Monitoring of Health-Recovery Processes with Control Charts

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***Abstract*—The paper presents a statistical process control method for monitoring health-recovery processes described by short non-stationary time series. The Shewhart control chart for residuals, based on model averaging approach, is built for differences between values of consecutive observations. The practical applicability of this new approach has been demonstrated using a real-life example of a recovery from a mild hypertension episode.**

Keywords–E-health; Control chart for residuals; Short time series; Non-stationary process; Stability of recovery process.

I. Introduction

Stability is an important feature of many processes. A process is considered stable, or under control, when its uncontrolled variation is purely random (e.g., due to random measurement errors). In 1924 W. Shewhart introduced a simple tool for monitoring stable processes - a control chart. In its initial stage, which is assumed to be in-control, monitored process characteristics are measured, and their mean value and standard deviation are recorded. These recorded values are used for the design of a control chart, known as the Shewhart control chart, which consists of control lines: central, and two (or one, when only deviation of a process level in one direction is interesting) control. The central line represents the mean value of the process level (or a certain target value for this process), and control lines are located at three standard deviations from a central line. The process is considered stable when its future observations are located inside control lines (limits). When an observation falls outside the control lines, an alarm signal is generated, and the process is considered as being possibly out of control (unstable). When a monitored process goes out of control, it is recommended to look for the reason of this, and take appropriate actions with the aim to revert it to the in-control state.

Basic control charts, used in over 90% practical applications, are designed under two main assumptions: statistical independence of consecutive observations, and the normal distribution of measured characteristics. However, in many practical cases, especially when individual process observations are monitored, these assumptions are not fulfilled. Thus, in the recent 40 years, many inspection procedures that do not rely on these assumptions have been proposed. They are usually described in scientific journal papers or in a few textbooks on statistical quality control, such as a famous book of Montgomery [1]. Some of these procedures have been applied in health-related services, and similar applications. A comprehensive review of different applications of control charts in health-care and public-health surveillance can be found in the paper by Woodall [2]. Since the time of the publication of this paper, many other papers on this topic have been published, mainly in journals related to medicine. For example, some recent applications of control charts in the analysis of health-related data can be found in [3].

Despite real popularity of control charts in many areas, such as industry, finance and business, the number of their applications in health care is relatively small. Probably the main reason of this situation is incompatibility of basic assumptions used for their construction, and the reality of health care. For example, consecutive observations of health-related characteristics are seldom independent. Moreover, they are often described by non-stationary random processes, and the runs of interesting observations are short. Therefore, control charts described in popular textbooks, and in the great majority of scientific papers, are not appropriate for monitoring such processes. Some new, more appropriate, approaches have been investigated quite recently. For example, the properties of control charts used for short runs for autocorrelated, but stationary, data have been discussed in [4].

In this paper, we are interested in a special kind of medical data, namely describing health-recovery processes. For many years, physicians have been prescribing certain treatments, and advances in the health recovery of a treated patient have been monitored during visits, e.g., in health care units. Therefore, possible failures of applied treatments were usually disclosed with delay. In many cases, such delays have had detrimental effects on patient's health. However, with the development of e-health systems based on telemedicine this situation has been dramatically changed. Nowadays, it is possible to monitor the state of patient's health even continuously. However, the main problem now is not related to measurements and transmission of data, but to processing of available information. When human's life is endangered, very expensive systems, e.g., in intensive care hospital units, are used. However, in many cases, the usage of all those sophisticated Information Technology (IT) systems is not necessary. It is quite sufficient to process data off-line, and to signal only these cases when consultancy or intervention of a physician is really needed. What is important in this context, it is the stability of health-recovery processes, understood as non-existence of abnormal and unpredictable changes of the monitored process. It has to be noted here, that an unstable process may be still inside some "normal limits", pre-established by physicians, but its revealed instability suggests the possibility of going beyond such limits. Monitoring of such stability can be achieved by the usage of appropriately designed control charts. The proposal of such monitoring processes, based on a control chart for so called residuals, is the main purpose of this paper.

The proposed approach is general, and may be applied in various contexts. For example, it is suitable to monitor the stability of the blood pressure measurements for patients suffering from hypotension, and to generate early warnings of the Acute Hypotensive Episode (AHE), in which patient's arterial blood pressure decreases to an abnormally low level, that may lead to severe complications or even death. Accurate long-term prediction in this case would allow doctors for timely and effective intervention. The 10th Annual PhysioNet/Computers in Cardiology Challenge 2009 was devoted to predicting the AHE. The results of this competition have been described in [5]. Its participants provided various complex solutions, including: neural networks [6], a rule-based approach [7], decision trees [8] or support vector machines [9]. A short review of other recent approaches for the AHE prediction can be found in [10]. The main aim of all those solution is to predict accidents of AHE for patient in intensive care hospital units. Thus, complex algorithms requiring large computational power could be implemented for this purpose. However, none of them focuses on the monitoring of stability of the processes, and generation of early warnings. The control-chart-based approach, proposed in this paper, yields such early warnings that the stability of the monitored process is disturbed, and that an abnormal episode may occur. Moreover, the proposed method can be easily implemented using limited computational resources.

The paper is organized as follows. In Section II, we describe a mathematical model of a stochastic process (a time series) that may be useful for the description of health-recovery data. Then, in Section III, we propose a control chart based procedure that may be used for monitoring non-stationary health-recovery processes. The problem of the monitoring of short time series using the sXWAM chart, proposed by us in [11], is considered in Section IV. The paper is concluded in its last section.

II. Mathematical model of a monitored process

Consider a real-life example of blood pressure measurements of a patient who is under treatment against mild blood hypertension. In Figure 1, we present results of one-a-day measurements for a period of 480 days.

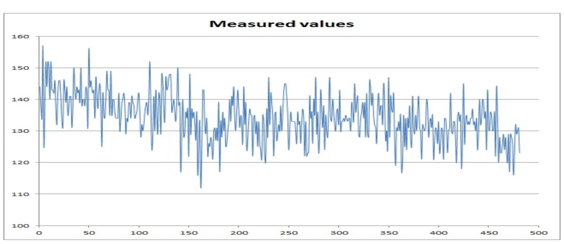

Figure 1. Measurements of blood pressure.

A specialist in time series analysis will find immediately that these measurements, because of a visible trend, may be described by a non-stationary time series (most frequently used methods of the statistical analysis of time series can be found, e.g., in the book by Brockwell and Davis [12]). An important model of such time series is the Autoregressive Integrated Moving Average (ARIMA) model, introduced in the seminal book by Box and Jenkins [13]. For an ARIMA non-stationary process of first order, differences between consecutive observations are described by a stationary Autoregressive Moving Average (ARMA) process, well described in many statistical textbooks. Now, let us look at Figure 2, where such differences are displayed. The process displayed in Figure 2 is definitely stationary. We have found that it may be described by an autoregression process of the fourth order $AR(-0.862, -0.713, -0.358, -0.207)$. Therefore, this real-life example has motivated us to consider in this papers time series described by models of a similar type.

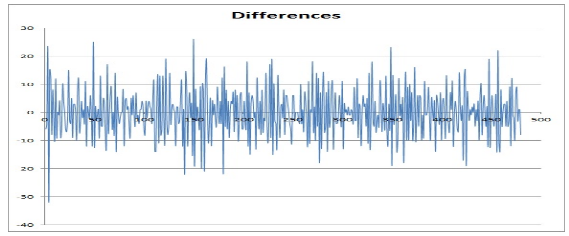

Figure 2. Differences between consecutive measurements.

Let $X_1, X_2, \ldots, X_n$ be a series of measurements obtained during a period of time when a monitored process may be considered (e.g., according to a physician who supervises the treatment) as stable. The process of first differences is now defined as follows: $D_i = X_{i+1} - X_i, i = 1, \ldots, n-1$. We assume the ith difference is related to the previous observations according to the equation

$$D_i = a_1 * d_{i-1} + a_2 * d_{i-2} + \cdots + a_p * d_{i-p} + \epsilon_i, i = p+1, \ldots, \tag{1}$$

where $\epsilon_i, i = p+1, \ldots$ are normally distributed independent random variables with the expected value equal to zero, and the same finite standard deviation.

Estimation of the model $AR(p)$, given by (1), is relatively simple when we know the order of the model p. In order to find estimators $\hat{a}_1, \ldots, \hat{a}_p$, we have to calculate first p sample autocorrelations $r_1, r_2, \ldots, r_p$, defined as

$$r_i = \frac{n \sum_{t=1}^{n-i} (d_t - \hat{\mu})(d_{t+i} - \hat{\mu})}{(n-i) \sum_{t=1}^{n} (d_t - \hat{\mu})^2}, i = 1, \ldots, p, \tag{2}$$

where n is the number of observations in the sample (usually, it is assumed that $n \geq 4p$), and $\hat{\mu}$ is the sample average. Then, the parameters $a_1, \ldots, a_p$ of the $AR(p)$ model are calculated by solving the Yule-Walker equations (see, [12])

$$\begin{array}{l} r_1 = a_1 + a_2 r_1 + \ldots + a_p r_{p-1} \\ r_2 = a_1 r_1 + a_2 + \ldots + a_p r_{p-2} \\ \ldots \\ r_p = a_1 r_{p-1} + a_2 r_{p-2} + \ldots + a_p \end{array} \tag{3}$$

The estimators obtained by solving the Yule-Walker equations are, unfortunately, not numerically stable, especially for small sample sizes. A better method was proposed by Burg. A good description of Burg's algorithm can be found in [14]. Burg's algorithm is used to solve the following optimization problem: for the set of observations $x_1, \ldots, x_N$ find the values $a_1^*, \ldots, a_k^*$ that minimize F_k defined as

$$F_k = \sum_{n=k}^{N} (x_n - (-\sum_{i=1}^{k} a_i x_{n-i}))^2 \tag{4}$$

The estimators of the $AR(p)$ model given by (1) are obtained by setting $k = p, N = n$, $x_i = d_i, i = 1, \ldots, n-1$, and $\hat{a}_i = -a_i^*, i = 1, \ldots, p$.

In practice, however, we do not know the order of the autoregression process, so we need to estimate p from data. In order to do this, we define a transformed random variable, called the *residual*. In the case of autoregression processes, considered in this paper, the residual is defined as

$$Z_i = D_i - (a_1 d_{i-1} + \ldots + a_p d_{i-p}), i = p+1, \ldots, n. \quad (5)$$

When we know exactly the autoregression model, the probability distribution of residuals is the same as the distribution of random variables $\epsilon_i, i = 1, \ldots$ in (1), and its variance can be used as a measure of the accuracy of predictions of future values of the process. For given sample data of size n, the variance of residuals is decreasing with the increasing values of p. However, the estimates of p model's parameters $a_1, \ldots, a_p$ become less precise, and thus the overall precision of prediction with future data deteriorates. As the remedy to this effect, several optimization criteria with a penalty factor, which discourages the fitting of models with too many parameters, have been proposed. In this research we have used the criterion proposed by Akaike [15], and defined as

$$\begin{aligned} BIC = \quad & (n-p)\ln[n\hat{\sigma}^2/(n-p)] + n(1 + \ln\sqrt{2\pi}) + \\ & p\ln[(\textstyle\sum_{t=1}^{n} d_t^2 - n\hat{\sigma}^2)/p], \end{aligned} \quad (6)$$

where d_t are our transformed process observations centered in such a way that their expected values are equal to zero, and $\hat{\sigma}^2$ is the observed variance of residuals. The fitted model, i.e., the estimated order p and parameters of the model $\hat{a}_1, \ldots, \hat{a}_p$ minimizes the value of BIC calculated according to (6). We will use this model in the construction of a control chart for monitoring health-recovery processes.

III. Control chart for process monitoring with autocorrelated data

A. Design of a chart

The design of a simple Shewhart control chart, in the case of a sufficiently large number of individual and mutually independent observations, is extremely simple. One has to collect data (a sample) from a period when the monitored process is stable, calculate average value $\bar{x}$ and standard deviation σ_x, and set the control limits, upper (CUP) and lower (CLO), to

$$\begin{aligned} CUP &= \bar{x} + 3\sigma_x \\ CLO &= \bar{x} - 3\sigma_x. \end{aligned} \quad (7)$$

When process deterioration is related only to increase (decrease) of a process level, one can use one-sided control charts with respective upper (lower) control limits. Usually, it is assumed that the monitored characteristic is normally distributed, and in this case the probability of observing the observation outside one control limit when the monitored process is stable (i.e., observing a false alarm) is very low, and equals 0.00135. It means, that the expected number of observations between consecutive false alarms is equal approximately to 740 (for a one-sided chart), or to 370 (for a two-sided chart).

When consecutive observations of a monitored process are statistically dependent, the situation becomes much more complicated. For example, when sample data are autocorrelated, the properties of a control chart designed using a simple algorithm described above may be completely different from those observed for independent data. To cope with this problem, statisticians have proposed two general approaches. In the first one, we chart the original data, but control limits are adjusted using the knowledge about the type of dependence. In the second general approach, originally introduced by Alwan and Roberts [16], a control chart is used for monitoring residuals. Their methodology is applicable for any class of processes, so it is also applicable for the autoregression process of differences D_i considered in this paper. According to the methodology proposed by Alwan and Roberts [16], the deterministic part of (1) is estimated from sample data of n elements, and used for the calculation of residuals according to (5). Then, these residuals are used for the construction of our control chart according to the algorithm described above.

It is worth noticing that the Shewhart control chart for individual observations, also known as the X chart, is not the only control chart used for monitoring stability of monitored processes. However, it is the simplest one. Moreover, it is easy to interpret by non-specialists. This second feature seems to us very important if we have to use it in a simple health-care monitoring procedure.

B. Operating procedure

Operating procedure of the proposed control chart for residuals, applied for differences between consecutive observations of the monitored process, is the same as in the case of a classical Shewhart control chart. Using the estimated process model, we calculate the predicted value of the difference between the next two observations of the monitored process. An alarm signal is generated when an observed residual (difference between an observed and predicted values) falls beyond control lines. In Figure 3, we present a one-sided (with an upper control limit) control chart for residuals calculated for the process of differences between consecutive measurements of blood pressure displayed in Figure 1. The model of the process of differences D_i was estimated using first 20 observations of the monitored process of blood pressure measurements. Using Burg's algorithm we found that it is the autoregression process of the fourth order $AR(-0.987, -0.805, -0.217, -0.133)$ (Note, that this model is different from the model estimated from larger amount of data presented in Figure 2). Then, residuals calculated for differences $D_5, \ldots, D_{19}$ have been used for the design of a control chart with the upper control limit equal to 20.29. The estimated model has been used for the calculation of residuals related to the next 80 observations. These residuals are displayed on the control chart. We can see

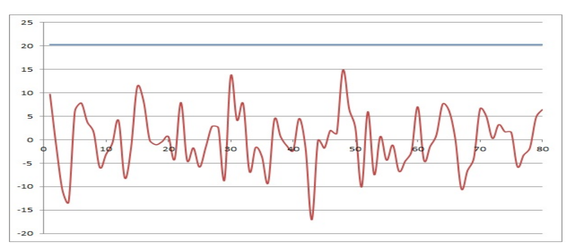

Figure 3. Control chart for residuals of differences of the first order related to measurements of blood pressure.

that the monitored process seems to be under control, as all calculated residuals are located below the upper control limit.

In comparison to a classical control chart for original observations, a control chart for residuals of differences has one important disadvantage: self-adaptation to a changed pattern of data. In order to explain this feature, let us transform our exemplary data by adding 20 to each observation starting from the 10th. The control chart in this case is presented in Figure 4. From Figure 4, we can see that starting from the 10th point until the 12th point on the chart the value of displayed residuals sharply increased, but does not exceed the control limit. Later on, it has returned to the previous level. It means that our chart is able to detect shifts of the monitored process only immediately after the jump. This is in sharp contrast to the classical Shewhart control chart (if it can be applied), where all data points observed after the shift indicate the deterioration of the monitored process. Thus, if the alarm is not generated immediately it will be generated in the future quite randomly, despite the obvious deterioration of the monitored process. Therefore, we have to add an additional mechanism that will increase the probability of detection just after the shift.

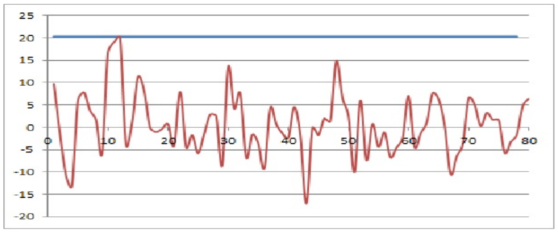

Figure 4. Control chart for residuals of differences of the first order related to measurements of blood pressure with a shifted process level.

One of possible solutions of the problem mentioned above is to use an additional control chart. It can be a control chart for residuals calculated for second order differences defined as $D2_i = X_{i+2} - X_i$. The methodology for the design of this chart is exactly the same as that already described in this paper. Additional advantage of this approach is due to a fact that differences of the second order decrease or even cancel the impact of short cycles in the observed time series. A "weak" alarm signal is generated if it is generated on only one of these two charts. A "strong" alarm signal, that detects possible persistent deterioration, is generated when two consecutive points on the second chart are located beyond its control limits.

In our numerical example of shifted data, the model of the time series of differences of the second order, estimated from the sample of 20 observations, is the autoregression process of the second order $AR(-0.444, -0.555)$. Using this model, we can calculate residuals and design a respective control chart, presented in Figure 5. We can see that in the case of this control chart, deterioration of the process has been revealed with a delay of one measurement. Thus, if we have used both charts, we would detect the change of the process.

Another possible solution which is simpler for implementation, but theoretically less justified, is to calculate an additional residual as the difference between the observed difference of the second order and the predicted difference of the first order, but calculated for the previous observation, and to plot the maximum of these two residuals on the chart designed for the case of differences of the first order. A "weak" alarm is generated when a point on the chart is located beyond the control limits. For a "strong" alarm it is necessary to observe at least two consecutive points on the chart situated beyond the control limits.

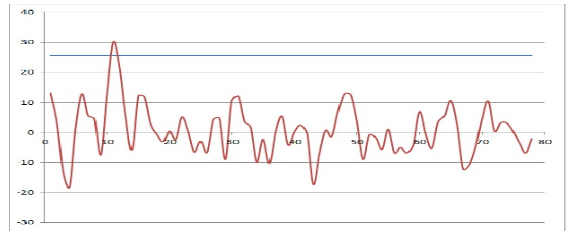

Figure 5. Control chart for residuals of differences of the second order related to measurements of blood pressure with a shifted process level.

It has to be stressed here, that the proposed procedures are based on rather heuristic reasoning, based on observations of a particular series of measurements. Unfortunately, closed mathematical formulae that describe statistical properties of a control chart when observed values of measurements are statistically dependent, as for now, do not exist (except for the simplest cases). Therefore, the properties of the proposed procedures have to be investigated in the future using complex simulation experiments.

IV. USING THE sXWAM CONTROL CHART FOR SHORT PROCESS RUNS

One of the most important characteristics of a control chart is its rate of false alarms. An alarm is considered false if it is generated in a period of time when a monitored process is stable. False alarm rate is usually accompanied with good abilities to detect process disorders, so if this falsity does not lead to serious consequences, higher false alarm rates may be considered acceptable. However, when an alarm cannot be neglected because of its serious consequences, the false alarm rate should very low. For example, in certain pharmaceutical production processes an alarm should trigger a stop of a process, and this may be very costly if the triggering alarm is false. In the case of a stable process, described by the model $AR(-0.987, -0.805, -0.217. -0.133)$ estimated from a sample of $n = 20$ observations, a chart presented in Figure 6 exhibits two false alarms.

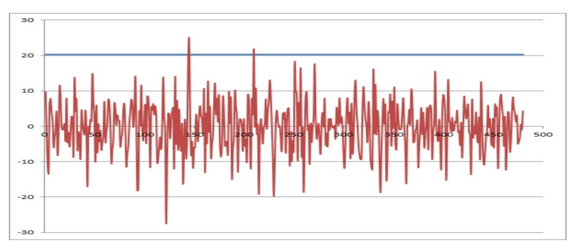

Figure 6. Control chart for residuals with two false alarms.

It has been observed by many authors (see [4], for more information) that control charts for autocorrelated data, especially those designed using small samples of observations, have elevated false alarm rates. Hryniewicz and Kaczmarek-Majer [4] have noted that this rather unfavorable property is somewhat related to the problem of bad predictability in short time series. Inspired by the very good properties of their prediction algorithm for short time series [17], they proposed in [4] a new control chart for residuals, named the XWAM control chart, based on the concept of model averaging.

Let us denote by M_0 the model of a monitored process estimated from a (usually) small sample, and describe its parameters by a vector $(a_{1,0},\ldots.a_{p_0,0})$. We assign to this estimated model a certain weight $w_0 \in [0,1]$. We also consider k alternative models $M_j, j=1,\ldots,k$, each described by a vector of parameters $(a^0_{1,j},\ldots,a^0_{p_j,j})$. In general, any model with known parameters can be used as an alternative one, but in this paper we restrict ourselves to the models chosen according to an extended version of the algorithm described in [11]. Let $w_1^{'},\ldots,w_k^{'}$ denote the weights assigned to models $M_1,\ldots,M_k$ by this algorithm when only alternative models are considered. Because the total weight of the chosen alternative models is $1-w_0$, to the estimated model we assign the weight w_0, and to each chosen alternative model we will assign a weight $w_j=(1-w_0)w_j^{'}, j=1,\ldots,k$.

When we model our process using $k+1$ models (one estimated from data, and k alternatives) each process observation generates $k+1$ residuals. In the case of differences of the first order considered in this paper, they are calculated using the following formula

$$z_{i,j}=d_i-(a_{1,j}d_{i-1}+\ldots+a_{p_j,j}d_{i-p_j}), j=0,\ldots,k; i=p_j+1,\ldots. \tag{8}$$

In (8), we have assumed that for a model with $p_j, j=0,\ldots,k$ parameters we need exactly p_j previous consecutive observations in order to calculate the first residual. Therefore, we need $i_{min}=\max(p_0,\ldots,p_k)+1$ observations for the calculation of all residuals in the sample. For the calculation of the parameters of the XWAM control chart we use $n-i_{min}+1$ weighted residuals calculated from the formula

$$z_i^\star=\sum_{j=0}^{k} w_j z_{i,j}, i=i_{min},\ldots,n. \tag{9}$$

The central line of the chart is calculated as the mean of $z_i^\star$, and the control limits are equal to the mean plus/minus three standard deviations of $z_i^\star$, respectively. The operation of the XWAM control chart is a classical one. First decision is made after i_{min} observations. The weighted residual for the considered observation is calculated according to (9), and compared to the control limits. An alarm is generated when the weighted residual falls beyond the control limits.

The method for the construction of the XWAM chart was firstly proposed by Hryniewicz and Kaczmarek in [4] where they proposed an algorithm for the calculation of the weights of alternative models. This algorithm is based on the methods of computational intelligence, namely the DTW (Dynamic Time Warping) algorithm for comparison of time series. Unfortunately, this algorithm is computationally demanding, so in [11] they proposed its simplification, coined as the sXWAM (simplified XWAM). In this approach, Hryniewicz and Kaczmarek proposed not to compare original time series (observed and alternative), but their summarizations in terms of the autocorrelation functions of the pth order. Let $r_1,r_2,\ldots,r_p$ be the consecutive p values of the sample autocorrelation function, calculated using (2). Similarly, let $r_{1,i},r_(2,i),\ldots,r_{p,i}, i=1,\ldots,J$ be the consecutive p values of the autocorrelation function of an alternative model. For given parameters of the alternative autoregression process $a_{1,i},\ldots,a_{p,i}, i=1,\ldots,J$ the values of $r_{1,i},r_(2,i),\ldots,r_{p,i}, i=1,\ldots,J$ can be found by solving the Yule - Walker equations (3). In general, the consecutive values of r_p can be computed using the following recursion equation

$$r_p=a_1r_{p-1}+a_2r_{p-2}+\cdots+a_p \tag{10}$$

Just like in [11], in this paper we consider only processes of the maximum fourth order. In such a case, explicit formulae for the first three autoregression coefficients are the following [11]:

$$r_1=A_1, \tag{11}$$

$$r_2=a_1A_1+a_2, \tag{12}$$

$$r_3=\frac{a_1B_1+a_3+(a_2+a_4)(A_1+A_2B_1)}{1-a_1B_2-(a_2+a_4)(A_2B_2+A_3)}, \tag{13}$$

where

$$A_1=\frac{a_1}{1-a_2}, \tag{14}$$

$$A_2=\frac{a_3}{1-a_2}, \tag{15}$$

$$A_3=\frac{a_4}{1-a_2}, \tag{16}$$

$$B_1=\frac{A_1(a_1+a_3)+a_2}{1-(a_1+a_3)A_2-a_4}, \tag{17}$$

$$B_2=\frac{A_3(a_1+a_3)}{1-(a_1+a_3)A_2-a_4}. \tag{18}$$

Hence, the consecutive values of $r_4,r_5,\ldots$ can be directly computed from (10).

As the measure of distance between the estimated autocorrelations $r_1,r_2,\ldots,r_p$ and the correlations calculated for the ith alternative model $r_{1,i},r_{2,i},\ldots,r_{p,i}, i=1,\ldots,J$ Hryniewicz and Kaczmarek-Majer [11] used a simple sum of absolute differences (called the Manhattan distance in the community of data mining)

$$dist_{i,MH}=\sum_{k=1}^{p}|r_k-r_{k,i}|, i=1,\ldots,J. \tag{19}$$

In this paper, we consider a slightly more general version of the sXWAM chart. As our alternative models, we consider those autoregression models with k lowest values of $dist_{i,MH}$. Their weights, after some standardization, are inversely proportional to the distances of the closest models. The design of the sXWAM chart for residuals is thus much simpler than the original XWAM chart. The values of the autoregression functions for different alternative models can be computed in advance, and stored in an external file. This file can be read by a computer program, and used for choosing the model that fits to the observed sample (and its estimated autoregression function).

The example of the sXWAM chart is presented in Figure 7 for the same original data that have been used for the construction of the control chart presented in Figure 6. For the design of this chart it was assumed that the weight for sample data is $w_0 = 0.7$. Five alternative process models have been found using the algorithm described above: $AR(-0.9, 0.5, 0.4, -0.3)$ with relative weight $w_1^{'} = 0.201$, $AR(0.8, 0.7, -0.5, -0.3)$ with relative weight $w_2^{'} = 0.201$, $AR(-0.9, 0.5, 0.4, -0.3)$ with relative weight $w_3^{'} = 0.200$, and $AR(-0.8, 0.7, 0.5, -0.3)$ with relative weight $w_4^{'} = 0.199$, and $AR(0.8, 0.5, -0.3, 0.4)$ with relative weight $w_5^{'} = 0.199$. We can see that in this case we have observed only one

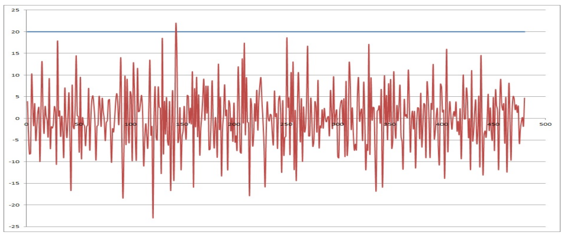

Figure 7. Control chart for weighted residuals with one false alarm.

alarm generated at the same time point as one of the alarms generated on the control chart with non-weighted residuals. Experiments with artificially shifted process levels have shown that the detection ability of the proposed sXWAM chart are similar to that observed for the chart with non-weighted residuals.

V. CONCLUSIONS

In this paper, we have considered monitoring stability of short and non-stationary processes using a simple tool such as a Shewhart control chart. Such processes are typical for health-recovery processes, where natural randomness of measured health-related characteristics is accompanied by random or deterministic trends. Statistical analysis of non-stationary processes is usually very difficult and costly for implementation, as it requires large amount of available data and sophisticated specialized software. It can be used at intensive-care hospital units or in cases when patient's life is endangered. However, in many cases, it is completely sufficient to monitor the state of health using personal measuring devices, and to alarm a patient (or his/hers physician) only in the case of unexpected events. We are of the opinion that this can be done using simple tools, like control charts, by simple software implemented in personal measurement equipments. For this reason we have decided to propose such a chart in this paper.

In our research, we assume that at its initial stage the monitored process is supervised (e.g., by a physician), and considered as stable. Data from this stable period, considered as our sample data, are used for the identification of the monitored process and construction of a control chart. Because simple methods for monitoring non-stationary processes do not exist, we propose to monitor differences of the first order (i.e,, differences between values of consecutive measurements). This approach is effective for linear or approximately linear trends. When we consider, as in this paper, short series of observations, this assumption seems to be rather realistic. However, it is possible to apply the proposed methodology for differences of a higher order. For example, in this paper, we also consider differences of the second order which can be used in the case of processes with alternating (e.g., morning and evening) process levels. In our investigations we have assumed that our series of observations are rather short, and the monitored process has to be identified using a small sample of measurements. This assumption reflects reality when health-recovery processes is evaluated by a physician for only short time, and the period in which the process has to be stable is also short (e.g., until a next treatment is applied). For this reason, we have proposed a novel statistical tool, sXWAM chart, developed recently by us.

The performance of the proposed method has been verified using real-life data. Unfortunately, the amount data, considered in this research, is rather limited, so the presented results should be viewed as a kind of proof of the concept. Further investigations using real and simulated data are needed for more precise evaluation of the statistical properties of the proposed monitoring procedure.

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