

Adaptive Scheduling of Smart Home Appliances Using Fuzzy Goal Programming

Honggang Bu and Kendall E. Nygard

Department of Computer Science

North Dakota State University

Fargo, ND, 58108, USA

Honggang.bu@my.ndsu.edu, kendall.nygard@ndsu.edu

Abstract-A smart electrical grid is highly instrumented and can be intelligently controlled. We describe a smart grid study in which we assume that utilities dynamically price electrical power to help regulate supply and demand balance, and that consumers have the ability to intelligently schedule times for the operation of their home appliances in response to prices. We present a mixed integer linear fuzzy goal programming with priorities imposed on different appliances. The goal programming formulation allows time preference constraints to be elastic rather than rigid. Another important feature of the model is flexibility of time slot delays for pairs of appliances for which the operation of one must follow the other (a washer/dryer pair for example). Numerical experimental results based on real spot prices for electricity are presented. In addition, computational time and the influence of time slot lengths and priorities are discussed.

Keywords-Smart grid; Dynamic Pricing; Adaptive Systems; Optimization.

I. INTRODUCTION

Dynamic electricity pricing on an hourly basis is increasingly common in the United States [1]. This pricing policy is intended to help reduce system peak demand and also shift some load to off-peak, less expensive time periods. This can achieve more balance between energy demand and generation. Hourly pricing provides customers with opportunities to reduce their costs by managing the times at which electricity is consumed in the home. Smart appliances that can be accessed and controlled under the expanded addressing space of Internet Protocol version 6 (IPv6) are becoming common, and older appliances can be IP controlled through devices such as smart power bars.

Several studies on optimal scheduling of home appliances have been reported. Using Markov chains to model both energy prices and residential device usage, an energy management system called CAES for residential demand response applications to reduce residential energy costs and smooth energy usage was proposed [2]. In developing a Mixed Integer Linear Programming (MILP) problem formulation for electricity management in multiple homes, Oliveira et al. considered both cost and variations in the availability of the power supply [3]. Sou et al. proposed an MILP formulation with discrete time-slots [4]. In that model, one execution period (e.g., one day) is discretized into a prescribed number of uniform time slots. Amounts of

energy are assigned to each time slot for each phase of appliance operation. Inspired by the model of Sou, Wu included the CO₂ footprint cost into the objective function by giving it a weight for modeling environmental concerns [5]. Giorgio developed a similar MILP formulation, but also included domestic renewable energy and batteries as energy sources [6].

In our work, we schedule home appliances using time slots and a MILP, based on portions of the existing work [4]. We expand the approach by adopting a fuzzy goal programming formulation [8]. Our model supports priority distinctions for the different appliances, and rigid time preference constraints are transformed into soft ones and included in the fuzzy goal programming framework with priorities. In addition, we devised constraints for modeling alternative delay times between running times of closely related appliances. The new method uses electricity prices known 24 hours in advance; so, the scheduling is exactly one day ahead.

Section II briefly introduces the concept of MILP and fuzzy goal programming; Section III presents the mathematical formulation for our mixed integer linear fuzzy goal programming model; Section IV provides the numerical experiments and results; and Section V presents the conclusions.

II. MILP AND GOAL PROGRAMMING

MILP is a widely used subset of mathematical programming in which the objective function is a linear function of the decision variables, which can be either integer or non-integer. Each constraint is formed from a linear combination of the decision variables [7].

When multiple, conflicting objectives or goals are involved in an optimization problem, goal programming is a powerful and effective tool. The two major differences of goal programming from conventional single-objective linear programming are the incorporation of flexibility in the constraint functions, and the satisficing approach that seeks a balanced and practical solution rather than an absolute optimal one [8]. To solve optimization problem with multiple conflicting goals using fuzzy goal programming, which is also called Chebyshev goal programming [8], each original single goal is first optimized to get the corresponding optimal goal value, then a solution that

minimizes the maximum deviation from any single optimized goal value is sought. Based on the degree of importance of each goal, priorities can be added to these deviations to reflect different penalties applied to different failures to meet the optimal goals [9]. A new general goal based on the weighted sum of deviations can therefore be formed and solved.

III. MATHEMATICAL FORMULATION

A. Assumptions and Parameters

We define an energy phase as an uninterruptible sub-process of the entire operation process of an appliance. Each appliance has a single phase or multiple energy phases that must be operated in sequence, with each using a pre-specified amount of electrical energy. The technical specifications of appliances defined by the manufacturers of appliances must be met. Constraints are employed to ensure the sequential operations of some appliances, to model the delay between the running of two closely related appliances, to ensure that the total energy consumed within a certain period does not exceed the peak energy allowed, and to ensure that user time preferences are met. The overall objective of the model is to produce the schedule for running the appliances that saves a consumer as much energy cost as possible, while meeting all of the constraints. Our MILP formulation is for a single 24-hour day. Each hour is uniformly discretized into h time slots, so that the number of total time slots in a day is $m = 24 * h$. N is the number of appliances, and for each appliance i ($i = 1, 2, \dots, N$), n_i is the number of uninterruptible energy phases for each appliance.

Parameters λ_g ($0 < \lambda_g < 1$, and $g = 1, 2, \dots, N + 1$), satisfying $\sum_{g=1}^{N+1} \lambda_g = 1$, are used to model the priorities assigned to each single deviation goal in the fuzzy goal programming model. Here, λ_g ($g = 1, 2, \dots, N$) is for the deviation goals for each corresponding appliance energy cost, and λ_{N+1} is the priority for the user time preference penalty deviation goal. λ_g ($0 < \lambda_g < 1$, $g = 1, 2, \dots, N + 1$) is specified by the user according to their preferences for different appliances.

T_{ij} ($i = 1, 2, \dots, N, j = 1, 2, \dots, n_i$) represents the nominal processing time for energy phase j for appliance i in minutes, $\underline{\gamma}$ and $\bar{\gamma}$ ($0.5 < \underline{\gamma} < 1 < \bar{\gamma} < 1.5$) are lower and upper processing time limit factors for energy phase j of appliance i . To denote the lower and upper limits of power assignment, respectively, to the corresponding energy phase, \underline{P}_{ij}^k and \bar{P}_{ij}^k are introduced. The delay between two energy phases of an appliance is restricted by \underline{D}_{ij} and \bar{D}_{ij} , the appliance technical specifications defining the lower and upper delay time, respectively, in minutes. E_{ij} is used to denote the total energy that a phase should use according to the technical specification.

B. Decision variables

Real (continuous) decision variables p_{ij}^k ($k = 1, 2, \dots, m; i = 1, 2, \dots, N; j = 1, 2, \dots, n_i$) are used to indicate the energy assigned to energy phase j of appliance i during the period of time slot k .

To indicate during time slot k whether a particular energy phase j of appliance i is being processed, a series of binary decision variables $x_{ij}^k \in \{0, 1\}$ are used, with $x_{ij}^k = 1$ indicating energy phase being processed, and otherwise not being processed.

Binary variables $s_{ij}^k \in \{0, 1\}$ are utilized to indicate whether the processing of a particular energy phase is already finished by a particular time slot. If and only if $s_{ij}^k = 1$, energy phase j of appliance i is complete by time slot k .

To indicate whether appliance i is making a transition between energy phase $j - 1$ to j at time slot k , binary variables t_{ij}^k ($j = 2, \dots, n_i$) are utilized. $t_{ij}^k = 1$ if and only if during time slot k , the appliance i has finished energy phase $j - 1$ in some earlier time slot, but the energy phase j has not yet started. These variables are useful for restricting the delay between energy phases of an appliance.

For the fuzzy goals, parameters δ_g ($\delta_g > 0, g = 1, 2, \dots, N + 1$) are introduced to denote the normalized maximum deviation between the best and the worst values of each single objective function. Specifically, δ_g ($g = 1, 2, \dots, N$) are for the corresponding appliances, and δ_{N+1} is for the user time preference.

C. Constraints

1) Single appliance energy cost objective function:

The total electricity cost for appliance i during the entire execution period, denoted by Z_i ($i = 1, 2, \dots, N$), is

$$\sum_{k=1}^m \sum_{j=1}^{n_i} c^k p_{ij}^k \quad (1)$$

where, c^k denotes the electricity price for time slot k .

2) Objective function for user time preference violation penalty:

Here, we consider a simple user time preference in which the household user divides the day into two general parts: one that can be used to run a certain appliance and the other one cannot. Rather than use rigid constraints to absolutely prohibit using an appliance during the non-preferred time, we allow the time period to be used but impose a penalty on doing so. Let $TP_i^k \in \{0, 1\}$ denote the user time preference interval, and $TP_i^k = 0$ if and only if none of the energy phase of appliance i is to be run during time slot k . Assume k_{start}^i , k_{mid}^i , and k_{end}^i is the first, middle, and the last slot number of the whole user prohibited time period (which is continuous) for appliance i , respectively; then the penalty for using prohibited time is expressed as

$$\sum_{i=1}^N \sum_{j=1}^{n_i} \sum_{k=k_{start}^i}^{k_{end}^i} x_{ij}^k \alpha^{-|k-k_{mid}^i|} \quad (2)$$

where, $\alpha > 1$ is a constant, and $k_{mid}^i = [(k_{start}^i + k_{end}^i)/2]$. This is the objective function for the violation penalty for a user time preference, and is denoted as Z_{N+1} . Note that this function is a weighted penalty in that the closer to the middle of the prohibited time zone, the higher penalty that results.

3) *Maximum single objective deviation constraints:* Let U_i and L_i be the best possible and worst possible values, respectively, for the k^{th} single objective, then we have the following constraints:

$$(\beta U_i - Z_i)/(\beta U_i - L_i) \leq \delta_i, i = 1, 2, \dots, N + 1 \quad (3)$$

$$\delta_i \geq 0 \in \mathbb{R}, i = 1, 2, \dots, N + 1 \quad (4)$$

Each $\delta_i (i = 1, 2, \dots, N + 1)$ represents the worst deviation level for the k^{th} objective. Each U_i and L_i are obtained by optimizing corresponding Z_i and $-Z_i$ alone, respectively, without regard to other objectives. The expression $(\beta U_i - L_i)$ in (3) helps normalize the objective deviation level and thus adjust different levels to similar fluctuation ranges. With the normalized deviation levels, applying desired priorities to different objectives is easier. In consideration of the interaction between or among appliances and user time preferences, U_i may be close to but not the real possible optimal single objective value. So, an auxiliary coefficient β is incorporated to U_i to help use a better objective value than the “false” best possible value. Since in this study the best single objective value is the minimum value, β should be a positive constant and less than 1.

4) *Sequential processing between appliances:* Suppose appliance \tilde{i} must be finished before appliance i starts (for example, the washing machine operations must be finished before the dryer starts), then the following constraint restricting the relationship between the last energy phase of the appliance \tilde{i} and the first energy phase of appliance i must be satisfied:

$$s_{in_i}^k \geq x_{i1}^k, \forall k \quad (5)$$

5) *Between-appliance delay:* In reality, some appliances are more closely related than just following the constraints restricting their sequential processing. For example, the dryer can start running only after the washing machine is finished, as specified by (5), and in practice the delay between the two appliances usually cannot be very large. Suppose, for example, that if the dryer must start working within 3 time slots after the washing machine is done, then the following constraints holds:

$$s_{in_i}^k - s_{in_i}^{k-1} - x_{in_i}^k \leq x_{i1}^k + x_{i1}^{k+1} + x_{i1}^{k+2} \quad \forall k = 2, 3, \dots, m - 2, \quad (6)$$

These constraints should be used together with (5), namely, appliance \tilde{i} and i must satisfy (5) first.

To establish that these constraints are theoretically correct, consider the logic below.

If k_0 is the first time slot after the last energy phase of appliance \tilde{i} is finished, then $k_0 - 1$ is the last slot when the last energy phase of appliance \tilde{i} is being processed. This also implies:

i) When $k < k_0$, $s_{in_i}^k = s_{in_i}^{k-1} = 0$, and $x_{in_i}^k = 0$ or 1, so, the left side of the constraints is always equal to or less than 0. In this case, the constraints hold. Also in this case, the appliances sequential processing constraints ensure that all $x_{i1}^k = 0, \forall k < k_0$.

ii) When $k = k_0$, $s_{in_i}^k = 1, s_{in_i}^{k-1} = 0$, and $x_{in_i}^k = 0$, so, the left side of the constraints is always equal to 1. In this case, the constraints require that at least one of the time slots right after the finishing of the previous appliance must be used to start processing of the second appliance.

iii) When $k > k_0$, $s_{in_i}^k = s_{in_i}^{k-1} = 1$, and $x_{in_i}^k = 0$, so, the left side of the constraints is always equal to 0. In this case, the constraints hold.

6) *Sequential processing between energy phases:* Usually an energy phase of an appliance cannot start working unless its preceding phases have finished. The following constraints specify this condition:

$$s_{i(j-1)}^k \geq x_{ij}^k, \forall i, k, \forall j = 2, 3, \dots, n_i \quad (7)$$

7) *Between-phase delay:* The delay between two energy phases of an appliance is restricted to a specific range. Suppose that \underline{D}_{ij} and \overline{D}_{ij} are the appliance technical specifications defining the lower and upper delay, respectively, in minutes, then the following constraints must be satisfied:

$$\left\lfloor \frac{\underline{D}_{ij}}{60} h \right\rfloor \leq \sum_{k=1}^m t_{ij}^k \leq \left\lfloor \frac{\overline{D}_{ij}}{60} h \right\rfloor, \forall i, \forall j = 2, 3, \dots, n_i \quad (8)$$

$$t_{ij}^k = s_{i(j-1)}^k - x_{ij}^k - s_{ij}^k, \forall i, k, \forall j = 2, 3, \dots, n_i \quad (9)$$

8) *Uninterruptible operation of an energy phase:* To ensure the integrity and continuity of an energy phase, the following constraints should be satisfied:

$$x_{ij}^k \leq 1 - s_{ij}^k, \forall i, j, k \quad (10)$$

$$x_{ij}^{k-1} - x_{ij}^k \leq s_{ij}^k, \forall i, j, \forall k = 2, 3, \dots, m \quad (11)$$

$$s_{ij}^{k-1} \leq s_{ij}^k, \forall i, j, \forall k = 2, 3, \dots, m \quad (12)$$

9) *Energy phase process time limits:* Process time limits are enforced by the following constraint:

$$\left\lfloor \frac{T_{ij}^h}{60} \underline{\gamma} \right\rfloor \leq \sum_{k=1}^m x_{ij}^k \leq \left\lfloor \frac{T_{ij}^h}{60} \overline{\gamma} \right\rfloor, \forall i, j \quad (13)$$

where h is the number of time slots in each hour, T_{ij} is the nominal processing time for energy phase j in appliance i in minutes, $\underline{\gamma}$ and $\overline{\gamma}$ ($0.5 < \underline{\gamma} \leq 1 \leq \overline{\gamma} < 1.5$) are the lower and upper processing time limits factor for energy phase j in appliance i .

10) *Energy phase energy assignment requirement and bounds:* Each energy phase uses a certain amount of energy E_{ij} specified by the manufacturer:

$$\sum_{k=1}^m p_{ij}^k = E_{ij}, \forall i, j \quad (14)$$

To ensure power safety, the total energy assigned in any time slot is not allowed to exceed the peak signal or in other words the total slot energy upper bound:

$$\sum_{i=1}^N \sum_{j=1}^{n_i} p_{ij}^k \leq PEAK^k, \forall k \quad (15)$$

The energy assignment in any time slot for each energy phase of each appliance should satisfy the following constraint:

$$\frac{P_{ij}^k}{h} x_{ij}^k \leq p_{ij}^k \leq \frac{\bar{P}_{ij}^k}{h} x_{ij}^k, \forall i, j, k \quad (16)$$

where, P_{ij}^k and \bar{P}_{ij}^k are the lower and upper limits of power (not energy) assignment, respectively, to the corresponding energy phase. These limits are specified by the appliance manufacturer.

11) Basic decision variable constraints:

$$p_{ij}^k \geq 0 \in \mathbb{R}, \forall i, j, k \quad (17)$$

$$x_{ij}^k \in \{0,1\}, \forall i, j, k \quad (18)$$

$$s_{ij}^k \in \{0,1\}, \forall i, j, k \quad (19)$$

$$t_{ij}^k \in \{0,1\}, \forall i, k \forall j = 2, \dots, n_i \quad (20)$$

D. Cost function

Finally, the following total cost function, which represents the weighted sum of the maximum objective deviation from each single goal, is specified:

$$\sum_{i=1}^{N+1} \lambda_i \delta_i \quad (21)$$

E. General formulation

The general formulation of the proposed framework is summarized as follows:

$$\text{minimize}_{p,x,s,t,\delta} \quad \text{Cost function (21)}$$

$$\text{Subject to:} \quad \text{Constraints (3)-(20)}$$

This is a MILP formulation transformed from the fuzzy goal programming formulation, and it can be solved using classical algorithms or heuristic search methods [4][5].

IV. NUMERICAL EXPERIMENTS

All experiments were conducted on a desktop computer with an Intel^R CoreTM 3.40GHz CPU and 16GB RAM. The optimization problem was solved using MATLAB interface of YALMIP and IBM ILOG CPLEX 12.5 solver [10][11][12].

The 24-hour ahead hourly electricity price data of Nov. 3rd, 2013, for Long Island of New York State used in this paper was taken from the NYISO [13]. From midnight to next midnight, these predicted pricing data in USD/MWh were 32.19, 27.63, 26.51, 24.6, 26.41, 22.57, 27.21, 28.6, 31.45, 35.64, 36.35, 36.86, 36.87, 36.21, 34.82, 35.17, 41.37, 57.86, 54.65, 55.44, 50.31, 45.73, 39.02, and 35.67. From these data it can be seen that the highest price (57.86) was 2.56 times the lowest price (22.57).

This study involved three controllable same smart home appliances including a dish washer, a washing machine, and a dryer, similar to those used by Sou et al. [4]. Three different lengths of time slot, 3 minutes, 5 minutes, and 10 minutes, were investigated in the numerical experiments. The dishwasher is not supposed to be run during midnight to 7 o'clock in the morning, and both the washing machine and

dryer are not supposed to be run during midnight to 6 o'clock in the morning. The parameter α , which is the penalty term for using user prohibited time, was set to 1.1. The dryer can only start working after the washing machine has finished, and the delay between them should be no more than 3 time slots. The parameters D_{jj} in (8) for all phases is assumed to be 0, and \bar{D}_{ij} in (8) for the dishwasher, washing machine and dryer were set to 5, 10, and 0 minutes, respectively. The parameters γ and $\bar{\gamma}$ in (13) were set to 0.8 and 1.2, respectively. The peak signals in (15) for 3-minute, 5-minute, and 10-minute time slots are assumed to be 3300 Wh, 5500 Wh, and 11000 Wh, respectively. The dishwasher, washing machine and dryer have 6, 8, and 1 energy phases, respectively. The parameter β in (3) was set to 0.5. The detailed technical specifications of the three appliances are shown in Table I through Table III. All of the rest of the parameter values can be found in these tables. Three representative user priority combinations $\lambda_g (g = 1,2,3,4)$ for the objective function were selected for study, as is listed in Table IV. In reality, all the priority choices are made by the users and completely up to them with regard to their preferences.

TABLE I. DISHWASHER TECHNICAL SPECIFICATIONS

energy phase	Energy required (Wh)	Min power (W)	Max power (W)	Nominal operation time (min)
pre-wash	16	6.47	140	14.9
Wash	751.2	140.26	2117.8	32.1
1st rinse	17.3	10.28	132.4	10.1
Drain	1.6	2.26	136.2	4.3
2nd rinse	572.3	187.3	2143	18.3
drain & dry	1.7	0.2	2.3	52.4

TABLE II. WASHING MACHINE TECHNICAL SPECIFICATIONS

energy phase	Energy required (Wh)	Min power (W)	Max power (W)	Nominal operation time (min)
movement	118	27.231	2100	26
pre-heating	5.5	5	300	6.6
Heating	2054.9	206.523	2200	59.7
Maintenance	36.6	11.035	200	19.9
Cooling	18	10.8	500	10
1st rinse	18	10.385	700	10.4
2nd rinse	17	9.903	700	10.3
3rd rinse	78	23.636	1170	19.8

TABLE III. DRYER TECHNICAL SPECIFICATIONS

energy phase	Energy required (Wh)	Min power (W)	Max power (W)	Nominal operation time (min)
Drying	2426.3	120.51	1454	120.8

TABLE IV. USER PRIORITIES FOR THREE APPLIANCES AND USER TIME PREFERENCE

Priority choice	1	2	3
Dishwasher	0.2	0.1	0.4
washing machine	0.2	0.2	0.3
Dryer	0.3	0.2	0.2
user time preference	0.3	0.5	0.1

V. DISCUSSION OF RESULTS

A. Computational time

Prematurely terminating the optimization process using the first feasible solution terminating condition can dramatically save computational time and at the same time have little influence on the final objective function value. Table V lists the relative extra time cost in using the default optimal solution terminating condition compared to using the first feasible solution terminating condition. Table VI shows the relative objective function error between the two terminating strategies. In view of this fact, our study adopted the first feasible terminating strategy in the remaining experiments.

TABLE V. RELATIVE EXTRA TIME COST (%)

Priority choice	10-min time slot	5-min time slot	3-min time slot
1	121.9013	290.0044	171.1622
2	49.74624	193.8439	158.5955
3	19.67994	225.2939	328.6174
average	63.77581	236.3807	219.4584

TABLE VI. RELATIVE OBJECTIVE ERROR (%)

Priority choice	10-min time slot	5-min time slot	3-min time slot
1	4.107487	3.16173	2.773725
2	2.172829	3.354763	4.889764
3	1.636755	1.867869	0.46734
average	2.639024	2.794787	2.710276

B. Influence of time slot length on electricity cost

The relative extra total electricity cost using the worst solution instead of the best solution is used to facilitate the investigation of the influence of time slot length on electricity cost, and the results are shown in Table VII. Here, the total electricity cost refers to the sum of the three single appliance energy cost objectives specified in (1). From Table VII, it can be seen that the worst-case total energy cost is approximately double that of the best case. The average relative extra energy cost for each time slot indicates no obvious cost savings between the 10-min time slot and the 5-min time slot, while the 3-min time slot can save significant money. This is because the smaller the time slot length, the more flexibility for appliances scheduling.

However, the computation time for the 3-min time slot case is more than 3 times and 10 times that for the 5-min and 10-min time slot case, respectively. This illustrates the tradeoff between time slot size and computational time.

TABLE VII. RELATIVE EXTRA TOTAL ELECTRICITY COST (%)

Priority choice	10-min time slot	5-min time slot	3-min time slot	average for each priority
1	98.25	103.91	112.22	104.79
2	101.40	102.01	110.13	104.52
3	113.56	106.73	113.64	111.31
average for each time slot	104.40	104.22	112.00	106.87 (overall average)

C. Influence of objective priority choice on electricity cost

The influence of the single objective priority choice made by the user can be seen from Table VII. Priority choice 3 (0.4, 0.3, 0.2, 0.1) saves more money because of its very low user time preference priority and hence more prohibited time used. Using Choice 1 (0.2, 0.2, 0.3, 0.3) and Choice 2 (0.1, 0.2, 0.2, 0.5), similar results were produced. There are three reasons for this. First, the user time preference deviation objective δ_4 has great influence on the general fuzzy goal objective relative to appliance-related deviation objectives. We observe that the average optimal value of the user time preference-related deviation objective is more than 10 times that any of the other single objective values. Although the use of the worst and best objective values in constructing the fuzzy objective dramatically resizes the fluctuation range to a range that is similar to that of the other objectives, the great difference still exists. Second, the user time preference objective priority level 0.3 and 0.5 have almost the same effects on this single objective value. Third, only three appliances were involved in this study and two of them are closely related, resulting in small scheduling flexibility. Separately selecting and treating the user time preference objective priority levels can produce better effects.

Illustrations of the price data (Fig. 1) used in this study and some typical energy assignment examples (Fig. 2 – Fig. 4) are given below. All the energy assignment examples are based on the 10-min time slot. With the increase of the user time preference-related objective priority, the violation of the user prohibited time decreases, and eventually no prohibited time is used when this priority is very high.

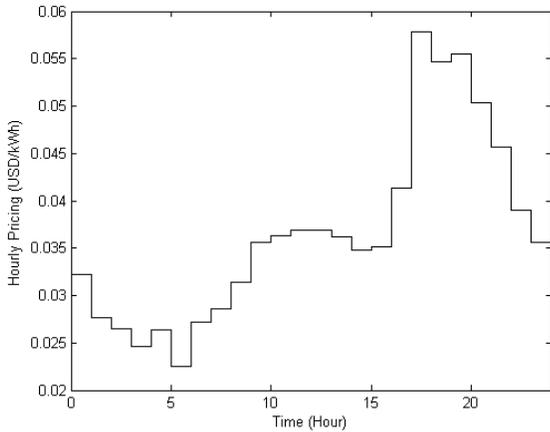


Figure 1. Hourly pricing data.

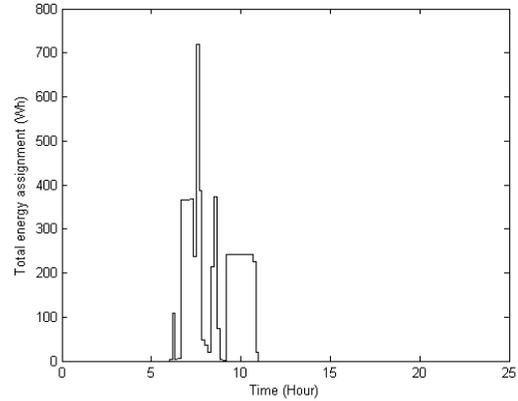


Figure 4. Total energy assignment under the priority choice (0.02, 0.04, 0.04, 0.9).

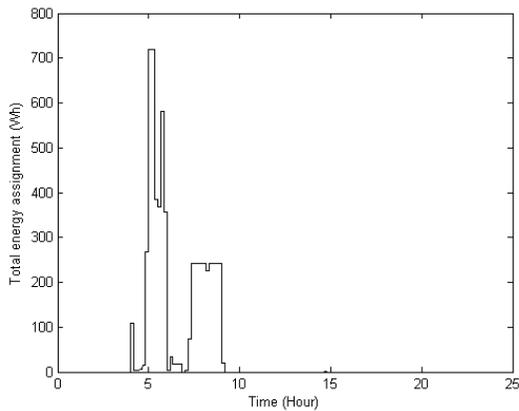


Figure 2. Total energy assignment under priority choice 3 (0.4, 0.3, 0.2, 0.1).

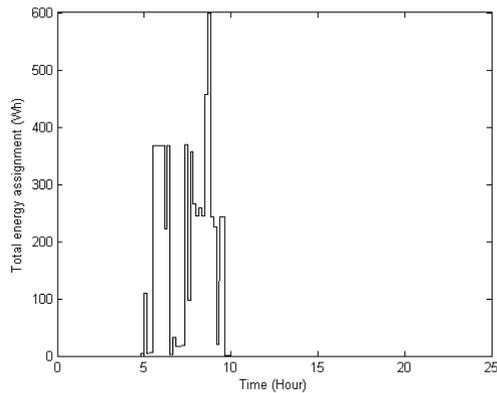


Figure 3. Total energy assignment under the priority choice (0.14, 0.23, 0.20, 0.43).

D. Comparative discussion

The research of applying adaptive fuzzy goal programming theory with priority considered to the area of optimal scheduling of smart home appliances is still quite new. Compared with the reported existing models, our model is more realistic and practical with the new developed between-appliance delay constraints, the soft user time preferences, and the priorities imposed on each single objective. In [4] and [14], only the plain MILP formulation was used to model the home appliance scheduling problem based on rigid user time preferences and without considering the priority of each appliance. A very simple household appliances scheduling formulation taking into account only the peak hourly load constraints was proposed [15]. Samadi et al. [16] proposed a real-time residential load scheduling that took consideration of the load uncertainty, but quite different energy phase concepts such as sleep, awake, active, finished, etc., were used. A type of semi-soft user time preference constraints were proposed in home appliances scheduling [17], however, there are two limitations in this study: the user time preference constraint under each discrete sensitivity level was still a rigid one; no energy phase concepts were adopted for detailed investigation. Direct comparison of the energy saving between the proposed model and other models would be totally meaningless as each model was established based on quite different assumptions, objective, and constraints. In many cases, electricity cost is only part of the general objective.

The proposed fuzzy goal programming model for home appliances scheduling does involve more variables and constraints than does a plain MILP model. The implementation time required for our model is 13% more on average than that for [3]. Since the proposed scheduling is supposed to make one day in advance and only a few minutes or even less than one minute is needed to finish the implementation, the extra time cost becomes marginal.

VI. CONCLUSION AND FUTURE WORK

The proposed mixed integer fuzzy goal programming model for adaptive scheduling of smart home appliances was shown to be effective in saving user's total electricity cost. The user time preferences were transformed from rigid constraints to soft violation penalty objectives and integrated into the fuzzy goal programming formulation. Our optimization solution also allows users to give preferred priorities to different appliances objectives and the user time preference objective as well. The newly introduced constraints that restrict the delay between two closely related appliances make the proposed framework practical. More appliances with same or different type are to be included in the future research to further investigate the performance of the proposed method. The general conclusion of the study is that a closed-form optimization model is an effective approach for adaptation of home appliance schedules to changing prices of electrical power. Future work is to include more common and frequently-used smart home appliances in the study to further test the validity of the proposed model.

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