Quantifying Network Heterogeneity by Using Mutual Information of the Remaining Degree Distribution

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***Abstract*—As the Internet becomes a social infrastructure, a network design method that has adaptability against the failure of network equipment and has sustainability against changes of traffic demand is becoming important. Since we do not know in advance when the environmental changes occur and how large the changes are, it is preferable to have heterogeneity in topological structures so that the network can evolve more easily. In this paper, we investigate the heterogeneity of topological structures by using mutual information of remaining degree distribution. We discuss and show that the mutual information represents the heterogeneity of topological structure through illustrative examples. Our results show that the mutual information is high at most of router-level topologies, which indicate that the route-level topologies are highly designed by, e.g., the network operators. We also compared topologies with different mutual information, and show that, when node failures occur, the alternative paths will less converge on some of the links in topology having low mutual information.**

Keywords-power-law network; router-level topology; topological structure; mutual information; network heterogeneity; degree distribution; node failure.

I. Introduction

As the Internet becomes the social infrastructure, it is important to design the Internet that has adaptability and sustainability against environmental changes [1], [2]. However, dynamic interactions of various network-related protocols make the Internet into a complicated system. For example, it is shown that interactions between routing at the network layer and overlay routing at the application layer degrade the network performance [2]. Therefore, a new network design method which has the adaptability against the failure of network equipment and has the sustainability against changes of traffic demand is becoming important. Since complex networks display heterogeneous structures that result from different mechanisms of evolution [3], one of the key properties to focus on is the network heterogeneity where, for example, the network is structured heterogeneous rather than homogeneous by some design principles of information networks.

Recent measurement studies on the Internet topology show that the degree distribution exhibits a power-law attribute [4]. That is, the probability P_x, that a node is connected to x other nodes, follows $P_x \propto x^{-\gamma}$, where γ is a constant value called scaling exponent. Generating methods of models that obey power-law degree distribution are studied widely, and Barabási-Albert (BA) model is one of it [5]. In BA model, nodes are added incrementally and links are placed based on the connectivity of topologies in order to form power-law degree distribution. The resulting topology has a large number of nodes connected with a few links, while a small number of nodes connected with numerous links. Topologies generated by BA model are used to evaluate various kinds of network performance [6], [7].

However, it is not enough to explain topological characteristics of router-level topologies by such models. It is because topological characteristics are hardly determined only by degree distribution [8], [9]. Li et al. [8] enumerated several different topologies with power-law, but identical degree distribution, and showed the relation between their structural properties and performance. They pointed out that, even though topologies have a same degree distribution, the network throughput highly depends on the structure of a topology. The lessons from this work suggest us that the heterogeneity of the degree distribution is insufficient to discuss the topological characteristics and the network performance of router-level topologies.

In this paper, we focus on the property, diversity. It is a property studied in biological systems. Biological systems are systems that evolve robustly under many kinds of environmental changes. They often studied with information networks in complex system field [10]–[13]. Many of their networks also exhibit power-law attribute. A study of a key mechanism for adapting to environment changes in biological systems [10] explained that, because the system components can contribute to required traits diversely, the system can getting traits required in a new environment by changing their contribution adaptively. Prokopenko et al. [14] considered the diversity changes in growing process of some complex systems. They said that an organized system, which we consider as a less diverse system here, with effectively less configurations available. They also said that the system configurations may be have and look more complex than a disorganized system, a diverse system, to which more configurations are available. From their words, we considered that a diverse system which more configurations are available to is easy to adapt to different environment. Therefore, we think that diversity is an interesting property to focus on in router-level topologies.

In [14], they used mutual information to measure the complexity, which we consider as diversity here. Inspired from their work, we investigate the topological diversity of router-level topologies by using mutual information. Here, the topological diversity means how diverse the interconnections are in any sub graphs chosen from the topology. Mutual information yields the amount of information that can obtain about one random variable X by observing another variable Y. The topological diversity can be measured by considering Y as some random variable of a part of the topology and X as the rest of it. Solé et al. [3] studied complex networks by using remaining degree distribution as the random variable. They calculated the mutual information of remaining degree distribution of biological networks and artificial networks such as software networks and electronic networks, and shown that both of them have higher mutual information than randomly connected networks. In this paper, we evaluate the mutual information of some router-level topologies, and show that the mutual information represents the topological diversity.

Heterogeneity of structures have also been studied by Milo et al. [15]. They have introduced a concept called Network Motif. The basic idea is to find several simple sub graphs in complex networks. Arakawa et al. [16] shows the characteristic of router-level topologies by counting the number of each kind of sub graph which consists of 4 nodes respectively. They conclude that router-level topologies have more sub graphs called "sector", that is removing one link from 4 nodes complete graph, than other networks. However, Network Motif is expected to evaluate the frequency of appearance of simple structure in a topology, and is not expected to measure the diversity of topology.

The rest of this paper is organized as follows. The definition of remaining degree and mutual information is explained in Section II. We investigate the topological characteristic and give some illustrative examples by changing the mutual information through a rewiring process in Section III. In Section IV, mutual information of several router-level topologies are calculated, and shown. Another topological characteristic, which is from the information network aspect, is shown in there too. Finally, we conclude this paper in Section V.

II. Definitions

Information theory was originally developed by Shannon for reliable information transmission from a source to a receiver. Mutual information measures the amount of information that can be obtained about one random variable by observing another. Solé et al. [3] used remaining degree distribution as the random variable to analysis complex networks. In this section, we explain the definitions of the mutual information of remaining degree with some example topologies shown in Table I.

Remaining degree k is defined as the number of edges leaving the vertex other than the one we arrived along, so that it is one less than the ordinary degree. The example is shown in Figure 1, where the remaining degree is set to two for the left node and three for the right node.

The distribution of remaining degree $q(k)$ is obtained from:

$$q(k) = \frac{(k+1)P_{k+1}}{\Sigma_k k P_k}, \tag{1}$$

where $P(P_1, \dots, P_x, \dots, P_K)$ is the ordinary degree distribution, and K is the maximum degree.

The mutual information of remaining degree distribution, $I(q)$, is

$$I(\text{q}) = H(\text{q}) - H_c(\text{q}|\text{q'}), \tag{2}$$

where q=($q(1)$, … , $q(i)$, … , $q(N)$) is the remaining degree distribution, and N is the number of nodes.

The first term $H(\text{q})$ is entropy of remaining degree distribution:

$$H(\text{q}) = -\sum_{k=1}^{N} q(k) \log(q(k)), \tag{3}$$

and the range of entropy is $0 \leq H(\text{q})$. Within the context of complex networks, it provides an average measure of network's heterogeneity, since it measures the dispersion of the degree distribution of nodes attached to every link. H is 0 in homogeneous networks such as ring topologies. As a network become more heterogeneous, the entropy H gets higher. Abilene inspired topology [8], that is shown in Figure 2, is heterogeneous in its degree distribution, as shown in Figure 3. Therefore, it has higher entropy as shown in Table I.

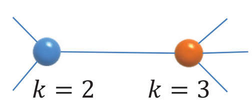

Figure 1. Example of remaining degree

Table I
Mutual information of example topologies

Topology	H	H_c	I
Ring topologies	0	0	0
Star topologies	1	0	1
Abilene-inspired topology	3.27	2.25	1.02
A random topology	3.22	3.15	0.07

The second term H_c(q|q') is the conditional entropy of the remaining degree distribution:

$$H_c(\text{q}|\text{q'}) = -\sum_{k=1}^{N}\sum_{k'=1}^{N} q(k')\pi(k|k')\log\pi(k|k'), \quad (4)$$

where $\pi(k|k')$ are conditional probability:

$$\pi(k|k') = \frac{q_c(k,k')}{q(k')}. \quad (5)$$

$\pi(k|k')$ give the probability of observing a vertex with k' edges leaving it provided that the vertex at the other end of the chosen edge has k leaving edges. Here, $q_c(k,k')$ is the joint probability, which gives the probability of existence of a link that connects a node with k edges and a node with k' edges, and it is normalized as:

$$\sum_{k=1}^{N}\sum_{k'=1}^{N} q_c(k,k') = 1. \quad (6)$$

The range of conditional entropy is $0 \leq H_c(\text{q}|\text{q'}) \leq H(\text{q})$. Ring topologies and star topologies have the lowest H_c, because, when knowing the degree of one side of a link, the degree of the node on the other side is always determined. For Abilene inspired topology, because of its heterogeneous degree distribution, it is hard to determine the degree of the other side of a link than ring topologies or star topologies. Therefore, the conditional entropy H_c(q|q') is higher than them. However, to compare with a random topology that have almost the same H(q) as Abilene-inspired topology, the H_c(q|q') of Abilene-inspired topology is lower than that of the random topology. That means the degree combination of a pair of nodes connected to a link is more biased in Abilene-inspired topology than in the random topology.

Finally, using the distribution and probability explained above, mutual information of the remaining degree distribution can also be expressed as follow:

$$I(\text{q}) = -\sum_{k=1}^{N}\sum_{k'=1}^{N} q_c(k,k')\log\frac{q_c(k,k')}{q(k)q(k')}. \quad (7)$$

The range of mutual information is $0 \leq I(\text{q}) \leq H(\text{q})$. It is higher in star topologies and Abilene-inspired topology, since it can get more information about the degree of a node by observing the node connected to it. And I(q) of ring topologies and the random topology is low, but the reason is different because of the difference in their H. In ring topologies, because of the homogeneous degree distribution, no information can be obtained. On the contrast, in the random topology, though the degree distribution is heterogeneous, because of the random connections, less information can be obtained. As we can see from these example topologies, I(q) is hard to discuss without considering about H(q). Hereafter in this paper, we mainly use H(q) and I(q) to discuss topologies.

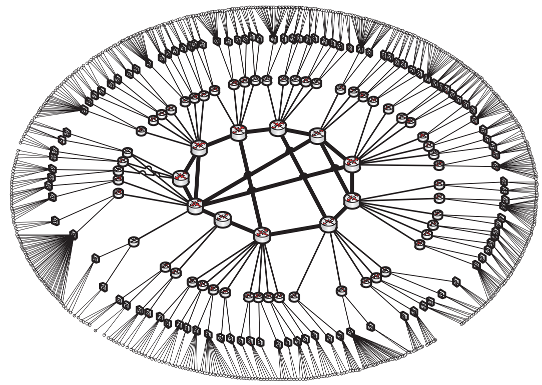

Figure 2. Abilene-inspired topology [8]

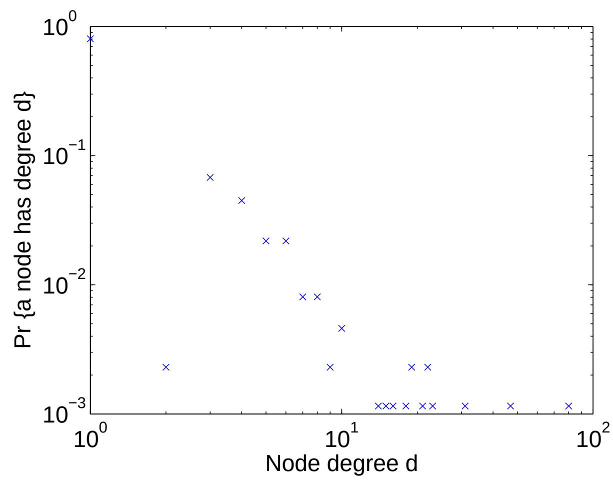

Figure 3. Degree distribution of Abilene-inspired topology

III. MUTUAL INFORMATION AND THE CHARACTERISTIC OF TOPOLOGIES

In this section, we explore the relationship between entropy and average hop distance. Then, we show some illustrative examples of some topologies with different mutual information.

A. Entropy H and average hop distance

To show the relationship between entropy and the characteristic of topologies, we generate topologies having different entropy, and compared their average hop distance and degree distribution.

Topologies are generated by simulated annealing that looks for a candidate network that minimize the potential function $U(G)$. Here, the temperature is set to 0.01, and the cooling rate is set to 0.0001. The simulation searched 450000 steps. The initial topology is set to a topology obtained by BA model which has 523 nodes and 1304 links, that is as same as AT&T explained in Section IV. Topologies

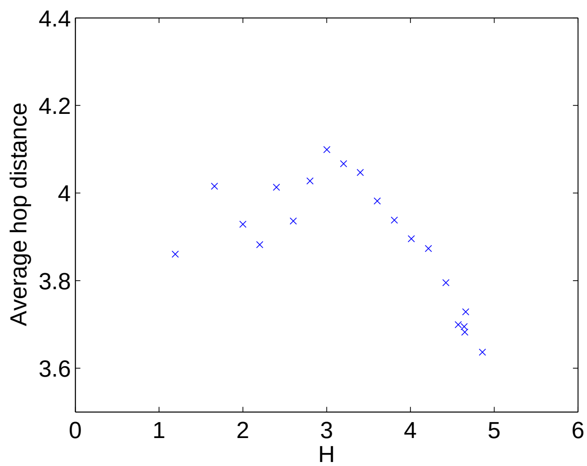

Figure 4. Average hop distance

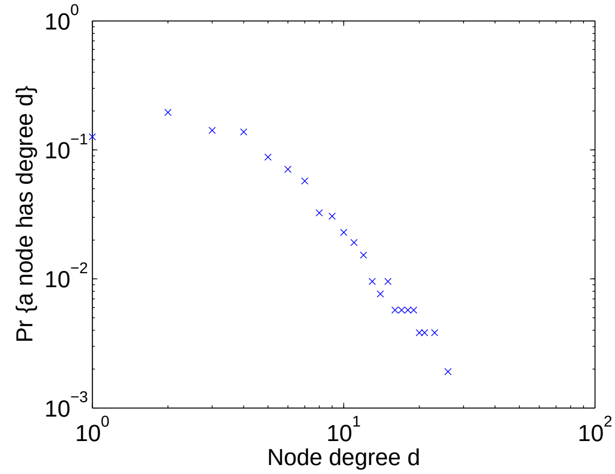

Figure 6. Degree distribution ($H = H_c = 4.2$)

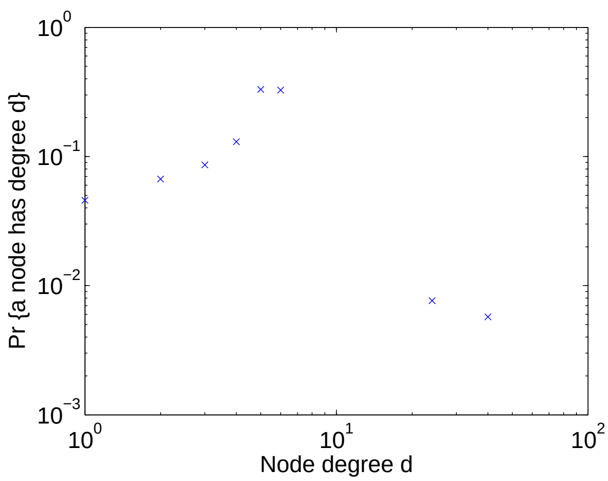

Figure 5. Degree distribution ($H = H_c = 2.2$)

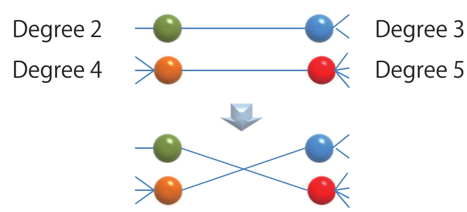

Figure 7. Rewiring method to leave the degree distribution unchanged

are changed by random rewiring, and try to minimize the following potential function:

$$U(G) = \sqrt{(H - H(G))^2 + (H_c - H_c(G))^2}. \quad (8)$$

Here H and H_c are pre-specified value of entropy and conditional entropy respectively. $H(G)$ and $H_c(G)$ are entropy and conditional entropy calculated by the topology G generated in the optimizing search process. We generated topologies by setting H, H_c as $H = H_c$ from 1 to 5. Every time in the search process, $U(G)$ converge to approximately 0. Therefore, entropy and conditional entropy of the generated topologies are almost equal, and their I are approximately 0.

Figure 4 shows the average hop distance of topologies we generated. Degree distribution of a topology generated by setting $H = H_c = 2.2$ is shown in Figure 5, $H = H_c = 4.2$ is shown in Figure 6. Here, average hop distance is defined as the average of hop distance between every node pairs. We calculate the hop distance by assuming the minimum hop routing. From the result, we can see that, when H increases higher than 3, the average hop distance decreases. This is because, as H increases, the degree distribution become biased, and it gets close to power-law around $H = 4$.

B. Mutual information I and topological diversity

Next, we show some illustrative examples of topologies with different mutual information. Because router-level topologies obey power-law, we compare topologies having high H.

Topologies are again generated by the simulated annealing. We set the same parameter and the same initial topology as we have used in the previous section. The different points are the way to rewire the topology and the potential function $U^I(G)$. For the first point, topology is changed by a rewiring method [17] that leaves the degree distribution unchanged, i.e., by exchanging the nodes attached to any randomly selected two links (Figure 7). For the second point, the potential function we used to minimize is $U^I(G)$ defined

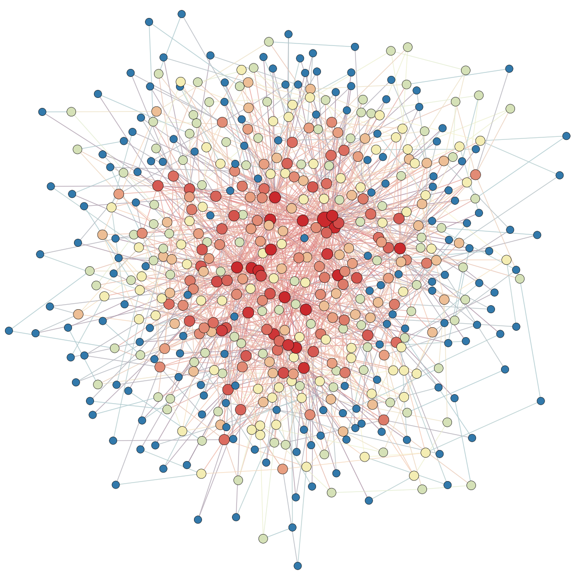

Figure 8. T_{Imin} with minimum mutual information

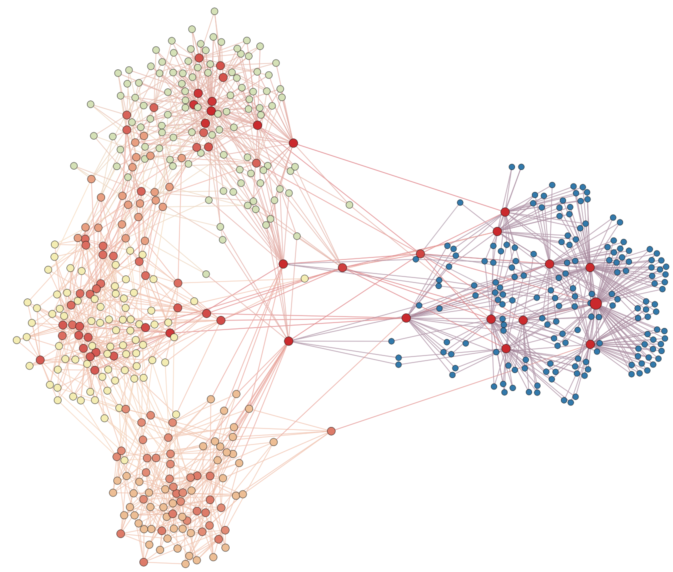

Figure 9. T_{Imax} with maximum mutual information

Table II
TOPOLOGIES OBTAINED BY SIMULATED ANNEALING

Topology	Nodes	Links	$H(G)$	$H_c(G)$	$I(G)$
BA	523	1304	4.24	3.98	0.26
T_{Imin}	523	1304	4.24	4.13	0.12
T_{Imax}	523	1304	4.24	1.54	2.70

as,

$$U^I(G) = |I - I(G)|, \tag{9}$$

where I is pre-specified mutual information, and $I(G)$ is mutual information calculated by the topology G generated in the optimizing search process. Note that looking for a pre-specified mutual information I is as the same as looking for a pre-specified conditional entropy H_c under the same entropy H. Because the entropy is same when the degree distribution unchanged, minimizing mutual entropy is identical to maximize conditional entropy.

To show the relationship between mutual information and topological diversity, we use two topologies: topology T_{Imin} with minimum mutual information and topology T_{Imax} with maximum mutual information. T_{Imin} is generated by setting $I = 0.0$ for simulated annealing, and the resulting mutual information is 0.12. The topology is shown in Figure 8. T_{Imax} is generated by setting $I = 3.0$ for simulated annealing, and the resulting mutual information is 2.70. The topology is shown in Figure 9. In both figures, colors represent node degrees. Nodes which have the same color have the same node degree. Topological characteristics of the initial topology, T_{Imin} and T_{Imax} are summarized in Table II.

From Figures 8 and 9, we can see that topology with high mutual information is less diverse, and have more regularity than the one with low mutual information. From Figure 10 to Figure 13, we show $\pi(k|k')$ dependent on remaining degree k. $\pi(k|k')$ is defined as the probability that observing a vertex with k' edges leaving it provided that the vertex at the other end of the chosen edge has k leaving edges. Figures 10 and 11 show $\pi(k|k')$ of nodes with the largest remaining degree and nodes with the smallest remaining degree in T_{Imin}, respectively. Figures 12 and 13 show $\pi(k|k')$ of nodes with the largest remaining degree and nodes with the smallest remaining degree in T_{Imax}, respectively. We can see that $\pi(k|k')$ of T_{Imax} is more biased than that of T_{Imin}. This also represents that the topology with high mutual information is less diverse than the one with low mutual information.

IV. TOPOLOGICAL DIVERSITY IN ROUTER-LEVEL TOPOLOGIES

In this section, we calculate the measurement for some router-level topologies. According to those measurements, we discuss the topological diversity of the router-level topologies. Next, we evaluate topologies with different mutual information from an information network aspect. We evaluate the amount of increment of the edge betweenness centrality under some node failure occurring situation, and evaluate the link capacity needed to deal with it.

A. Mutual information of router-level topologies

In this section, we show the mutual information of some router-level topologies. We calculated mutual information for topologies: Level3, Verio, AT&T, Sprint and Telstra. The

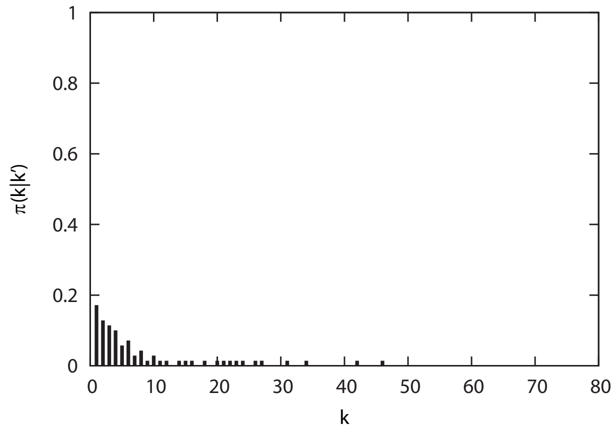

Figure 10. $\pi(k|k')$ of nodes with the largest remaining degree in T_{Imin}

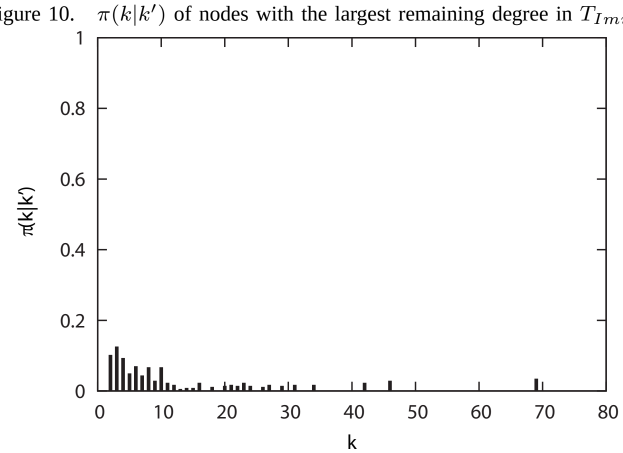

Figure 11. $\pi(k|k')$ of nodes with the smallest remaining degree in T_{Imin}

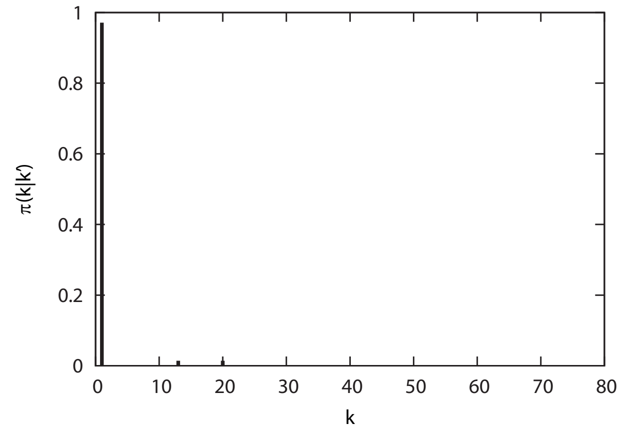

Figure 12. $\pi(k|k')$ of nodes with the largest remaining degree in T_{Imax}

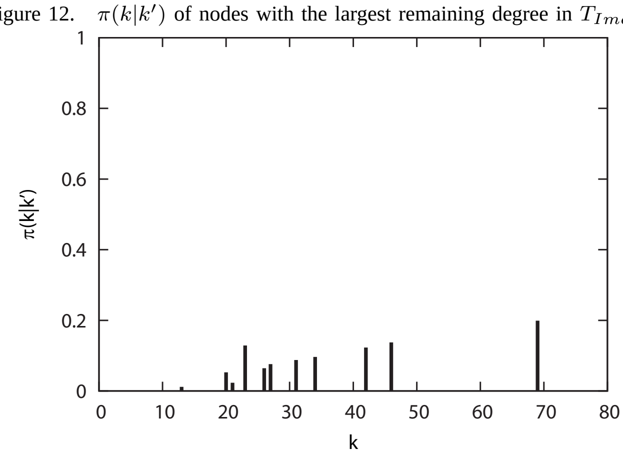

Figure 13. $\pi(k|k')$ of nodes with the smallest remaining degree in T_{Imax}

Table III
MUTUAL INFORMATION OF ROUTER-LEVEL TOPOLOGIES

Topology	Nodes	Links	$H(G)$	$H_c(G)$	$I(G)$
Level3	623	5298	6.04	5.42	0.61
Verio	839	1885	4.65	4.32	0.33
ATT	523	1304	4.46	3.58	0.88
Sprint	467	1280	4.74	3.84	0.90
Telstra	329	615	4.24	3.11	1.13
BA	523	1304	4.24	3.98	0.26

router-level topologies are measured by Rocketfuel tool [18]. To compare with those router-level topologies, a topology made by BA model [5] which has the same number of nodes and links with AT&T is also calculated. The results are summarized in Table III and Figure 14.

From Table III, we can see that, all the router-level topologies have high H, which means they have heterogeneous degree distribution. Level3 topology has higher H than others. This is because the measured topology includes many MPLS paths. These paths made the topology having high heterogeneity in degree distribution. Except Level3 topology, other router-level topologies shown in Table III has almost the same H.

Comparing those topologies with BA topology that also have almost the same H, we can see that, the mutual information of router-level topologies are higher than that of the model-based topology. This can be explained by a design principle of router-level topologies. Because router-level topologies are designed under the physical and technological constraints such as the number of switching ports and/or maximum switching capacity of routers, there are some restrictions and a kind of regulations on constructing the topologies, so that they are less diverse. Note, however, that of Verio topology is low. This can be explained by its growing history. Because Verio grows big with small ISPs [19], it contains various kinds of design principles conducted in each ISP. Therefore, Verio topology is more diverse than other router-level topologies.

B. Link capacity needed for topologies with different mutual information

In this section, we generated several topologies with different mutual information, but having the same entropy, and compared their characteristics in an information network

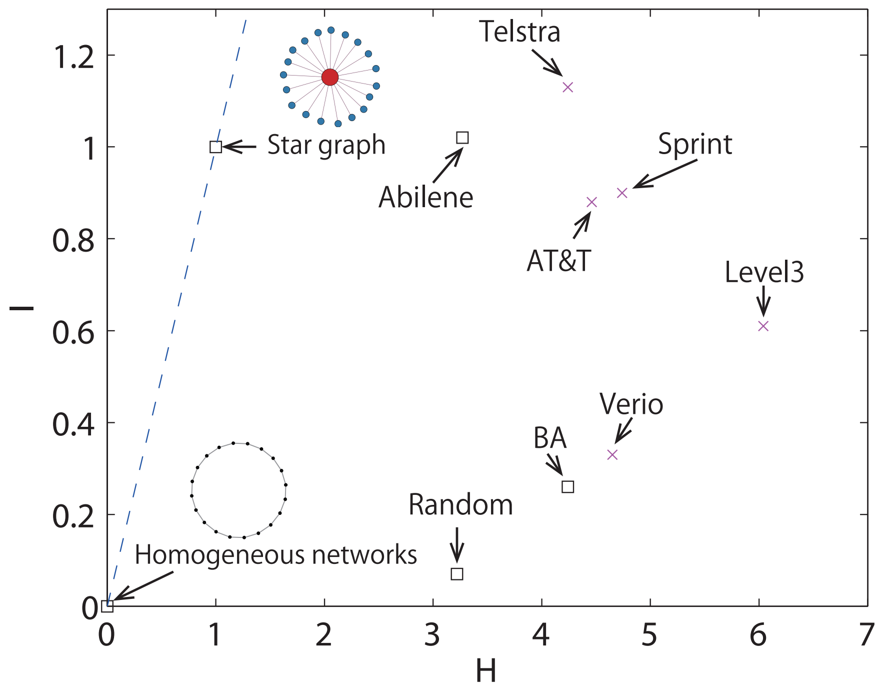

Figure 14. Entropy and mutual information

Table IV
MUTUAL INFORMATION OF TOPOLOGIES REWIRED FROM AT&T

Topology	AT&T$_{0.3}$	AT&T$_{0.4}$	AT&T$_{0.5}$	AT&T$_{0.6}$	AT&T$_{0.7}$	AT&T$_{0.8}$	AT&T
H	4.45583	4.45583	4.455834	4.45583	4.45583	4.45583	4.45583
H_c	4.17594	4.07697	3.97701	3.87589	3.77558	3.67903	3.57515
I	0.27989	0.37886	0.47882	0.57994	0.68025	0.77680	0.88068
Average hop distance	3.57439	3.56669	3.64005	3.74615	3.92027	4.18759	5.06338

aspect. To investigate the adaptability against environmental changes, we evaluate changes in edge betweenness centrality under some node failures occurring situation. When considering about the information network, it is preferable to have fewer changes in load on links even when node failures occur, because the load increment would lead to high link usage, that would increase delay, or high link capacity cost, that is needed to deal with it. To evaluate it simply, we regard edge betweenness centrality as load on links, and evaluate the minimum link capacity needed to cover node failures. Note that the edge betweenness centrality does not reflect the actual load on links. Nevertheless, we use the edge betweenness centrality to characterize ISP topologies because it gives a fundamental characteristic to identify the amount of traffic flow on topologies.

Topologies we used to compare this time are generated by rewiring AT&T randomly. The rewiring method leaves the degree distribution unchanged, which is as same as explained in Section III-B. Because the topological diversity become lower as the rewiring proceed, we calculated mutual information for every topology, and pick out topologies every time when the mutual information decreases 1 than the previous picked out one. The entropy, conditional entropy and mutual information of all the selected topologies are summarized in Table IV. AT&T$_{0.3}$ is the last topology possible to generate by this method with a long time of simulation. The average hop distance of each topology is also shown in it.

The failure we consider here is a single node failure. First, we evaluate the minimum link capacity needed to cover every pattern of single node failures. The link capacity $C(i)$ on link i is calculated as follow:

- Step 0: For all links i, set the initial edge betweenness centrality $E(i)$ as the link capacity $C(i)$:

$$C(i) = E(i). \tag{10}$$

- Step 1: When node j fails, calculate the new edge betweenness centrality $E_j(i)$ for every link. Renew the

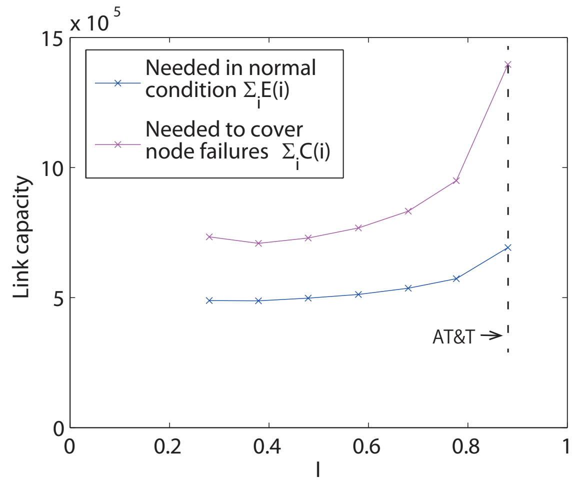

Figure 15. Link capacity

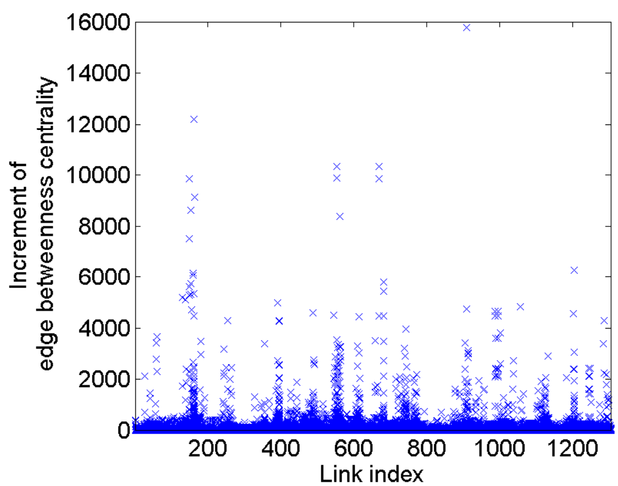

Figure 16. Increament of edge betweenness centrality (AT&T)

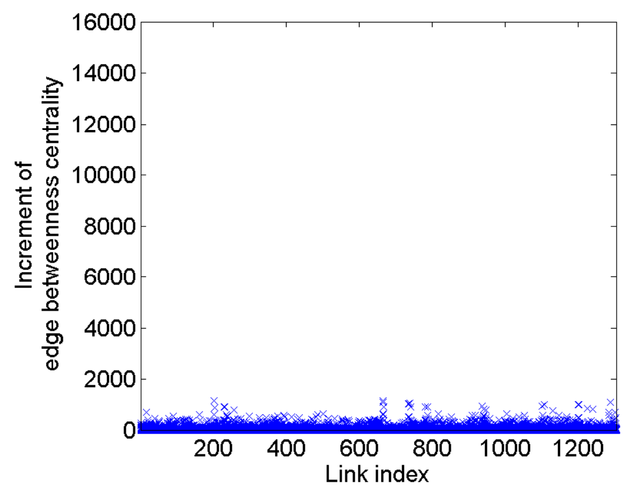

Figure 17. Increament of edge betweenness centrality (AT&T$_{0.3}$)

link capacity as (11) for every link:

$$\begin{cases} C(i) = E_j(i) & \text{if } (E_j(i) > C(i)) \\ C(i) = C(i) & \text{otherwise.} \end{cases} \tag{11}$$

- Step 2: Go back to Step1, select a new j until every node has been selected.

The total of edge betweenness centrality $\Sigma_i E(i)$ and the total of link capacity needed to cover every pattern of single node failure $\Sigma_i C(i)$ is shown in Figure 15. Because $\Sigma_i E(i)$ is directly affected by average hop distance, the difference of $\Sigma_i E(i)$ in each topology is not important. What we want to see from this figure is, the extra amount of link capacity needed to cover the node failures, which is not needed in normal condition. We can see that for the original AT&T, about twice as much as $\Sigma_i E(i)$ is needed for $\Sigma_i C(i)$. When mutual information of the topology decrease, $\Sigma_i C(i)$ tends to decrease.

We next evaluate the changes of edge betweenness centrality on each link. The increment in edge betweenness centrality is also calculated for every failure node j :

$$\begin{cases} A_j(i) = E_j(i) - E(i) & \text{if } (E_j(i) > E(i)) \\ C_i = 0 & \text{otherwise.} \end{cases} \tag{12}$$

$A_j(i)$ for all the j sorted by link index i is shown in Figures 16 and 17. Figure 16 is calculated for original AT&T, and Figure 17 is calculated for AT&T$_{0.3}$. We can see that, in AT&T, load in some of the links are highly increased compared to AT&T$_{0.3}$. This means many alternative paths tend to converge on some of the links when node failures occur. In the contrast, for AT&T$_{0.3}$, the variation of increment of edge betweenness centrality on every link is small. This can be considered because the alternative paths are balanced on many links.

From these evaluations, we conclude that link capacity needed to deal with node failures decrease when the topology becomes diverse because alternative paths less convergences in such topology.

V. Conclusion and future work

In this paper, we investigated the network heterogeneity of router-level topologies by using mutual information. We mainly discussed topologies using entropy H and mutual information I.

In Section II, we used ring topologies, star topologies, Abilene-inspired topology and a random topology for examples to explain the measurements. H indicates the heterogeneity of degree distribution in complex networks, and I indicates the amount of information about the node degree that can be obtain by observing a node connected to it.

In Section III, we generated topologies between $(H, I) = (1, 0)$ and $(H, I) = (5, 0)$, and showed that, when H increases higher than 3, the average hop distance decreases. We also generated topologies that having the same H with BA model but with different I, and showed that the topology is diverse when mutual information is high, and the topology has regularity when mutual information is low.

In Section IV, from calculating mutual information of some router-level topologies, we found that most of the router-level topologies have higher mutual information than a model-based topology. From comparing the topology with different mutual information generated from AT&T, we find that link capacity needed to deal with node failures decrease when the topology becomes diverse because alternative paths less convergences in the topology with high topological diversity.

Our next work is to evaluate network performance of topologies with different mutual information also considering physical distance, and to apply this measure to designing information network that has adaptability and sustainability against environmental changes.

Acknowledgment

This research was supported in part by Grant-in-Aid for Scientific Research (A) 24240010 of the Japan Society for the Promotion of Science (JSPS) in Japan. Thanks to Hajime Nakamura, Shigehiro Ano, Nagao Ogino and Hideyuki Koto from KDDI for their helpful advice.

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