. TDCS Optimum Receiver

We assume that the receiver and the emitter have the same waveforms buffered in their memory and also that they have the same mapping between a waveform and a bits word. To demodulate data, the receiver makes a correlation of the incoming waveform with every buffered waveform and the decision is taken by considering the maximum correlation value. Although this demodulation requires a huge amount of computations, this way of demodulating was chosen to ensure a fair comparison between all the modulation schemes. However, it is worth noting that a computation-efficient implementation is possible to demodulate CSK signals by using DFT and IDFT as stated in [12]. Now that the TDCS System is introduced, let us clarify the role that plays the dimensionality parameter $N = 2W_uT$ on the system's performance.

D. Impacts of the dimensionality N on TDCS performance

Since the symbol period is $T = \frac{1}{\Delta_f}$, and since the useful spectrum is described by $K_{\text{used}} = \frac{W_u}{\Delta_f}$ frequency components, the signal dimensionality is equal to $N = 2K_{\text{used}}$. Thus, it is possible to write the spectrum efficiency of the system in a simple manner in Eq. 6.

$$\eta_{\text{eff}} = \frac{\log_2(M_{mod})}{K_{\text{used}}} \quad (6)$$

A first tradeoff follows up: when defining the spectrum mask avoiding jammers, care should be taken not to choose a high granularity that would imply a leakage in spectrum efficiency.

As previously stated, dimensionality of a TDCS signal is set by the spectrum mask properties (the used bandwidth W_u and the frequency sampling space Δ_f). In the following, we assume these parameters are already set. However the number of used dimensions remains designer's choice and depends on the modulation stage. As explained in II-B, choosing a good modulation is of paramount importance: the system has to use as many symbols as possible to maximize the spectrum efficiency, but meanwhile it also has to occupy as many dimensions as possible to minimize the BER. In the following subsection, we study how well the standard modulations performs in TDCS.

IV. Performance of TDCS Modulation

In the studied TDCS system, the signal bandwidth is defined from 1kHz to 7MHz and avoids interferences that are present on the range from 2 to 3MHz. As a consequence, its total available bandwidth is $W_a = 6.999$ Mhz, but its used bandwidth is $W_u = 5.999$ MHz. $K_{\text{used}} = 256$ frequency components describe the used bandwidth, and thus the dimensionality of our system is $N = 512$. These figures are summarized in the spectrum mask of Fig. 3.

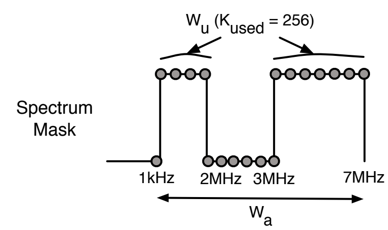


Figure 3. Spectrum description of the signal

A. Phase Shift Keying

Described in III-B, this modulation makes the symbol occupy only two dimensions as the usual PSK signals. As a consequence, the standard PSK waterfalls curves, fit perfectly PSK-TDCS BER results, as shown on Fig. 4.

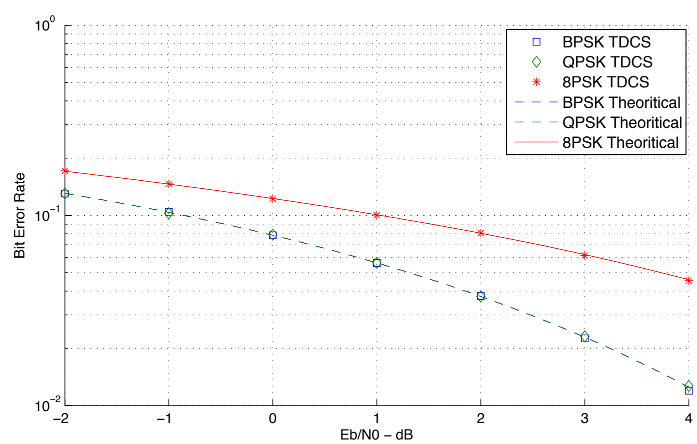


Figure 4. BER performance TDCS using M_{PSK}-Ary PSK Modulation

The modulation and demodulation processes can be easily implemented [7] and the spectrum efficiency is :

$$\eta_{\text{eff}} = \frac{\log_2(M_{\text{PSK}})}{K_{\text{used}}} \tag{7}$$

The problem is that the system is sub-optimal with regards to the attainable BER because only 2 dimensions over $2K_{\text{used}}$ are used. The BER leakage is obvious when looking at Fig. 5b.

B. Cyclic Shift Keying

As stated in [12], CSK can be seen as equivalent to M_{CSK}-Ary Orthogonal Signaling (MOS). This is effectively the case, as we can see on Fig. 5a : the theoretical BER performance of a MOS system match the simulated CSK-TDCS results.

Of course, since the cyclic shifted versions of the waveforms are inherently correlated, CSK is theoretically sub-optimal in terms of BER in comparison with MOS, but this is not detectable when the dimensionality and the E_b/N_0 are high enough.

In Fig. 5b, we observe that BER of TDCS-CSK is much lower than any PSK systems and keeps decreasing when the CSK constellation grows. The spectral efficiency follows Eq. (8) and is also higher than TDCS-PSK systems. Moreover the demodulation process of CSK has the advantage of being easily implementable as described in [12] and done in [8].

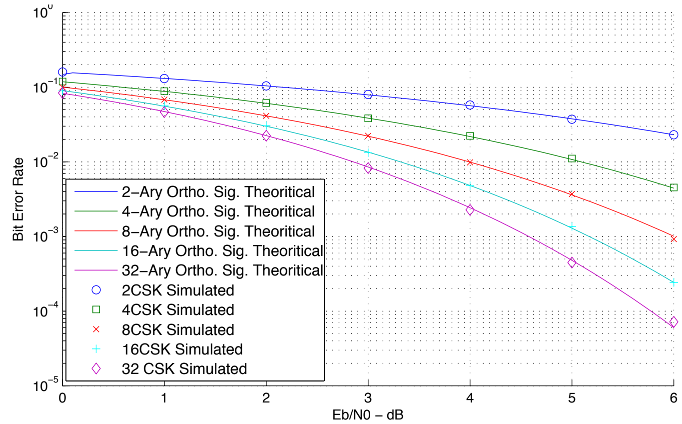
$$\eta_{\text{eff}} = \frac{\log_2(M_{\text{CSK}})}{K_{\text{used}}} \tag{8}$$

The main difference between Eq.(7) and Eq.(8) is that increasing M_{CSK} can lead to BER and spectrum efficiency improvements whereas increasing M_{PSK} leads to a BER degradation.

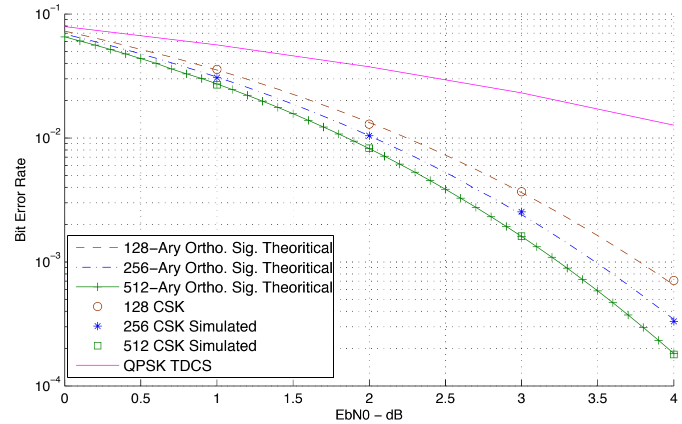
It is worth noting that in the literature, system configurations have always been $M_{\text{CSK}} \leq K_{\text{used}}$ without further explanation. It is possible that people fear that using CSK TDCS with $M_{\text{CSK}} > K_{\text{used}}$ would not provide enough orthogonality between CSK symbols. Yet, experimental results show that it can actually provide enough orthogonality to enhance the BER according to Fig. 5b and as thoroughly investigated in the next subsection.

We can note that using M_{CSK}-Ary CSK Modulation with $M_{\text{CSK}} = 512$, improves the spectral efficiency while achieving the same $3e^{-4}$ BER with 0.3dB $\frac{E_b}{N_0}$ less than the system using $M_{\text{CSK}} = 256$.

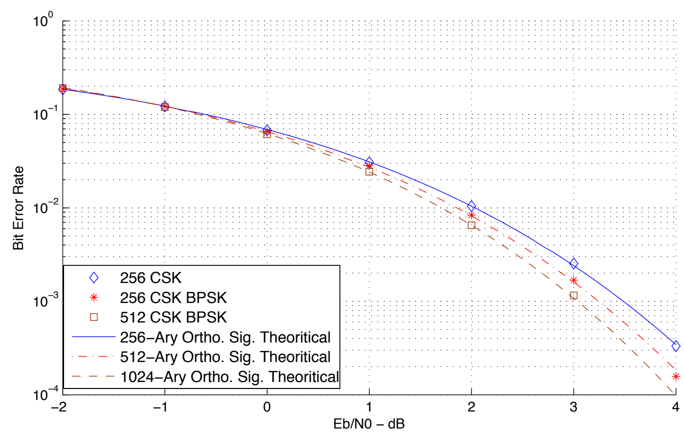
Moreover, it has never been tried to combine Antipodal Signaling and CSK with TDCS Systems. This results in bi-orthogonal signaling. MOS and M-Ary Bi-Orthogonal Signaling have almost identical BER functions when M is large, but Bi-Orthogonal Signaling occupies two times less dimensions (and thus bandwidth). As a consequence, it is possible to use $2K_{\text{used}}$-Ary CSK with antipodal signaling to further enhance the spectrum efficiency and the BER performance. Indeed, in Fig. 5c, we can observe a 0.5 dB $\frac{E_b}{N_0}$ gain over $M_{\text{CSK}} = 256$-Ary CSK TDCS when using $M_{\text{CSK}} = 512$ with antipodal signaling at the same $1e^{-3}$ BER. Note that the constellation size is then four times higher.



(a) BER performance of TDCS System using M_{CSK}-Ary CSK Modulation



(b) BER performance of TDCS System using M_{CSK}-Ary CSK Modulation with $M_{\text{CSK}} \geq K_{\text{used}}$



(c) BER performance of TDCS system using an Antipodal M_{CSK}-Ary CSK Modulation

Figure 5. BER performance of several CSK configurations

C. Combination of PSK and CSK

In [9], a system is presented where CSK and PSK modulations are combined. In this article, the information streams mapped by PSK or CSK are distinguished, and the BER for both streams are computed. Let first consider the spectral efficiency of such a combined M_{PSK}-Ary PSK / M_{CSK}-Ary

 ISBN: 978-1-61208-123-6

CSK TDCS system, we have:

$$\eta_{\text{eff}} = \frac{\log_2(M_{\text{PSK}} M_{\text{CSK}})}{K_{\text{used}}} \quad (9)$$

Budiarjo et al. were thus right when stating that this modulation leads to a better spectrum utilization but when considering the total BER of the whole system (both streams taken into account) rises up another advantage, not studied in [9]. This is what we computed for different system configurations in order to highlight the effect of dimensionality. Results are plotted in Fig. 6. Note that we use the X-axis scale E_b/N_0 with E_b computed as $E_b = \frac{E_s}{M_{mod}}$ with $M_{mod} = M_{\text{PSK}} M_{\text{CSK}}$ the total constellation size.

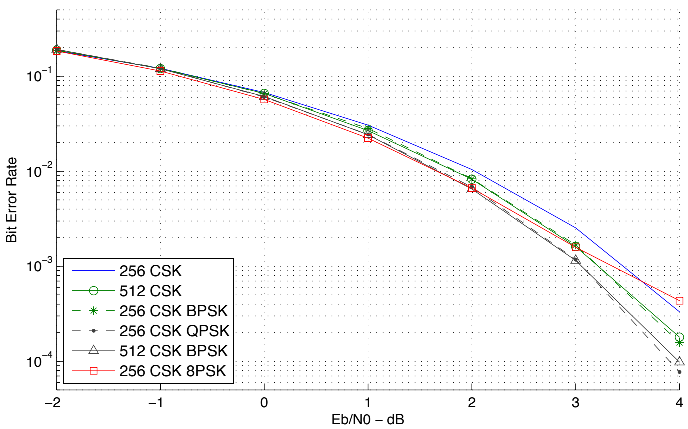


Figure 6. BER performance of PSK-CSK TDCS

As observed in Fig. 6, 512-Ary CSK and 256-Ary CSK-BPSK (equivalent to CSK with antipodal signaling) curves are overlapping. It is noteworthy to highlight that 256-CSK-QPSK has almost the same BER performance than 512-CSK-BPSK. As explained in the previous sections, it comes from the fact that in both cases, all dimensions $2K_{\text{used}}$ are used.

To provide a more accurate comparison of these modulations, Table I sums up intercorrelation statistics of the waveforms for each modulation technique:

Intercorr. Statistics	32 CSK	256 CSK	256 CSK QPSK	512 CSK BPSK	256 CSK 8PSK
mean	8.7e-5	6.9e-4	6.6e-4	6.8e-4	1.6e-3
min	1.1e-8	1.3e-9	8e-38	9.1e-10	8e-38
max	1.6e-3	8.5e-2	8.5e-2	8.5e-2	.5
var	1.6e-7	6.6e-5	4.8e-5	4.5e-5	7.6e-4

TABLE I. Intercorrelation Coefficient Statistics of WF generated by different techniques

First of all, the maximum intercorrelation coefficient increases as the constellation size does. This comes from the fact that the constellation size approaches or even exceeds the dimensionality limit. It is thus not further possible to generate more orthogonal waveforms. Concerning the minimum value, the QPSK-CSK modulation shows much better performance since QPSK generates 2 purely orthogonal waveforms for each CSK-generated one. There is no obvious rules for the other cases, since it highly depends on the autocorrelation function of the waveform : if the time shift occurs on one of its lobe or not. However, we can note that 256-CSK-QPSK average intercorrelation coefficient is slightly lower than 512-CSK-BPSK. As a matter of fact, it should results in better BER performance noticeable at higher E_b/N_0.

To end section IV, let us summarize in Table II the different rules to follow to take the most of TDCS' spectral efficiency. These guidelines are independant from the chosen spectrum mask.

Rule #	Guideline
1	Use every dimension the system offers to decrease the BER and enhance the spectral efficiency.
2	Use a spectrum granularity high enough to avoid jammers, but as low as possible to enable good spectrum efficiency.
3	Between two modulations having the same number of symbols, choose the one offering the largest intersymbol distance.

TABLE II. Guidelines to follow to improve TDCS' spectral efficiency

V. Conclusion

In this paper, using the often forgotten fact that TDCS is a multidimensional communication system enabled us to show the maximum attainable BER and spectral efficiency we can expect from any TDCS system. We show that TDCS is inherently a "power-efficient" communication system as previously suggested in [8]. However, Han et al. did not take benefit of every available dimension. We were thus able to improve their performance by using 512-Ary CSK modulation instead of 256-Ary CSK. We also further investigated the implementation proposed in [9] and show the global BER benefit of this implementation. This asset relies on the major fact we proved: the same BER can be obtained at lower $\frac{E_b}{N_0}$ when using every single dimension while enhancing the spectrum efficiency of a standard CSK Modulation. By using every dimension, the minimum BER and the maximum throughput are simultaneously gathered. This article provided guidelines to avoid further under-utilization of TDCS systems

If it is now obvious that TDCS systems fit well to low throughput transmissions in a context of poor SNR or long-haul communications, the next step is to extend the use of TDCS to high throughput communications, while ensuring a good reliability. This can be done by ensuring a better orthogonality between the dimensions and by using several dimensions at one time to transmit more bits simultaneously.

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