Ocean Vessel Trajectory Estimation and Prediction Based on Extended Kalman Filter

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Abstract — The accurate estimation and prediction of the trajectories of maneuvering vessels in ocean navigation are important tools to improve maritime safety and security. Therefore, many conventional ocean navigation systems and Vessel Traffic Management & Reporting Services are equipped with Radar facilities for this purpose. However, the accuracy of the predictions of maneuvering trajectories of vessels depends mainly on the goodness of estimation of vessel position, velocity and acceleration. Hence, this study presents a maneuvering ocean vessel model based on a curvilinear motion model with the measurements based on a linear position model for the same purpose. Furthermore, the system states and measurements models associated with a white Gaussian noise are also assumed. The Extended Kalman Filter is proposed as an adaptive filter algorithm for the estimation of position, velocity and acceleration that are used for prediction of maneuvering ocean vessel trajectory. Finally, the proposed models are implemented and successful computational results are obtained with respect to prediction of maneuvering trajectories of vessels in ocean navigation in this study.

Keywords- Trajectory estimation; Trajectory prediction; Target tracking; Extended Kalman Filter; Curvilinear motion model.

I. INTRODUCTION

The European Union (EU) is surrounded by a busy and complex set of sea routes. Furthermore, over 90% of EU external trade transports by the sea and over 3.7 billion tones of freight per year are transferred through the EU ports. In addition, passenger traffic in the seas around the regions of the EU is present approximated to 350 million passenger journeys per year [1]. With the increased demand for maritime transportation of passengers and freight, the increase maritime safety and security issues are highlighted in this region. Therefore, the proposal for local community vessel traffic monitoring and information systems has been considered by the EU Directive 2002/59 [2] for highly dense maritime traffic regions to equip with the regional Vessel Traffic Monitoring & Reporting (VTMR) systems to improve the safety and security.

The detection, tracking, trajectory estimation and trajectory prediction of maneuvering vessels are important facilities for navigation systems as well as the VTMR systems to improve safety, security and survivability in ocean navigation. However, conventional ocean navigation and VTMR systems are equipped with several marine instruments for the same purpose: Radar, Laser, Automatic Radar Plotting Aid (ARPA), and Automatic Identification System (AIS). Even though the first experimental Radar systems were envisioned, around 1920, for ship collision avoidance [3], advanced Radar facilities were developed for the land and air navigation systems in later stages. Furthermore, Laser systems are proposed by recent studies [4] that will be important part of the target detection in close proximity.

ARPA provides accurate information of range and bearing of nearby vessels. AIS is capable of giving all the information on vessel structural data, position, course, and speed. The AIS simulator and marine traffic simulator have been implemented on several experimental platforms to perform navigation safety and security studies [5]. However, there are many challenges faced by the ocean surveillance [6]: The larger surveillance volume, synchronization of targets and sensors, noisy signal propagation environment and multi-target situation observations.

Furthermore, the effective estimation and prediction of trajectories of maneuvering ocean vessels have not been facilitated with the present navigation and VTMR systems. Therefore, main objective in this study is to propose a methodology for the navigation and VTMR system to estimate the present position, velocity, and acceleration of the vessels by observing or measuring only the noisy vessel positions. Furthermore, the estimated position, velocity, and acceleration can be used to predict the future navigation trajectories of the ocean vessel, which is another advantage of this proposed study.

However, the effective prediction of maneuvering trajectories of ocean vessels depends on the accuracy of data that are extracted from observations of the positions from the respective ocean vessels and the adaptive capabilities of the estimation algorithm. Therefore, accurate instruments with low sensor noise, as well as the capable optimal/sub-optimal adaptive estimation algorithm, should be formulated to archive accurate prediction in ocean vessel navigation. Several methods for the estimation and prediction of the maneuvering trajectories have been proposed by recent studies with respect to the land, air and ocean navigation systems. However, almost all the target tracking methods that are used for trajectory estimation and prediction are model based with respect to the recent studies [7].
The models that are used in the estimation and prediction of maneuvering target tracking models can be grouped into two general categories: Continuous-time and Discrete-time models. The continuous-time model in maneuvering target tracking that includes the dynamic system model as well as the measurement model can be formulated as [7]:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)) + w_x(t) \\
z(t) &= f(x(t)) + w_z(t)
\end{align*}
\]  

where \(x(t), u(t)\) and \(z(t)\) represent the system states, control inputs and measurements in continuous-time, respectively. Furthermore, \(w_x(t)\) and \(w_z(t)\) are the process and measurement noise of the system in continuous-time, respectively. The discrete-time model in maneuvering target tracking that includes the dynamic system model as well as the measurement model can be formulated as [7]:

\[
\begin{align*}
\dot{x}(k+1) &= f(x(k), u(k)) + w_x(k) \\
z(k) &= f(x(k)) + w_z(k)
\end{align*}
\]

where \(x(k), u(k)\) and \(z(k)\) represent the system states, control inputs and measurements in discrete-time, respectively. Furthermore, \(w_x(k)\) and \(w_z(k)\) are the process and measurement noise of the system in discrete-time, respectively.

However, combination of continuous-time and discrete-time models in maneuvering target tracking approaches are also been used in recent studies. A nonlinear kinematic system model associated with a white Gaussian state noise for the ocean vessel in considered in this study as further described in Section III.

Furthermore, a linear model for the measurement system associated with a white Gaussian measurement noise is also considered in this study. An estimation algorithm of the Extended Kalman Filter (EKF) is proposed for the prediction of the future navigational trajectories of ocean vessels. However, the EKF algorithm is working in this study as an adaptive filter that estimates the system states of position, velocity and acceleration.

The work presented in this study is part of the on-going effort to formulate an intelligent collision avoidance system in ocean navigation, described in [8] and [9]. The organization of this paper is as follows: The recent developments in the Detection and Tracking of Moving Objects are discussed in Section II. The detail view of the Estimation and Prediction of Ocean Vessel Trajectories are presented in the Section III. Finally, the Computational Simulations and Conclusions are presented in Section IV and V, respectively.

II. RECENT DEVELOPMENTS IN DETECTION AND TRACKING

The Detection and Tracking of Moving Objects (DTMO) and Simultaneous Localization and Mapping (SLAM) are important divisions that are developed under the autonomous navigation systems. Hence, the tools developed under the DTMO and SLAM can be adopted for the estimation and prediction of maneuvering trajectories of land, air and ocean navigation systems. However, the SLAM assumes that the unknown environment is static and the moving objects are as noise sources.

The study of DTMO is the main part of the estimation and prediction of maneuvering trajectories in ocean navigation because moving targets are the major concern in its analysis. Even though the DTMO and SLAM are developed as two independent research directions, they can be complementary to each other in navigation systems [10].

The main functionalities of the DTMO systems are divided into three sections in recent studies [11]: Scan unit, Target Classification unit and Target Tracking & Behavior Prediction unit. The Scan unit consists of the instrumentations that are used for identification of the targets. The Target Classification unit consists of a classification of the targets with respect to the geometrical shapes and sizes. Finally, the Target Tracking & Behavior Prediction is used for estimation of the target current states and prediction of the target future states.

Identification of an accurate mathematical model for the maneuvering target is an important step in estimation and prediction of future trajectories in ocean navigation. When a single model cannot capture the required behavior of the target, the multiple model approaches is also proposed in several studies [12]. In general, maneuvering target tracking models that are used in recent literature can be divided into three categories considering the dimensional space [7]: 1D, 2D and 3D models. While 3D models are popular applications of the air and submersible navigation systems and 1D and 2D models are used in land and ocean navigation systems.

Nevertheless, a formulation of an effective estimation algorithm for maneuvering target is also an important step in prediction of the future maneuvering trajectories in ocean navigation. However, the accuracy of trajectory prediction of a target depends on the adaptive capabilities in the estimation algorithm and there are several approaches can be indentified in recent literature.

A multiple model approach, a constant velocity model and constant speed turn model, with the unscented Kalman filter for curvilinear motion for tracking of maneuvering vehicle is proposed in [13]. Further 2D Laser based obstacle motion tracking in unconstrained environments, with the Kalman Filter algorithm and predicting obstacles future motion, is presented in [14].

The target tracking methods, in combination with the Particle filter and Kalman filters using the radar information is presented in [15]. Furthermore, the Neural Kalman filter for target tracking is illustrated in the study of [16].

A people tracking system that is based on the Laser range data, a multi-hypothesis Leg-Tracker, using a Kalman filter with a constant velocity model, is proposed by [17]. The 2D Laser based obstacle motion tracking in dynamic unconstrained environments using the Kalman filter algorithm [18], and Particle Filters and Probabilistic Data
Associations [19] to predict targets motions are presented in the respective studies.

III. ESTIMATION AND PREDICTION OF OCEAN VESSEL TRAJECTORIES

The main objective in this section is to develop mathematical tools for the estimation and prediction of navigation trajectories of ocean vessels. Therefore, this section is divided into three sections [20]: Target Motion Model (TMM), Measurement Model and Associated Techniques (MAT) and Trajectory Tracking and Estimation (TTE).

A. Target Motion Model

A suitable mathematical model for the vessel maneuvering in ocean navigation is considered in this section. The 2D kinematic model that can capture the navigation capabilities of an ocean vessel is considered during model selection process. In general, ocean vessels always follow parabolic shaped maneuvering trajectories rather than sudden motions as observed in the land and air navigation systems. Furthermore, a vessel maneuvering model is assumed to be a point target with negligible dimensions in this study. Considering the above requirements the continuous-time Curvilinear Motion Model [21] is proposed as the TMM.

The continuous-time Curvilinear Motion Model that is formulated for ocean vessel navigation is presented in Figure 1. The vessel is located in the point A. The vessel x and y positions are represented by \( x(t) \) and \( y(t) \) in continuous-time with respect to the XY coordinate system. Furthermore, the continuous-time velocity components along the x and y axis are represented by \( v_x(t) \) and \( v_y(t) \). The heading angle is presented by \( \chi_a(t) \) and it is assumed that the vessel course and heading conditions are similar. The vessel total continuous velocity is presented by \( v_a(t) \), (where \( v_a^2(t) = v_x^2(t) + v_y^2(t) \)) as illustrated in the figure.

On should note that there are some important features that can be observed from the Curvilinear Motion Model. As presented in the figure, when the normal acceleration \( a_n(t) \) is 0 the model performs the straight line motion, when the tangential acceleration \( a_t(t) \) is 0 the model performs circular motions. Furthermore \( a_t(t) > 0 \) and \( a_n(t) < 0 \) the acceleration conditions that produce parabolic navigation trajectories are also presented in the figure.

Therefore, the Curvilinear Motion Model capabilities of capturing the multi-model features are other advantages in this approach. The standard continuous-time Curvilinear Motion model can be written as:

\[
\begin{align*}
\dot{x}(t) &= f(x(t)) + w_x(t) \\
\dot{y}(t) &= f(y(t)) + w_y(t) \\
\dot{\chi}_a(t) &= \frac{a_t(t)}{V_a(t)} \\
\dot{V}_a(t) &= a_a(t)
\end{align*}
\]

where

\[
\begin{align*}
&x(t) = \begin{bmatrix} x(t) \\ v_x(t) \\ y(t) \\ v_y(t) \\ a_t(t) \\ a_n(t) \end{bmatrix}, &f(x(t)) = \begin{bmatrix} v_x(t) \\ a_t(t)f_x + a_n(t)f_y \\ v_y(t) \\ a_t(t)f_y - a_n(t)f_x \\ 0 \\ 0 \end{bmatrix} \\
f_x &= \frac{v_x(t)}{\sqrt{v_x^2(t) + v_y^2(t)}}, &f_y &= \frac{v_y(t)}{\sqrt{v_x^2(t) + v_y^2(t)}}
\end{align*}
\]

and \( w_x(t) \) is the process noise that is considered as a white Gaussian distributions with 0 mean value and \( Q(t) \) covariance.

\[
Q(t) = \text{diag} \left[ Q_{xx}(t) \ Q_{vy}(t) \ Q_{yy}(t) \ Q_{at}(t) \ Q_{an}(t) \right]
\]

where \( Q_{xx}(t), Q_{yy}(t), Q_{vy}(t), Q_{at}(t) \) and \( Q_{an}(t) \) are respective system state covariance values. Furthermore, the tangential \( a_t(t) \) and normal \( a_n(t) \) accelerations are formulated as:

\[
\begin{align*}
\dot{a}_t(t) &= w_{at}(t) \quad \text{with} \quad E[\dot{a}_t(t)] = a_{t0} \\
\dot{a}_n(t) &= w_{an}(t) \quad \text{with} \quad E[\dot{a}_n(t)] = a_{n0}
\end{align*}
\]

where \( a_{t0} \) and \( a_{n0} \) are mean acceleration values that are constants, and \( w_{at}(t) \) and \( w_{an}(t) \) are tangential and normal
acceleration derivatives that are modeled as white Gaussian
distributions with 0 mean and, \( Q_k \) and \( Q_{an} \) covariance
values, respectively. The Jacobian of \( f(x(k)) \) can be
expressed as:

\[
\frac{\partial}{\partial x}(f(x(t))) =
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & a_{1}(t)f_{vx}^{yy} + a_{n}(t)f_{vy}^{yy} & 0 & a_{1}(t)f_{vy}^{yy} + a_{n}(t)f_{vy}^{yy} & f_{vx}^{yy} & f_{vy}^{yy} \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & a_{1}(t)f_{vx}^{yy} - a_{n}(t)f_{vx}^{yy} & 0 & a_{1}(t)f_{vy}^{yy} - a_{n}(t)f_{vy}^{yy} & f_{vx}^{yy} - f_{vy}^{yy} & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]  

(7)

where

\[
f_{vx}^{yy} = \frac{v_{x}^{2}(t)}{\left(v_{x}^{2}(t) + v_{y}^{2}(t)\right)^{3/2}}, \quad f_{vy}^{yy} = \frac{v_{y}^{2}(t)}{\left(v_{x}^{2}(t) + v_{y}^{2}(t)\right)^{3/2}}
\]

\[
f_{vx}^{yy} = \frac{v_{x}^{2}(t)v_{y}(t)}{\left(v_{x}^{2}(t) + v_{y}^{2}(t)\right)^{3/2}}, \quad f_{vy}^{yy} = \frac{v_{y}^{2}(t)}{\left(v_{x}^{2}(t) + v_{y}^{2}(t)\right)^{3/2}}
\]

B. Measurement Models and Associated Technique

The measurement model is formulated as a discrete-time
linear model due to availability of the ocean vessel positions
usually in discrete time instants. The position values of ocean
vessels can be captured by Radar or Laser based
measurement systems. It is assumed that the vessel position
measurement sensor is located in the position \( O (0,0) \), as
presented in Figure 1. Even though the Radar or Laser based
measurement systems initially capture the Polar coordinates of
ocean vessels, it is assumed that the Cartesian coordinates
of the position coordinates can be derived and no correlation
between the position measurements. The vessel position
measurements in discrete-time can be written as:

\[
z(k) = h(x(k)) + w_z(k)
\]  

(8)

where

\[
z(k) = \begin{bmatrix} z_x(k) \\ z_y(k) \end{bmatrix}, \quad h(x(k)) = \begin{bmatrix} x(k) & 0 & 0 & 0 & 0 \\ 0 & 0 & y(k) & 0 & 0 \end{bmatrix}
\]

and \( z_x(k) \) and \( z_y(k) \) are measurements of \( x \) and \( y \) positions of
the target vessel, and \( w_z(k) \) is a white Gaussian measurement
noise with zero mean and covariance \( R(k) \). The covariance
\( R(k) \) can be written as:

\[
R(k) = \text{diag}[R_{x}(k) \ R_{y}(k)]
\]

where \( R_{x}(k) \) and \( R_{y}(k) \) are respective measurements
covariance values. The Jacobian of measurement model can be written as:

\[
\frac{\partial}{\partial x}h(x(k)) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}
\]  

(9)

C. Trajectory Tracking and Estimation

The development of the Trajectory Tracking and
Estimation (TTE) can elaborate into several directions in
recent studies [20]: The single model based Kalman Filter
(KF), Extended Kalman Filter (EKF), Adaptive Kalman
Filter (AKF), etc. However, the Extended Kalman Filter is
proposed as an adaptive algorithm for the TTE in this study,
due to the EKF capabilities of capturing the nonlinear system
states of the ocean vessel navigation.

In 1960, R.E. Kalman formulated a method of
minimization of a mean-least square error-filtering problem
using a state space system model. The two main features of
the Kalman formulation and solutions of systems are
associated with the Vector modeling of the random
processes under consideration and recursive processing of
the noisy measurements data [22]. However, these conditions
are associated with most of the engineering problems.

The general KF algorithm is limited for application to
linear systems. Therefore, the Extended Kalman Filter (EKF)
is considered as the standard technique for a number of non-
linear system applications. The summarized Extended
Kalman Filter [23] algorithm can be written as:

1) System Model

\[
\dot{x}(t) = f(x(t)) + w_x(t)
\]  

(10)

\[
w_x(t) \sim N(0, Q(t))
\]

\[
E[w_x(t)] = 0, \quad E[w_x(t); w_x(t)] = Q(t)
\]

2) Measurement Model

\[
z(k) = h(x(k)) + w_z(k)
\]  

(11)

\[
w_z(k) \sim N(0, R(k)), \quad k = 1,2,\ldots
\]

\[
E[w_z(k)] = 0, \quad E[w_z(k); w_z(k)] = R(k)
\]

3) Error Conditions

\[
\tilde{x}(k) = \hat{x}(k) - \bar{x}(k)
\]  

(12)

where \( \tilde{x}(t) \) is the state error and \( \hat{x}(t) \) the estimated states
of the system.
4) **System Initial States**

\[ x(0) \sim N(\hat{x}(0), P(0)) \]  

where \( x(0) \) is the initial estimated values and \( P(0) \) is the initial estimated error covariance of the system states.

5) **Other Conditions**

\[ \mathbb{E}[w_x(t); w_x(k)] = 0 \quad \text{for all } k, t \]  

6) **State Estimation Propagation**

\[ \frac{d}{dt} \hat{x}(k) = f(\hat{x}(k)) \]  

7) **Error Covariance Extrapolation**

\[ \frac{d}{dt} P(k) = F(\hat{x}(k))P(k) + P(k)F^T(\hat{x}(k)) + Q(k) \]

\[ F(\hat{x}(k)) = \frac{\partial}{\partial \hat{x}(k)} F(x(k)) \bigg|_{x(k) = \hat{x}(k)} \]

where \( P(k) \) is the estimated error covariance with

\[ P(k) = \text{diag} \left[ P_x(k), P_{v_x}(k), P_y(k), P_{v_y}(k), P_{a_x}(k), P_{a_y}(k) \right] \]

and \( P_x(k), P_{v_x}(k), P_y(k), P_{v_y}(k), P_{a_x}(k) \) and \( P_{a_y}(k) \) are respective estimated state error covariance values.

8) **State Estimate Update**

\[ \hat{x}(k^-) = \hat{x}(k^-) + K(k)[z(k) - h_x(\hat{x}(k^-))] \]  

where \( x(k^-) \) and \( x(k^+) \) are the prior and posterior estimated system states respectively, and \( K(k) \) is the Kalman gain.

9) **Error Covariance Update**

\[ P(k^+) = \left[ I - K(k)H_x(\hat{x}(k^-)) \right] P(k^-) \]  

where \( P(k^-) \) and \( P(k^+) \) are the prior and posterior error covariance of the system state respectively.

10) **Kalman Filter Gain**

\[ K(k) = \left[ P(k^-)H(\hat{x}(k^-))H(\hat{x}(k^-))P(k^-)H(\hat{x}(k^-)) + R(k) \right]^{-1} \]
IV. COMPUTATIONAL SIMULATIONS

This section contains a detail description of the software architecture and initial state values that are considered for the simulations. The proposed EKF algorithm is tested on the MATLAB software platform and simulations are presented in Figures of 2, 3 and 4.

The values that are considered in the computational simulations of the EKF simulations can be presented as: The initial start position \( x(0) = 0 \) (m) and \( y(0) = 0 \) (m) of the ocean vessel is considered. Then the initial velocity components of \( v_x(0) = 1 \) (ms\(^{-1}\)) and \( v_y(0) = 2 \) (ms\(^{-1}\)) are assigned and the actual mean accelerations \( a_{0x} = 2 \) (ms\(^{-2}\)) and \( a_{0y} = 4 \) (ms\(^{-2}\)) are assumed. The initial estimated position as \( \hat{x}(0) = 3 \) (m) and \( \hat{y}(0) = 0 \) (m) are considered. The estimated initial velocity components of \( \hat{v}_x(0) = 2 \) (ms\(^{-1}\)) and \( \hat{v}_y(0) = 0 \) (ms\(^{-1}\)) values are considered for the EKF algorithm. Furthermore, the initial estimated acceleration components of \( \hat{a}_{0x}(0) = 0 \) (ms\(^{-2}\)) and \( \hat{a}_{0y}(0) = 0 \) (ms\(^{-2}\)) are considered. The sampling time used in the EKF estimation is 0.01 (s).

The system state covariance values are assigned as \( Q_x(t) = Q_{x0} = Q_{x0} = 0.1 \) and \( Q_{x0}(0) = Q_{x0}(0) = 0.01 \), with the assumptions of position, velocity and acceleration component covariance values are uncorrelated. Similarly the initial estimated error covariance values are assigned as \( P_x(0) = P_{x0}(0) = P_{x0}(0) = P_{x0}(0) = P_{x0}(0) = 0.01 \), with assumptions of position, velocity and acceleration estimation error covariance components are uncorrelated. The covariance values for the measurements are assigned as \( R_x(t) = R_{x0}(t) = 10 \) with assumption of position measurement covariance components are uncorrelated.

The computational simulations of the trajectory estimations for a maneuvering target vessel using the EKF algorithm are presented in Figure 2. The figure represents the actual trajectory (Act. Traj.), Measured trajectory (Mea. Traj.) and Estimated trajectory (Est. Traj.) of the ocean navigation. As noted from the figure, the EKF estimates the vessel maneuvering trajectory successfully. The vessel velocity components of \( v_x(t) \) and \( v_y(t) \) of actual and estimated are presented in Figure 3. Furthermore, the figure represents the Actual (Act.) and Estimated (Est.) velocities for each velocity components. The successful velocity estimation values are also achieved by the EKF algorithm as presented in the figure within 15 (s) of time interval.

The Estimated (Est.) accelerations of \( a_x(t) \) and \( a_y(t) \) values are presented in Figure 4 with respect to the Actual (Act.) acceleration values. Furthermore, the figure represents the convergence of the Estimated accelerations into the Actual accelerations for normal and tangential acceleration components within 15 (s).

V. CONCLUSION

The satisfactory prediction of ocean vessel positions, velocities and accelerations are achieved by the EKF estimation that is working as an adaptive filter incorporated with the Curvilinear Motion Model and linear measurement model. As presented in Figure 4, the convergence of the estimated accelerations into actual accelerations within approximate time interval of 15(s). Therefore, the estimated velocities and acceleration components can be used for the future maneuvering trajectory prediction of ocean vessel navigation.

The estimated values of the velocity components have small variations around the actual values and that affect on the acceleration estimations. Hence, smoothing techniques can be used for better convergence of the estimated values of system states into the actual values. The improved system states can be used for better prediction of ocean vessel navigation trajectories within smaller time intervals.

Furthermore, it is assumed that the mean acceleration components are constant values with a white Gaussian noise in this study. However, this assumption may not always be realistic and changing acceleration conditions can be observed in ocean navigation. Even when real ocean vessel navigation consists of changing acceleration conditions, the formulations presented in this paper can still hold with the assumptions of constant acceleration within short time intervals.

One should note that the velocity and acceleration estimation values are achieved by only the noisy position measurements that are collected from the vessel navigation, which is the main contribution in this approach. However, the improved formulation of the EKF algorithm, with the smoothing techniques for the fast convergence into actual accelerations, is proposed as the further developments in this study.

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